The 8th International Workshop on

Differential Geometry

February 5-10, 2020 | Niji-Matsubara Hotel, Karatsu, Japan Supported by Grant-in-Aid for Scientific Research from JSPS



Invited Speakers

Chen Daguang	(Tsinghua University)
Cheng Qing-Ming	(Fukuoka University)
Hu Yingxiang	(Tsinghua University)
Li Haizhong	(Tsinghua University)
Wang Xianfeng	(Nankai University)
Wei Guoxin	South China Normal University)
Xia Chao	(Xiamen University)
Yang Dan	(Liaoning University)
Zhou Tailong	(Tsinghua University)

Organizing Committee

Cheng Qing-Ming	(Fukuoka University)
Li Haizhong	(Tsinghua University)
Wei Guoxin	(South China Normal University)

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Program

Feb. 5

17:00 -- 17:50 Cheng Qing-Ming (Fukuoka University) A conjecture on complete self-shrinkers

18:00 --19:30 Dinner time

08:30 --10:00 Chen Daguang (Tsinghua University) Lecture 1. Hypersurface in warped product manifolds

10:20 -- 11:50 Xia Chao (Xiamen University)

Lecture 1. Hsiung-Minkowski's formula and Rigidity for CMC hypersurfaces

Lunch time

13:30 -- 15:00 Hu Ying Xiang (Tsinghua University) Lecture 1. Curvature Flows for AF inequalities in Euclidean space

15:20 -- 16:50 Wang Xianfeng (Nankai University) Lecture 1. On the Pinkall-Sterling conjecture

17:10 -- 17:55 Chen Daguang (Tsinghua University) Lecture 2-1. Minkowski inequalities for hypersurfaces in the AdS manifold and RNAdS manifold

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18:00 --19:30 Dinner time

08:30 --09:50 Weiguoxin (South China Normal University) Complete λ -hypersurfaces in R^{n+1}

09:40-10:30 Zhou Tailong (Liaoning University) Shifted inverse curvature flows in hyperbolic space

11:00-- 11:50 Yang Dan (Liaoning University) Minimal surface in a unit sphere

Lunch time

13:30 -- 15:00 Xia Chao (Xiamen University)

Lecture 2. Guan-Li type flow and Alexandrov-Fenchel's inequalities I

15:20 -- 16:50 Wang Xianfeng (Nankai University) Lecture 2. On the proof of the Pinkall-Sterling conjecture

17:10 -- 17:55 Chen Daguang (Tsinghua University) Lecture 2-2. Minkowski inequalities for hypersurfaces in the AdS manifold and RNAdS manifold

18:00 --19:30 Dinner time

08:30 --10:00 Hu Ying Xiang (Tsinghua University) Lecture 2. The AF inequalities in hyperbolic space---expanding flows

10:20 -- 11:50 Xia Chao (Xiamen University)

Lecture 3. Guan-Li type flow and Alexandrov-Fenchel's inequalities II

Lunch time

13:30 -- 15:00 Chen Daguang (Tsinghua University) Lecture 3. The isoperimetric problem in warped product spaces

15:20 -- 16:50 Wang Xianfeng (Nankai University) Lecture 3. On the classification of embedded constant mean curvature tori in the three-sphere

17:10 -- 17:55 Hu Ying Xiang (Tsinghua University) Lecture 3-1. The AF inequalities in hyperbolic space--volume preserving flows

18:00 --19:30 Dinner time

08:30 --10:00 Xia Chao (Xiamen University)

Lecture 4. Weighted Reilly type formula and Applications

10:20 -- 11:50 Chen Daguang (Tsinghua University) Lecture 4. The isoperimetric inequality for a minimal submanifold in Euclidean space

Lunch time

13:30 -- 14:15 Hu Ying Xiang (Tsinghua University) Lecture 3-2. The AF inequalities in hyperbolic space--volume preserving flows

14:40 -- 16:10 Wang Xianfeng (Nankai University) Lecture 4. On the proof of the Lawson conjecture

16:30 -- 18:00 Hu Ying Xiang (Tsinghua University) Lecture 4. The AF inequalities in hyperbolic space--contracting flows

18:00 --19:30 Dinner time

08:30 --10:00 Li Haizhong (Tsinghua University) Harmonic mean curvature flow

The 8th International Workshop on Differential Geometry

Abstracts

The isoperimetric problem in warped product manifolds

Chen Daguang (Tsinghua University)

Lecture 1: Hypersurface in warped product manifolds

I will introduce some basic notations, definitions and properties of hypersurface flows in warped product manifolds.

Lecture 2: Minkowski inequalities for hypersurfaces in the AdS manifold and RNAdS manifold

I will introduce the inverse mean curvature flow in warped product manifold. As applications, we will give the proof of the Minkowski inequalities for hypersurfaces in the AdS manfiolds [BHW16] and Reissner-Nordström-anti-de Sitter manifold [CLZ19].

Lecture 3: The isoperimetric problem in warped product spaces

I will introduce a volume preserving flow and the isoperimetric problem in warped product spaces in [GLW19]. This flow preserves the volume of the bounded domain enclosed by a graphical hypersurface and monotonically decreases the hypersurface area. As an application, an isoperimetric problem is solved in warped product spaces [GLW19].

Lecture 4: The isoperimetric inequality for a minimal submanifold in Euclidean space

I will review the isoperimetric inequality in Euclidean space and introduce the isoperimetric inequality for a minimal submanifold in Euclidean space proved by S. Brendle [Brendle19].

[Brendle19] S. Brendle, The isoperimetric inequality for a minimal submanifold in Euclidean space}, arXiv:1907.09446v3.

[BHW16] S. Brendle, P.-K. Hung, and M.-T. Wang, A Minkowski-type inequality for hypersurfaces in the Anti-deSitter-Schwarzschild manifold, Comm. Pure Appl. Math. 69(2016), no. 1, 124-144.

[CLZ19] D.G. Chen, H. Z. Li and T.L. Zhou, A Penrose type inequality for graphs over Reissner-Nordström-anti-deSitter manifold, J. Math. Phys. 60 (2019), no. 4, 043503, 12 pp.

[GLW19] P. Guan, J. Li and M.-T. Wang, A volume preserving flow and the isoperimetric problem in warped product spaces, Trans. Amer. Math. Soc. 372 (2019), no. 4, 2777–2798.

A conjecture on complete self-shrinkers Cheng Qing-Ming (Fukuoka University)

It is a well-known conjecture that n-dimensional complete self-shrinkers of mean curvature flow in Euclidean space \mathbb{R}^{n+1} are isometric to a sphere, Euclidean space or a cylinder if the norm of the second fundamental form is constant. It is our purpose to study this conjecture on complete selfshrinkers of mean curvature flow in Euclidean spaces.

Curvature flows and Alexandrov-Fenchel inequalities

Hu Ying Xiang (Tsinghua University)

Lecture 1: Curvature Flows for AF inequalities in Euclidean space I will introduce the previous work to prove geometric inequalities in Euclidean space by hypersurface flows, including contracting flows, volume preserving flows and expanding flows.

Then I will deduce the evolution equations along the curvature flows in space forms (for our later purpose).

Lecture 2: The AF inequalities in hyperbolic space--expanding flows I will introduce the results via inverse curvature flows (Li-Wei-Xiong, Ge-Wang-Wu);

Lecture 3: The AF inequalities in hyperbolic space--volume preserving flows I will introduce the results via quermassintegral preserving flows (Wang-Xia, Andrews-Chen-Wei);

Lecture 4: The AF inequalities in hyperbolic space--contracting flows I will introduce the recent results via contracting flows, in particular, the harmonic mean curvature flow.

Harmonic mean curvature flow

Li Haizhong (Tsinghua University)

In this talk, we use the harmonic mean curvature ow to prove Alexandrov-

Fenchel type inequalities for strictly convex hypersurfaces in hyperbolic space. Using the new Alexandrov-Fenchel type inequalities and the inverse mean curvature ow, we show that the Alexandrov-Fenchel inequality for the total curvature in terms of the area for strictly convex hypersurfaces. This is a joint work with Ben Andrews and Yingxiang Hu.

Title: On the Lawson conjecture and the Pinkall-Sterling conjecture

Wang Xianfeng (Nankai University)

Lecture 1: On the Pinkall-Sterling conjecture

I will first review some history about constant mean curvature tori in the three-sphere involving the Lawson conjecture and the Pinkall-Sterling conjecture. We will focus on the proof of the Pinkall-Sterling conjecture given by Prof. Ben Andrews and Prof. Haizhong Li in [AL2015]. In Lecture 1, I will show the proof of the conclusion that for any embedded constant mean curvature torus in the three-sphere, the interior ball curvature equals maximum principal curvature.

Lecture 2: On the proof of the Pinkall-Sterling conjecture

In Lecture 2, I will finish the proof of the Pinkall-Sterling conjecture given by Prof. Ben Andrews and Prof. Haizhong Li in [AL2015] by proving that embedded constant mean curvature tori in the three-sphere have rotational symmetry.

Lecture 3: On the classification of embedded constant mean curvature tori in the three-sphere

In Lecture 3, based on the conclution that ``any embedded constant mean curvature torus in the three-sphere is axially symmetric'' proved in Lecture 2,

we will give a complete classification of embedded constant mean curvature torus in the three-sphere for any given value of the mean curvature, which is given by Prof. Ben Andrews and Prof. Haizhong Li in [AL2015].

Lecture 4: On the proof of the Lawson conjecture

In Lecture 4, I will give a brief proof of the Lawson conjecture given by Prof. Simon Brendle in [B2013].

References:

[AL2015] Ben Andrews and Haizhong Li, Embedded constant mean curvature tori in the three-sphere. J. Differential Geom. 99 (2015), no. 2, 169–189.

[B2013] Simon Brendle, Embedded minimal tori in S3 and the Lawson conjecture. Acta Math. 211 (2013), no. 2, 177–190.

Complete λ-hypersurfaces in Rⁿ⁺¹ Weiguoxin (South China Normal University)

In this talk, we introduce rigidity theorems, examples, the stability and classification theorems of complete ¥lambda-hypersurfaces in Euclidean space

Hsiung-Minkowski's formula and Reilly's formula

-- Applications to geometric rigidity and geometric inequalities

Xia Chao (Xiamen University)

In this series of lectures, I will talk about two kinds of important integral formulas - Hsiung-Minkowski's formula and Reilly's formula - in global

diffential geometry. I focus on a new Hsiung-Minkowski type formula and a weighted Reilly type formula obtained by my collaborators and me recently. With these formulas, we study rigidity problems for CMC hypersurfaces and Alexandrov-Fenchel type inequalities in several settings.

The lectures are based on my joint works with Junfang Li, Guohuan Qiu, Julian Scheuer, Guofang Wang, Changwei Xiong.

Lecture 1. Hsiung-Minkowski's formula and Rigidity for CMC hypersurfaces

Lecture 2. Guan-Li type flow and Alexandrov-Fenchel's inequalities I

Lecture 3. Guan-Li type flow and Alexandrov-Fenchel's inequalities II

Lecture 4. Weighted Reilly type formula and Applications

Minimal surface in a unit sphere Yang Dan (Liaoning University)

In this talk, I would like to consider some classical pinching problems on the minimal surface in a unit sphere. A nice orthonormal frame is built and the pinching concerning the gauss curvature and the normal curvature for minimal surfaces in a unit sphere is given.

Shifted inverse curvature flows in hyperbolic space

Tailong Zhou (Tsinghua University)

We introduce the shifted inverse curvature flow in hyperbolic space. This is a family of hypersurfaces in hyperbolic space expanding by f^{-p} with positive power p for a smooth, symmetric, strictly increasing and 1homogeneous curvature function f of the shifted principal curvatures with some concavity properties. We study the maximal existence and asymptotical behavior of the flow for horo-convex hypersurfaces. In particular, for 0 < p¥leq 1 we show that the limiting shape of the solution is always round as the maximal existence time is approached. This is in contrast to the asymptotical behavior of the (non-shifted) inverse curvature flow, in which case the limiting shape of inverse curvature flow in hyperbolic space is not necessarily round.