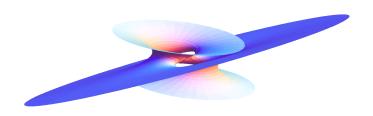
## The 7th International Workshop on Differential Geometry

Dedicated to Professor Tian Gang for his 60th birthday

March 23-27, 2017 | Niji-Matsubara Hotel, Karatsu, Japan

Supported by Grant-in-Aid for Scientific Research from JSPS



#### **Invited Speakers**

Chen Daguang (Tsinghua University)
Fan Huijun (Peking University)
Feng Huitao (Nankai University)
Fu Jixiang (Fudan University)
Futaki Akito (University of Tokyo)

Guo Zhen (Yunnan Normal University)

Kunikawa Keita (Nagoya University) Li Haizhong (Tsinghua University)

Li Jiayu (University of Science and Technology of China

Liu Xiaobo (Peking University)
Mabuchi Toshiki (Osaka University)
Saito Shunsuke (University of Tokyo)
Tamaru Hiroshi (Hiroshima University)

Tian Gang (Peking University and Princeton University)

Xu Guangbo (Princeton University) Yamaguchi Takao (Kyoto University)

Yan Wenjiao (Beijing Normal University)

Zhang Weiping (Nankai University) Zhu Xiaohua (Peking University)

**Organizing Committee** 

Cheng Qing-Ming (Fukuoka University), Sano Yuji (Fukuoka University)

# The 7th International Workshop on Differential Geometry

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March 23-27, 2017 | Niji-Matsubara Hotel, Karatsu, Japan

#### Program

#### March 23

Chair Sano Yuji (Fukuoka University)

17:10 -- 18:00 Mabuchi Toshiki (Osaka University)

Extremal K\u00e4"ahler versions of the Yau-Tian-Donaldson Conjecture

18:10 --20:20 Dinner time

20:30 -- 21:30

Discussion on Differential Geometry

#### March 24



Chair Ding Qing (Fudan University)

14:50 -- 15:40

Fu Jixiang (Fudan University)

A survey on balanced metrics

16:00 -- 16:50

Kunikawa Keita (Nagoya University)

Some nonexistence results for translating solitons

17:10 -- 18:00

Li Jiayu (University of Technology and Sciences of China)

Energy identity for a sequence of Sacks-Uhlenbeck maps to a sphere

18:10 --20:20 Dinner time

20:30 -- 21:30

Discussion on special topics

#### March 25

hypersurfaces

Chair Mabuchi Toshiki (Osaka University) 08:50 --09:40 Fan Huijun (Beijing University) Fukaya category of Laudao-Ginzburg Model 10:00 - 10:50Saito Shunsuke (University of Tokyo) Stability of anti-canonically balanced metrics 11:10 -- 12:00 Xu Guangbo (Princeton University) Gauged linear sigma model and adiabatic limit Lunch time Chair Liu Xiaobo (Beijing University) 16:00 -- 16:50 Yan Wenjiao (Beijing Normal University) Normal scalar curvature inequality on the focal submanifolds of isoparametric

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17:10 -- 18:00
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Guo Zhen (Yunnan Normal University)

Characterization of a class of typical Willmore submanifolds

18:10 --20:20 Reception

20:30 -- 21:30

Discussion on special topics

#### March 26

Chair Fan Huijun (Beijing University)

08:50 --09:40

Liu Xiaobo (Beijing University)

Connecting the Kontsevich-Witten and Hodge tau-functions by the Virasoro operators

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10:00 - 10:50
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Feng Huitao (Nankai University)

On a Conjecture of S.S. Chern

11:10 -- 12:00

Yamaguchi Takao (Kyoto University)

Comparison angles and volume

Lunch time

Chair Li Haizhong (Tsinghua University)

14:50 -- 15:40

Zhu Xiaohua (Beijing University)

Properness of F-functional

16:00 -- 16:50

Chen Daguang (Tsinghua University)

The estimates of the gaps of consecutive eigenvalue of Laplacian

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17:10 -- 18:00
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Futaki Akito (University of Tokyo)

Volume minimizing principle for conformally Kaehler Einstein-Maxwell metrics

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18:10 --20:20
Dinner time
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20:30 -- 21:30
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Discussion on special topics

#### March 27

Chair Zhang Weiping (Nankai University)

08:30 -- 09:20

Tian Gang (Beijing University and Princeton University)

TBA

## The 7th International Workshop on Differential Geometry

#### Abstracts

The estimates of the gaps of consecutive eigenvalue of Laplacian

Chen Daguang (Tsinghua University)

In this talk, we will talk about the estimates of the gaps of consecutive eigenvalue of Laplacian. For the eigenvalue problem of the Dirichlet Laplacian on the bounded domain in Euclidean space  $R^n$ , we obtain the estimates for the upper bounds of the gap of consecutive eigenvalues, which are the best possible in the meaning of the orders of eigenvalues. This is the joint work with Professor Hongcang Yang and Dr. Tao Zheng.

Fukaya category of Laudao-Ginzburg Model

Fan Huijun (Beijing University)

In this report, I will give a rigorous mathematical construction of the Fukaya category of Laudau-Ginzburg model, which were described briefly by recent work of Gaiotto-Moore-Witten and Kapranov-Kontsevich-Soibelman on the algebra of the infraed. This is a moduli problem about Witten equation with Lefschetz boundary condition. This is a joint work with Dingyu Yang and Wenfeng Jiang.

On a Conjecture of S.S. Chern

Huitao Feng (Nankai University)

In this talk, we will present a proof of an old conjecture of Prof. S.S. Chern on affine manifolds.

A survey on balanced metrics

Fu Jixiang (Fudan University)

In this talk, I will focus on a few aspects of balanced metrics on a compact hermitian metrics.

Volume minimizing principle for conformally Kaehler Einstein-Maxwell metrics

Futaki Akito (University of Tokyo)

We discuss on the volume minimization principle for conformally Kaehler Einstein-Maxwell metrics in the similar spirit as the Kaehler-Ricci solitons and Sasaki-Einstein metrics. This talk is base on a joint work with Hajime Ono.

Characterization of a class of typical Willmore submanifolds

Guo Zhen (Yunnan Normal University)

In this talk, We present an integral inequality on a compact Willmore hypersurface. In particular, the conformal classes of Willmore tori are characterized by using a conformal invariant function. In additional, we introduce a class of special Willmore submanifolds. Since this class of Willmore submanifolds includes known interesting examples, we call it typical Willmore submanifolds. We present rich concrete examples of this class Willmore submanifolds.

Some nonexistence results for translating solitons

Kunikawa Keita (Nagoya University)

Translating solitons (eternal solutions) are type II singularity models of mean curvature flow. In my talk, I will show some Bernstein type results for translating solitons in higher codimensions.

Uniqueness of closed self-similar solutions to  $\sigma_{\,\mathbf{k}}^{\ \alpha}$  -curvature flow

Li Haizhong (Tsinghua University)

In recent two papers, Choi-Daskaspoulos and Brendle-Choi-Daskaspoulos have proved that any n-dimensional strictly convex closed self-similar solutions to  $\sigma_n^{\ \alpha}$  --curvature flow in  $R^{n+1}$  with  $\alpha \le 1/n+2$ , must be an ellipsoid or round sphere.

By adapting their test functions and exploring properties of the k-th elementary symmetric functions  $\sigma_k$  intensively, we show that for any fixed k with  $1 \le k \le n-1$ , any strictly convex closed self-similar solutions to  $\sigma_k^{\alpha}$ -curvature flow in  $R^{n+1}$  must be a round sphere. This is joint work with Hui Ma and Shanze Gao.

Energy identity for a sequence of Sacks-Uhlenbeck maps to a sphere

Li Jiayu (University of Technology and Sciences of China)

### Energy identity for a sequence of Sacks-Uhlenbeck maps to a sphere

Li, Jiayu

#### Abstract

For a map u from a Riemann surface M to a Riemannian manifold and  $\alpha>1$ , the  $\alpha$  energy functional is defined as

$$E_{\alpha}(u) = \int_{M} |\nabla u|^{2\alpha} dx.$$

We call  $u_{\alpha}$  a sequence of Sacks-Uhlenbeck maps if  $u_{\alpha}$  is a critical point of  $E_{\alpha}$  and

$$\sup_{\alpha>1} E_{\alpha}(u_{\alpha}) < \infty.$$

In this talk, when the target manifold is a standard sphere  $S^K$ , we prove the energy identity for a sequence of Sacks-Uhlenbeck maps during blowing up.

### Connecting the Kontsevich-Witten and Hodge tau-functions by the Virasoro operators

Liu Xiaobo (Beijing University)

Kontsevich-Witten tau-function and the Hodge tau-function are generating functions for two types of intersection numbers on moduli spaces of stable curves. Both of them are tau functions for the KP hierarchy. In this talk, I will describe how to connect these two tau-functions by differential operators belonging to the  $\Psi$  idehat  $GL(\Psi infty)$  group. Indeed, these two tau-functions can be connected using Virasoro operators. This proves a conjecture posted by Alexandrov. This is a joint work with Gehao Wang.

Extremal K\u00e4"ahler versions of the Yau-Tian-Donaldson Conjecture

Mabuchi Toshiki (Osaka University)

In K\u00e4"ahler-Einstein cases, the Yau-Tian-Donaldson conjecture on polarized algebraic manifolds was solved affirmatively by Tian and Chen-Donaldson-Sun. However, for general polarizations, or more generally for extremal K\u00e4"ahler cases, the generalized versions of the conjecture are still open. In this talk, I'll discuss such versions of the conjecture focusing on the existence problem.

Stability of anti-canonically balanced metrics

Saito Shunsuke (University of Tokyo)

We study the asymptotic behavior of quantized Ding functionals along Bergman geodesic rays and prove that the slope at infinity can be expressed in terms of Donaldson-Futaki invariants and Chow weights. Based on the slope formula we introduce a new algebro-geometric stability on Fano manifolds and show that the existence of anti-canonically balanced metrics implies our stability. Therelation between our stability and others is also discussed. As another application of the slope formula, we get the lower bound estimate on the Calabi like functionals on Fano manifolds.

This is a joint work with R. Takahashi.

#### Left-invariant metrics and submanifold geometry

Tamaru Hiroshi (Hiroshima University)

For a left-invariant metric on a given Lie group, we can construct a submanifold, where the ambient space is the space of all left-invariant metrics on that Lie group. We expect that nice left-invariant metrics are corresponding to nice submanifolds. In this talk, we introduce our framework, and mention some results related to such correspondence.

Gauged linear sigma model and adiabatic limit

Xu Guangbo (Princeton University)

The gauged linear sigma model (GLSM), introduced by Witten, has been very influential in string theory. It unifies the Gromov-Witten theory and the Landau-Ginzburg theory, and provides a global picture of mirror symmetry. In a series of papers, we have made efforts in constructing a mathematical theory of GLSM using symplectic geometric methods. In this talk, I will present our recent construction in the so-called geometric phase of GLSM. In particular, I will explain how to define a Gromov-Witten type invariant for GLSM. If time permits, I will explain the relation between the GLSM invariant and Gromov-Witten invariant via adiabatic limit. This is a Joint work with Prof. Tian.

Comparison angles and volume

Yamaguchi Takao (Kyoto University)

We introduce a new geometric invariant of spaces with curvature bounded from below by means of comparison angles. Though this is a distance-invariant in nature, it turns out that it is closely related with the volume. For instance we can completely determine if the space is collapsed by using this invariant. This is a joint work with Ayato Mitsuishi.

### Normal scalar curvature inequality on the focal submanifolds of isoparametric hypersurfaces

Yan Wenjiao (Beijing Normal University)

An isoparametric hypersurface in unit spheres has two focal submanifolds. Condition A plays a crucial role in the classification theory of isoparametric hypersurfaces. This paper determines  $C_A$ , the set of points with Condition A in focal submanifolds. It turns out that the points in  $C_A$  reach an upper bound of the normal scalar curvature, (sharper than that in DDVV inequality). We also determine the sets  $C_P$  (points with parallel second fundamental form) and  $C_E$  (points with Einstein condition), which achieve two lower bounds for the normal scalar curvature. This talk is based on joint work with J.Q.Ge and Z.Z.Tang.

Positive scalar curvature on manifolds and foliations

Zhang Weiping (Nankai University)

The famous Lichnerowicz vanishing theorem states that the Hirzebruch A-hat genus of a closed Riemannian spin manifold of positive scalar curvature equals to zero. We discuss various generalizations of this classical result.

Properness of F-functional

Zhu Xiaohua (Beijing University)

F-functional plays an important role in the study of K-stability of Kaehler manifolds. In 1997, Tian gave an approach to prove the properness of F-functional by establishing a smooth lemma from Ricci flow. In this talk, we give another proof of Tian's result by using the regularity of complex Monge-Ampere equation. Our new approach can be also generalized to study twisted F-functional associated to singular Kaehler-Einstein metrics. This is a joint work with Tian.