

# The 5th International Workshop on Differential Geometry and Analysis

Supported by Graduate Study Advancement Project of Graduate School of Science,  
Fukuoka University and Grant-in-Aid for Scientific Research from JSPS

May 31–June 4, 2014 | Niji-Matsubara Hotel, Karatsu, Japan

Program

May 31

Chair Suyama Yoshihiko (Fukuoka University)

17:10 -- 18:00

Miyaoka Reiko (Tohoku University)

Stability of non-compact minimal Lagrangian submanifolds and L2 harmonic 1 forms

18:10 --20:20

Dinner time

20:30 -- 21:30

Discussion on Differential Geometry (幾何学に関する研究打ち合せ)

June 1

Chair Ding Qing (Fudan University)

08 : 50 -- 09 : 40

Zhang Weiping (Nankai University)

On the Connes fibration over a foliation

10 : 00 -- 10 : 50

Koiso Norihito (Osaka University)

Bi-harmonic hypersurfaces in Euclidean spaces

11 : 10 -- 12 : 00

Li Jiayu (University of Technology and Sciences of China)

Progress on asymptotic behavior of the Yang-Mills-Higgs flow

Lunch time

Chair Chen Qun (Wuhan University)

14:50 -- 15:40

Wei Guoxin (South China Normal University)

Complete  $\lambda$ -hypersurfaces of weighted volume-preserving mean curvature flow

16:00 -- 16:50

Shoda Toshihiro (Saga University)

The existence of non-holomorphic hyperelliptic minimal surfaces in flat 4-tori and their Morse indices

17:10 -- 18:00

Xin Yuanlong (Fudan University)

Area-minimizing hypersurfaces in a manifold with non-negative Ricci curvature

18:10 --20:20

Dinner time

20:30 -- 21:30

Discussion on Differential Geometry (幾何学に関する研究打ち合せ)

June 2

Chair Wu Dianhua (Guangxi Normal University)

08:50 --09:40

Hong Jiaying (Fudan University)

Regularity of Solutions to Degenerate Elliptic Monge-Ampere Equations

10 : 00 -- 10 : 50

Kasue Atsushi (Kanazawa University)

Kuramochi boundaries of nonlinear resistive networks

11 : 10 -- 12 : 00

Han Qing (University of Notre Dame)

Boundary expansions of minimal surfaces in the hyperbolic space

Lunch time

Chair Feng Huitao (Nankai University)

14:50 -- 15:40

Zhou Jiazu (Southwest University)

Steiner symmetrization and isoperimetric inequality

16:00 -- 16:50

Sano Yuji (Kumamoto University)

On computations on a bilinear form on toric Fano manifolds

17:10 -- 18:00

Li Haizhong (Tsinghua University)

A geometric inequality on hypersurface in hyperbolic space

18:10 --20:20

Reception

20:30 -- 21:30

Discussion on Differential Geometry (幾何学に関する研究打ち合せ)

June 3

Chair Guo Zhen (Yunnan Normal University)

08 : 50 --09 : 40

Li Anmin (Sichuan University)

Some fourth order PDE in affine geometry and toric geometry

10 : 00 -- 10 : 50

Koiso Miyuki (Kyushu University)

On bifurcation and local rigidity of triply periodic minimal surfaces in  $\mathbb{R}^3$

11 : 10 -- 12 : 00

Chen Binglong (Sun Yat-Sen University)

Moduli spaces of PIC metrics on four-manifolds

Lunch time

Chair He Gang (Zunyi Normal College)

15:10 -- 16:00

Fujimori Shoichi (Okayama University)

Zero mean curvature surfaces in the Lorentz-Minkowski 3-space

16:20 -- 16:50

Ogata Shiho (Fukuoka University)

Rigidity theorems of complete  $\lambda$ -hypersurfaces

17:10 -- 18:00

Dai Xianzhe (University of California, Santa Barbara)

An eigenvalue problem for manifolds with conical singularity

18:10 -- 20:20

Dinner time

20:30 -- 21:30

Discussion on Differential Geometry (幾何学に関する研究打ち合せ)

June 4

Chair Kawakubo Satoshi (Fukuoka University)

08:50 -- 09:40

Tian Gang (Princeton University and Peking University)

Kahler-Einstein metrics on Fano manifolds

10 : 00 -- 10 : 50

Kobayashi Ryoichi (Nagoya University)

Localization via parabolic translation and collective Cohn-Vossen inequality

11 : 10 -- 12 : 00

Wang Changping (Fujian Normal University)

Wintgen submanifolds of codimension 2 in  $S^{m+2}$

12:00-13:00

Lunch time

Chair Cheng Qing-Ming (Fukuoka University)

13 : 10 -- 14 : 00

Li Jun (Stanford University)

TBA

# The 5th International Workshop on Differential Geometry and Analysis

## Abstracts

### Moduli spaces of PIC metrics on four-manifolds

Chen Binglong (Sun Yat-Sen University)

Abstract: It is well-known that the moduli space of positive scalar curvature metrics on two sphere is path-connected. In dimension 3, this is still true by recent work of Marques. In dimension 4, this is an extremely difficult problem. In this talk, I will discuss the path-connectedness of moduli spaces of positive isotropic curvature metrics on four-manifolds. A byproduct is that the moduli space of PIC metrics on four-sphere is path-connected. This talk is based on a joint work with X.T. Huang.

### An eigenvalue problem for manifolds with conical singularity

Dai Xianzhe (University of California, Santa Barbara)

Abstract: I would like to report on our preliminary work with my student Changliang Wang in which we develop a theory of Perelman's functional for manifolds with conical singularity. This is essentially an eigenvalue problem and the asymptotic behavior of the eigenfunctions near the singularity plays an important role.



Zero mean curvature surfaces in the Lorentz-Minkowski 3-space

Fujimori Shoichi (Okayama University)

Abstract: Spacelike maximal surfaces and timelike minimal surfaces in Lorentz-Minkowski 3-space are both characterized as zero mean curvature surfaces. In this talk we consider the case where the zero mean curvature surface changes type from spacelike to timelike at a given non-degenerate null curve.

Boundary expansions of minimal surfaces in the hyperbolic space

Han Qing (University of Notre Dame)

Abstract: The minimal surface equation in the hyperbolic space is given by a quasilinear elliptic equation, which is non-uniformly elliptic and becomes singular on the boundary. The focus of this talk is to use the expansion near the boundary to discuss the regularity of solutions.

Regularity of Solutions to Degenerate Elliptic Monge-Ampere Equations

Hong Jiaying (Fudan University)

Abstract: In this talk two kinds of degenerate elliptic Monge-Ampere equations arising from study of isometric embeddings in Differential Geometry will be introduced and some existence and regularity theorem will be given. As far as the classification of boundary degeneracy be concerned, one is non-characteristic and another is characteristic. Their linearized operators correspond to Tricomi type and Keldys type respectively and their behaviors are quite different. A priori estimates for such two modeling boundary value problems are very useful for getting higher regularity solutions to the relevant degenerate elliptic Monge-Ampere equations.

Kuramochi boundaries of nonlinear resistive networks

Kasue Atsushi (Kanazawa University)

Abstract: We deal with nonlinear resistive networks (or weighted graphs) in the framework of modular sequence spaces, introduced by De Michele and Soardi in 1990. After introducing a compactification of a network, called the Kuramochi compactification, we discuss Dirichlet boundary value problems for solutions of Poisson equations on the compactification.

Localization via parabolic translation and collective Cohn-Vossen inequality

Kobayashi Ryoichi (Nagoya University)

Abstract : I will report on recent progress on a Cohn-Vossen type inequality for certain collection of truncated fundamental domains of free Fuchsian groups. I will focus on the origin of the problem, the reason why localization principle via parabolic translation is important in the collective Cohn-Vossen inequality. I will explain how the constant in this inequality behaves by looking at various examples.

On bifurcation and local rigidity of triply periodic minimal surfaces in  $\mathbb{R}^3$

Koiso Miyuki (Kyushu University)

Abstract: We construct a bifurcation theorem for triply periodic minimal surfaces in  $\mathbb{R}^3$ . By using this theorem, we determine the existence of infinitely many new examples of triply periodic minimal surfaces in  $\mathbb{R}^3$ , which form branches issuing from the H-family, the rPD-family, the tP-family, and the tD-family, that converge to some degenerate embedding of the families. As to nondegenerate triply periodic minimal surfaces, we prove a perturbation result using an equivariant implicit function theorem. Joint work with P. Piccione and T. Shoda.

## Bi-harmonic hypersurfaces in Euclidean spaces

Koiso Norihito (Osaka University)

Abstract : A map  $f: (M, g) \rightarrow (X, h)$  is called bi-harmonic map if it is a solution of variational problem defined by square integral of tension field  $\tau$ . A submanifold  $Y$  of  $(X, h)$  is called bi-harmonic submanifold if the inclusion map is a bi-harmonic map with respect to the induced metric. The equation of bi-harmonic submanifold is over-determined, but has many solutions: all minimal submanifolds are bi-harmonic submanifold. B. Y. Chen conjectured that there are no bi-harmonic submanifold in Euclidean spaces except minimal submanifolds. In this talk, we show that the conjecture is true for hypersurfaces under a certain generic condition. (Joint work with H. Urakawa)

## A geometric inequality on hypersurface in hyperbolic space

Li Haizhong (Tsinghua University)

Abstract: In this talk, we use the inverse curvature flow to prove a sharp geometric inequality on star-shaped and two-convex hypersurface in hyperbolic space.

## Progress on asymptotic behavior of the Yang-Mills-Higgs flow

Li Jiayu (University of Technology and Sciences of China)

Abstract: I shall talk about my joint work with Zhang Xi on the asymptotic behavior of the Yang-Mills-Higgs flow for Higgs pairs at infinity.

Stability of non-compact minimal Lagrangian submanifolds and  $L^2$  harmonic 1 forms

Miyaoka Reiko (Tohoku University)

Abstract: We show that there exist no non-trivial  $L^2$  harmonic 1-forms on a complete non-compact stable minimal Lagrangian submanifold in a Kähler manifold with positive Ricci curvature. In surface case, we give more details. Also, we mention the H-stability problem given by B. Palmer.

Rigidity theorems of complete  $\lambda$ -hypersurfaces

Ogata Shiho (Fukuoka University)

Abstract: Since  $n$ -dimensional  $\lambda$ -hypersurfaces in the Euclidean space are critical points of the weighted area functional for the weighted volume-preserving variations, in this talk, we talk about the rigidity properties of complete  $\lambda$ -hypersurfaces. We give a gap theorem of complete  $\lambda$ -hypersurfaces with polynomial area growth. By making use of the generalized maximum principle for  $L$  of  $\lambda$ -hypersurfaces, we prove a rigidity theorem of complete  $\lambda$ -hypersurfaces. This is a joint work with Professor Q.-M. Cheng and Professor G. Wei.

On computations on a bilinear form on toric Fano manifolds

Sano Yuji (Kumamoto University)

Abstract: Futaki-Mabuchi introduced a bilinear form on the set of holomorphic vector fields on a compact Kähler manifold. I will talk about its localization formula on the case of toric Fano manifolds.

The existence of non-holomorphic hyperelliptic minimal surfaces in flat 4-tori and their Morse indices

Shoda Toshihiro (Saga University)

Abstract: In this talk, I would like to introduce the existence of non-holomorphic hyperelliptic minimal surfaces of even genus in flat 4-tori. Moreover, we compute Morse indices of the minimal surfaces. It is joint work with Norio Ejiri, Meijo University.

Wintgen submanifolds of codimension 2 in  $S^{m+2}$

Wang Changping (Fujian Normal University)

Abstract: The important DDVV inequality concerning submanifolds in space forms was proved by Ge/Tang and Lu independently. Wintgen submanifolds are submanifolds attaining DDVV equality. In this talk we classify the Wintgen Submanifolds  $M^m$  in  $S^{m+2}$  of codimension 2. Using the framework of Moebius geometry, we show that the mean curvature sphere of Wintgen submanifold is an isotropic holomorphic curve in the complex quadric  $Q^{m+2}$ , where  $Q^{m+2}$  is the moduli space of all  $m$ -spheres in  $S^{m+2}$ . Conversely, we show that any isotropic complex curve in  $Q^{m+2}$  determines a Wintgen submanifold  $M^m$  of codimension 2 in  $S^{m+2}$ .

Complete  $\lambda$ -hypersurfaces of weighted volume-preserving mean curvature flow

Wei Guoxin (South China Normal University)

Abstract: We prove that  $\lambda$ -hypersurface is a critical point of the weighted area functional for the weighted volume-preserving variations, give classifications of complete  $\lambda$ -hypersurfaces with  $H - \lambda \geq 0$  and study F-stable results of  $\lambda$ -hypersurfaces. This is a joint work with Professor Qing-Ming Cheng.

# Steiner symmetrization and isoperimetric inequality

Zhou Jiazuo (Southwest University)

Abstract: The circle is uniquely characterized by property that among all closed planar curves of given length  $L$ , the circle of radius  $\frac{L}{2\pi}$  encloses maximum area. It can be most succinctly expressed as

$$\begin{equation}$$

$$L^2 - 4\pi A \geq 0,$$

$$\end{equation}$$

where  $A$  is the area enclosed by a curve  $C$  of length  $L$ , and where equality holds when and only when  $C$  is a circle.

The isoperimetric inequality for domains in space states that if  $K$  is an arbitrary domain in the Euclidean space  $\mathbb{R}^n$ , its volume  $V(K)$  and surface area  $S(K)$  are related by

$$\begin{equation}$$

$$S(K)^n \geq n \omega_n V(K)^{n-1} \quad \forall K \subset \mathbb{R}^n,$$

$$\end{equation}$$

where  $\omega_n$  is volume of the unit ball, and where equality holds when and only when  $K$  is a standard ball.

The isoperimetric problem attracted interests from geometers and analysts and many beautiful proofs, varying from very elementary to quite technical, appeared in the past. In this talk, we provide a simplified proof of the isoperimetric inequality by Steiner symmetrization. The Steiner symmetrization may be applied to solve some long-standing problems in convex geometric analysis, integral geometry and geometric probability.