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EMBEDDED HYPERSURFACES WITH CONSTANT mTH MEAN CURVATURE IN A UNIT SPHERE

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In this paper, we study n-dimensional hypersurfaces with constant mth mean curvature in a unit sphere $S^{n+1}(1)$ and construct many compact nontrivial embedded hypersurfaces with constant mth mean curvature $H_m > 0$ in $S^{n+1}(1)$, for $1 \leq m \leq n-1$. Moreover, if the 2nd mean curvature H_2 takes value between $\frac{1}{(\tan\frac{\pi}{k})^2}$ and $\frac{k^2-2}{n}$ for any integer $k \geq 2$ and $n \geq 3$, then there exists an n-dimensional compact nontrivial embedded hypersurface with constant H_2 (i.e. constant scalar curvature) in $S^{n+1}(1)$; If the 4th mean curvature H_4 takes value between $\frac{1}{(\tan\frac{\pi}{k})^4}$ and $\frac{k^4-4}{n(n-4)}$ for any integer $k \geq 3$ and $n \geq 5$, then there exists an n-dimensional compact nontrivial embedded hypersurface with constant H_4 in $S^{n+1}(1)$.

Keywords: Constant mth mean curvature; embedded hypersurfaces.

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1. Introduction

It is well known that Alexandrov ([1]) and Montiel–Ros ([10]) proved that the standard round spheres are the only possible oriented compact embedded hypersurfaces with constant mth mean curvature H_m in a Euclidean space $\mathbb{R}^{n+1}$, for $m \geq 1$. On the other hand, one knows that standard round spheres and Clifford hypersurfaces

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$S^l(a)\times S^{n-l}(b)$, $1\le l\le n-1$ are compact embedded hypersurfaces in a unit sphere $S^{n+1}(1)$. Hence, it is natural to ask the following:

Question. *Do there exist compact embedded hypersurfaces with constant mth mean curvature H_m in $S^{n+1}(1)$ other than the standard round spheres and Clifford hypersurfaces?*

When $m=1$, namely, when the mean curvature is constant, Ripoll ([13]) has proved the existence of compact embedded hypersurfaces of $S^3(1)$ with constant mean curvature ($H\neq 0,\pm\frac{\sqrt{3}}{3}$) other than the standard round spheres and the Clifford hypersurfaces. Then, Brito–Leite ([2]) have proved that there exist compact embedded hypersurfaces with constant mean curvature H in $S^{n+1}(1)$, which are not isometric to the standard round spheres and the Clifford hypersurfaces. Recently, Perdomo ([12]) has proved that there exists an n-dimensional compact nontrivial embedded hypersurface with constant mean curvature $H>0$ in $S^{n+1}(1)$ if mean curvature H takes value between $\frac{1}{(\tan\frac{\pi}{k})}$ and $\frac{(k^2-2)\sqrt{n-1}}{n\sqrt{k^2-1}}$, where any $n\ge 2$ and any integer $k\ge 2$.

For $m=2$, that is, when the scalar curvature is constant, Leite ([6]) has proved that there exist compact nontrivial embedded hypersurfaces with constant scalar curvature R satisfying $(n-1)(n-2)<R<n(n-1)$ in $S^{n+1}(1)$. Furthermore, Li–Wei ([8]) have proved that there exist many compact nontrivial embedded hypersurfaces with constant scalar curvature R satisfying $R>n(n-1)$ in $S^{n+1}(1)$, recently. But for $m>2$, one knows little about existence of compact embedded hypersurfaces with constant mth mean curvature H_m in $S^{n+1}(1)$. In this paper, we prove that there exist many compact nontrivial embedded hypersurfaces with constant mth mean curvature $H_m>0$ in $S^{n+1}(1)$, for $1\le m\le n-1$. In particular, for $m=4$, we prove that there exist a lot of compact embedded hypersurfaces with constant 4th mean curvature H_4 in $S^{n+1}(1)$ if it takes value between $\frac{1}{(\tan\frac{\pi}{k})^4}$ and $\frac{k^4-4}{n(n-4)}$ for any integer $k\ge 3$. Furthermore, for $m=1$, our results reduce to the conclusion of Brito–Leite ([2]). For $m=2$, we prove that there are many new compact embedded hypersurfaces with constant scalar curvature satisfying $R>n(n-1)$ in $S^{n+1}(1)$, other than ones of Li–Wei ([8]).

2. Preliminaries

Let M be an n-dimensional hypersurface of a unit sphere $S^{n+1}(1)$ with constant mth mean curvature H_m. We choose a local orthonormal frame $\{e_A\}_{1\le A\le n+1}$ in S^{n+1}, with dual coframe $\{\omega_A\}_{1\le A\le n+1}$, such that, at each point of M, $e_1,\ldots,e_n$ are tangent to M and e_{n+1} is the positively oriented unit normal vector. We shall make use of the following convention on the ranges of indices:

$$1\le A,B,C,\ldots,\le n+1;\quad 1\le i,j,k,\ldots,\le n.$$

Then the structure equations of S^{n+1} are given by

$$d\omega_A = \sum_{B=1}^{n+1} \omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0, \tag{2.1}$$

$$d\omega_{AB} = \sum_{C=1}^{n+1} \omega_{AC} \wedge \omega_{CB} - \omega_A \wedge \omega_B. \tag{2.2}$$

When restricted to M, we have $\omega_{n+1} = 0$ and

$$0 = d\omega_{n+1} = \sum_{i=1}^{n} \omega_{n+1i} \wedge \omega_i. \tag{2.3}$$

By Cartan's lemma, there exist functions h_{ij} such that

$$\omega_{in+1} = \sum_{j=1}^{n} h_{ij}\omega_j, \quad h_{ij} = h_{ji}. \tag{2.4}$$

This gives the second fundamental form of M, $B = \sum_{i,j} h_{ij}\omega_i\omega_j e_{n+1}$. The mean curvature H is defined by $H = \frac{1}{n}\sum_i h_{ii}$. From (2.1)–(2.4), we obtain the structure equations of M (see [4, 7])

$$d\omega_i = \sum_{j=1}^{n} \omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0, \tag{2.5}$$

$$d\omega_{ij} = \sum_{k=1}^{n} \omega_{ik} \wedge \omega_{kj} - \frac{1}{2}\sum_{k,l=1}^{n} R_{ijkl}\omega_k \wedge \omega_l. \tag{2.6}$$

and the Gauss equations

$$R_{ijkl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk} + (h_{ik}h_{jl} - h_{il}h_{jk}), \tag{2.7}$$

$$R - n(n-1) = n(n-1)(r-1) = n^2H^2 - S, \tag{2.8}$$

where R_{ijkl} denotes the components of the Riemannian curvature tensor of M, $R = n(n-1)r$ is the scalar curvature of M and $S = \sum_{i,j=1}^{n} h_{ij}^2$ is the square norm of the second fundamental form of M.

Let h_{ijk} denote the covariant derivative of h_{ij}. We then have

$$\sum_k h_{ijk}\omega_k = dh_{ij} + \sum_k h_{kj}\omega_{ki} + \sum_k h_{ik}\omega_{kj}. \tag{2.9}$$

Thus, by exterior differentiation of (2.4), we obtain the Codazzi equation

$$h_{ijk} = h_{ikj}. \tag{2.10}$$

We choose $e_1, \ldots, e_n$ such that

$$h_{ij} = \lambda_i\delta_{ij}. \tag{2.11}$$

Let H_m be mth mean curvature of M, then we have

$$C_n^m H_m = \sum_{1 \le i_1 < i_2 < \cdots < i_m \le n} \lambda_{i_1} \ldots \lambda_{i_m}, \tag{2.12}$$

where $C_n^m = \frac{n!}{m!(n-m)!}$.

In [11], Otsuki proved the following

Lemma 2.1 ([11]). *Let M be an n-dimensional hypersurface in a unit sphere $S^{n+1}(1)$ such that the multiplicities of principal curvatures are all constant. Then the distribution of the space of principal vectors corresponding to each principal curvature is completely integrable. In particular, if the multiplicity of a principal curvature is greater than 1, then this principal curvature is constant on each integral submanifold of the corresponding distribution of the space of principal vectors.*

From Lemma 2.1, we can easily obtain the following theorem.

Theorem 2.1. *Let M be an n-dimensional oriented complete hypersurface in a unit sphere $S^{n+1}(1)$ with constant mth mean curvature H_m and with two distinct principal curvatures. If the multiplicities of these two distinct principal curvatures are greater than* 1, *then M is isometric to Riemannian product $S^k(a) \times S^{n-k}(b)$, $2 \le k \le n-2$.*

3. A Representation Formula of Principal Curvatures

Now, let us consider that M is an n-dimensional oriented hypersurface with constant mth mean curvature H_m and with two distinct principal curvatures in $S^{n+1}(1)$. If multiplicities of these two distinct principal curvatures are all great than 1, then we can deduce from Theorem 2.1 that M is isometric to $S^k(a) \times S^{n-k}(b)$, $2 \le k \le n-2$. Hence, we shall assume that one of these two distinct principal curvatures is simple, that is, we assume

$$\lambda_1 = \lambda_2 = \cdots = \lambda_{n-1} = \lambda, \quad \lambda_n = \mu. \tag{3.1}$$

Since H_m is constant, we obtain from (2.12) that

$$C_n^m H_m = C_{n-1}^m \lambda^m + C_{n-1}^{m-1} \lambda^{m-1} \mu. \tag{3.2}$$

By Lemma 2.1, let us denote the integral submanifold through $x \in M$, corresponding to λ by $M_1^{n-1}(x)$. We write

$$d\lambda = \sum_i \lambda_{,i} \omega_i, \quad d\mu = \sum_j \mu_{,j} \omega_j. \tag{3.3}$$

If $m \ge 2$ and $\lambda = 0$ at some point p, then $H_m \equiv 0$, it follows that $\lambda \equiv 0$ on M (also see [14]). If $\lambda \equiv 0$, the sectional curvature of M is not less than 1 from Gauss

Eq. (2.7), then we have from [15] that M is totally umbilical. This is a contradiction. Hence we can assume that $\lambda > 0$ on M if $m \geq 2$. Then (3.2) yields

$$\mu = \frac{C_n^m H_m - C_{n-1}^m \lambda^m}{C_{n-1}^{m-1} \lambda^{m-1}} = \frac{nH_m - (n-m)\lambda^m}{m\lambda^{m-1}}. \tag{3.4}$$

From Lemma 2.1 and (3.4), one has

$$\lambda_{,1} = \cdots = \lambda_{,n-1} = 0. \tag{3.5}$$

From the formula

$$\lambda - \mu = \frac{n(\lambda^m - H_m)}{m\lambda^{m-1}}, \tag{3.6}$$

we obtain that

$$\lambda^m - H_m \neq 0. \tag{3.7}$$

By means of (2.9) and (2.11), we obtain

$$\sum_k h_{ijk}\omega_k = \delta_{ij} d\lambda_j + (\lambda_i - \lambda_j)\omega_{ij}. \tag{3.8}$$

We adopt the notational convention that

$$1 \leq a, b, c, \ldots \leq n - 1.$$

From (3.1), (3.2) and (3.8), we have

$$h_{ijk} = 0, \quad \text{if } i \neq j, \quad \lambda_i = \lambda_j, \tag{3.9}$$

$$h_{aab} = 0, \quad h_{aan} = \lambda_{,n}, \tag{3.10}$$

$$h_{nna} = 0, \quad h_{nnn} = \mu_{,n}. \tag{3.11}$$

Combining this with (2.10) and the formula

$$\sum_i h_{ani}\omega_i = dh_{an} + \sum_i h_{in}\omega_{ia} + \sum_i h_{ai}\omega_{in} = (\lambda - \mu)\omega_{an}, \tag{3.12}$$

we obtain from (3.10) and (3.6)

$$\omega_{an} = \frac{\lambda_{,n}}{\lambda - \mu}\omega_a = \frac{m\lambda^{m-1}\lambda_{,n}}{n(\lambda^m - H_m)}\omega_a. \tag{3.13}$$

Therefore we have

$$d\omega_n = \sum_a \omega_{na} \wedge \omega_a = 0. \tag{3.14}$$

Notice that we may consider λ to be locally a function of the parameter s, where s is the arc length of an orthogonal trajectory of the family of the integral submanifolds corresponding to λ. We may put

$$\omega_n = ds.$$

Thus, for $\lambda = \lambda(s)$, we have

$$d\lambda = \lambda_{,n} ds, \quad \lambda_{,n} = \lambda'(s). \tag{3.15}$$

From (3.6) and (3.13), we get

$$\omega_{an} = \frac{m\lambda^{m-1}\lambda_{,n}}{n(\lambda^m - H_m)}\omega_a = \frac{m\lambda^{m-1}\lambda'(s)}{n(\lambda^m - H_m)}\omega_a = \{\log|\lambda^m - H_m|^{1/n}\}'\omega_a, \tag{3.16}$$

which shows that the integral submanifolds $M_1^{n-1}(x)$ corresponding to λ is umbilical in M and $S^{n+1}(1)$.

On the other hand, we can deduce from (3.16) that

$$\nabla_{e_n} e_n = \sum_{k=1}^{n} \omega_{ni}(e_n)e_i = 0.$$

According to the definition of geodesic, we know that the integral curve of the principal vector field e_n corresponding to the principal curvature μ is a geodesic.

This proves the following result:

Lemma 3.1. *If M is an n-dimensional oriented complete hypersurface* ($n \geq 3$) *in $S^{n+1}(1)$ with constant mth mean curvature H_m and with two distinct principal curvatures, one of which is simple, then*

(1) *the integral submanifold $M_1^{n-1}(x)$ through $x \in M$ corresponding to λ is umbilical in M and $S^{n+1}(1)$,*
(2) *the integral curve of the principal vector field e_n corresponding to the principal curvature μ is a geodesic.*

Now we state our Theorem 3.1 as follows:

Theorem 3.1. *If M is an n-dimensional oriented complete hypersurface* ($n \geq 3$) *in $S^{n+1}(1)$ with constant mth mean curvature H_m and with two distinct principal curvatures one of which is simple, then M is isometric to a complete hypersurface of revolution $S^{n-1}(c(s)) \times M^1$ (i.e. the warped product of $S^{n-1}(c(s))$ and M^1), where $S^{n-1}(c(s))$ is of constant curvature $[(\log | \lambda^m - H_m |^{1/n})']^2 + \lambda^2 + 1$. And $w =| \lambda^m - H_m |^{-1/n}$ satisfies the following ordinary differential equation of order* 2:

$$\frac{d^2w}{ds^2} - w\left\{\frac{(n-m)(w^{-n}+H_m)^{(2-m)/m}}{mw^n} - H_m(w^{-n}+H_m)^{(2-m)/m} - 1\right\} = 0. \tag{3.17}$$

Proof. According to the structure equations of $S^{n+1}(1)$ and (3.16), we may compute

$$\begin{aligned} d\omega_{an} &= \sum_{b=1}^{n-1} \omega_{ab} \wedge \omega_{bn} + \omega_{an+1} \wedge \omega_{n+1n} - \omega_a \wedge \omega_n \\ &= (\log|\lambda^m - H_m|^{1/n})' \sum_{b=1}^{n-1} \omega_{ab} \wedge \omega_b - \lambda\mu\omega_a \wedge ds - \omega_a \wedge ds, \\ d\omega_{an} &= d[(\log|\lambda^m - H_m|^{1/n})'\omega_a] \\ &= \{\log|\lambda^m - H_m|^{1/n}\}'' ds \wedge \omega_a + \{\log|\lambda^m - H_m|^{1/n}\}' d\omega_a \\ &= \{-(\log|\lambda^m - H_m|^{1/n})'' + [(\log|\lambda^m - H_m|^{1/n})']^2\}\omega_a \wedge ds \\ &\quad + (\log|\lambda^m - H_m|^{1/n})' \sum_{b=1}^{n-1} \omega_{ab} \wedge \omega_b. \end{aligned}$$

Then we obtain from two equalities above that

$$\{\log|\lambda^m - H_m|^{1/n}\}'' - [(\log|\lambda^m - H_m|^{1/n})']^2 - \lambda\mu - 1 = 0. \tag{3.18}$$

Combining (3.18) with (3.6), we have

$$\{\log|\lambda^m - H_m|^{1/n}\}'' - [(\log|\lambda^m - H_m|^{1/n})']^2 + \frac{(n-m)\lambda^m - nH_m}{m\lambda^{m-2}} - 1 = 0. \tag{3.19}$$

We know that $\lambda^m - H_m \neq 0$. If $\lambda^m - H_m < 0$, from (3.6), we have

$$\lambda^2 - \lambda\mu = \frac{n(\lambda^m - H_m)}{m\lambda^{m-2}} < 0. \tag{3.20}$$

According to the Gauss equation (2.7), we know that the sectional curvature of M is not less than 1. From [3, 15], we know that M is isometric to a totally umbilical hypersurface. This is impossible because M has two distinct principal curvatures. Hence, $\lambda^m - H_m > 0$. Let us define a positive function $w(s)$ over $s \in (-\infty, +\infty)$ by

$$w = (\lambda^m - H_m)^{-1/n}, \tag{3.21}$$

then (3.19) reduces to

$$\frac{d^2w}{ds^2} - w\left\{\frac{(n-m)(w^{-n} + H_m)^{(2-m)/m}}{mw^n} - H_m(w^{-n} + H_m)^{(2-m)/m} - 1\right\} = 0. \tag{3.22}$$

Integrating (3.22), we obtain

$$\left(\frac{dw}{ds}\right)^2 = C - w^2(w^{-n} + H_m)^{\frac{2}{m}} - w^2, \tag{3.23}$$

where C is the constant of integration.

We consider the frame $\{x, e_1, e_2, \ldots, e_n, e_{n+1}\}$ in the Euclidean space $\mathbb{R}^{n+2}$. Then, by (2.4), (3.13) and (3.18), we obtain

$$\begin{aligned} de_a &= \sum_{b=1}^{n-1} \omega_{ab} e_b + \omega_{an} e_n + \omega_{an+1} e_{n+1} - \omega_a e_{n+2} \\ &= \sum_{b=1}^{n-1} \omega_{ab} e_b + (\log|\lambda^m - H_m|^{1/n})' \omega_a e_n - \lambda \omega_a e_{n+1} - \omega_a e_{n+2} \\ &= \sum_{b=1}^{n-1} \omega_{ab} e_b + \{(\log|\lambda^m - H_m|^{1/n})' - \lambda e_{n+1} - e_{n+2}\} \omega_a \end{aligned}$$

$$\begin{aligned} & d\{(\log|\lambda^m - H_m|^{1/n})' - \lambda e_{n+1} - e_{n+2}\} \\ &\quad = \{(\log|\lambda^m - H_m|^{1/n})'' - \lambda' e_{n+1}\} ds \\ &\qquad + (\log|\lambda^m - H_m|^{1/n})' \left(\sum_{a=1}^{n-1} \omega_{na} e_a + \omega_{nn+1} e_{n+1} \right) \\ &\qquad - \lambda \left(\sum_{a=1}^{n-1} \omega_{n+1a} e_a + \omega_{n+1n} e_n \right) - \sum_{a=1}^{n-1} \omega_a e_a - \omega_n e_n \\ &\quad \equiv \{(\log|\lambda^m - H_m|^{1/n})'' - \lambda\mu - 1\} e_n \omega_n \\ &\qquad - \{\lambda' + (\log|\lambda^m - H_m|^{1/n})' \mu\} e_{n+1} \omega_n \quad (\bmod \{e_1, \ldots, e_{n-1}\}) \\ &\quad = (\log|\lambda^m - H_m|^{1/n})' \{(\log|\lambda^m - H_m|^{1/n})' e_n - \lambda e_{n+1} - e_{n+2}\} ds. \end{aligned}$$

By putting

$$W = e_1 \wedge e_2 \wedge \cdots \wedge e_{n-1} \wedge \{(\log|\lambda^m - H_m|^{1/n})' e_n - \lambda e_{n+1} - e_{n+2}\}, \tag{3.24}$$

we can show that

$$dW = (\log|\lambda^m - H_m|^{1/n})' W ds. \tag{3.25}$$

(3.25) shows that n-vector W in $\mathbb{R}^{n+2}$ is constant along $M_1^{n-1}(x)$. Hence there exists an n-dimensional linear subspace $E^n(s)$ in $\mathbb{R}^{n+2}$ containing $M_1^{n-1}(x)$. (3.25) also implies that the n-vector field W only depends on s and by integrating it, we get

$$W = \left\{ \frac{\lambda^m(s) - H_m}{\lambda^m(s_0) - H_m} \right\}^{1/n} W(s_0). \tag{3.26}$$

Theorefore, we have that $E^n(s)$ is parallel to $E^n(s_0)$ in $\mathbb{R}^{n+2}$ for every s.

From the calculation

$$d\omega_{ab} - \sum_{c=1}^{n-1} \omega_{ac} \wedge \omega_{cb} = \omega_{an} \wedge \omega_{nb} + \omega_{an+1} \wedge \omega_{n+1b} - \omega_a \wedge \omega_b$$
$$= -\{[(\log|\lambda^m - H_m|^{1/n})']^2 + \lambda^2 + 1\}\omega_a \wedge \omega_b,$$

we see that the curvature of $M_1^{n-1}(x)$ is $[(\log|\lambda^m - H_m|^{1/n})']^2+\lambda^2+1$ and $M_1^{n-1}(x)$ is locally isometric to $S^{n-1}(c(s))$. Therefore, M is isometric to a complete hypersurface of revolution $S^{n-1}(c(s)) \times M^1$ (i.e. the warped product of $S^{n-1}(c(s))$ and M^1). This proves Theorem 3.1. □

4. A Representation Formula of Period

One knows that the following immersion:

$$x : M^n \hookrightarrow S^{n+1}(1) \subset R^{n+2},$$
$$(s, t_1, \ldots, t_{n-1}) \mapsto (y_1(s)\varphi_1, \ldots, y_1(s)\varphi_n, y_{n+1}(s), y_{n+2}(s)). \tag{4.1}$$
$$\varphi_i = \varphi_i(t_1, \ldots, t_{n-1}), \quad \varphi_1^2 + \cdots + \varphi_n^2 = 1 \tag{4.2}$$

is a parametrization of a rotational hypersurface generated by a curve $(y_1(s), y_{n+1}(s), y_{n+2}(s))$, called the profile curve. Since the curve $(y_1(s),\ y_{n+1}(s),\ y_{n+2}(s))$ belongs to $S^2(1)$ and the parameter s can be chosen as its arc length, we have

$$y_1^2(s) + y_{n+1}^2(s) + y_{n+2}^2(s) = 1, \quad \dot y_1^2(s) + \dot y_{n+1}^2(s) + \dot y_{n+2}^2(s) = 1 \tag{4.3}$$

where the dot denotes the derivative with respect to s and from (4.3) we can obtain $y_{n+1}(s)$ and $y_{n+2}(s)$ as functions of $y_1(s)$. In fact, we can write (see [5, 9])

$$y_1(s) = \cos\vartheta(s), \quad y_{n+1}(s) = \sin\vartheta(s)\cos\theta(s), \quad y_{n+2}(s) = \sin\vartheta(s)\sin\theta(s). \tag{4.4}$$

We can deduce from (4.3) that

$$\dot\vartheta^2 + \dot\theta^2 \sin^2\vartheta = 1. \tag{4.5}$$

It follows from Eq. (4.5) that $\dot\vartheta^2 \le 1$. Combining these with $\dot\vartheta^2 = \frac{\dot y_1^2}{1-y_1^2}$, we have

$$\dot y_1^2 + y_1^2 \le 1. \tag{4.6}$$

We can get the plane curve ζ from α by projection of $S_+^2 = \{(y_1, y_{n+1}, y_{n+2}) \,|\, y_1 \ge 0, y_1^2 + y_{n+1}^2 + y_{n+2}^2 = 1\}$ onto the unit disk $E = \{(y_{n+1}, y_{n+2}) \,|\, y_{n+1}^2 + y_{n+2}^2 \le 1\}$. Then the plane curve ζ can be written as

$$y_{n+1}(s) = \sin\vartheta(s)\cos\theta(s), \quad y_{n+2}(s) = \sin\vartheta(s)\sin\theta(s). \tag{4.7}$$

Writing $r(s) = y_1(s)$, (4.5) can be written as

$$\dot\theta^2 = \frac{1-\dot\vartheta^2}{\sin^2\vartheta} = \frac{1-r^2-\dot r^2}{(1-r^2)^2}. \tag{4.8}$$

Do Carmo and Dajczer proved the following

Lemma 4.1 ([3]). *Let M^n be a rotational hypersurface of $S^{n+1}(1)$. Then the principal curvatures λ_i of M^n are*

$$\lambda_i = \lambda = \frac{\sqrt{1 - r^2 - \dot{r}^2}}{r} \tag{4.9}$$

for $i = 1, \ldots, n-1$, and

$$\lambda_n = \mu = -\frac{\ddot{r} + r}{\sqrt{1 - r^2 - \dot{r}^2}}. \tag{4.10}$$

On the other hand, let us fix a point $p_0 \in M$, let $\gamma(u)$ be the only geodesic in M such that $\gamma(0) = p_0$ and $\gamma'(0) = e_n(p_0)$. From (3.16), we know that $\gamma(u) = e_n(\gamma(u))$. Note that $\gamma(u)$ is also a line of curvature. Let us denote by $g(u) = w(\gamma(u))$. Since H_m is constant, we know from (3.23) that

$$(g')^2 + g^2(g^{-n} + H_m)^{\frac{2}{m}} + g^2 = C. \tag{4.11}$$

From (4.11), we have $C > 0$. Moreover, it is not so difficult to know that

$$q(x) = C - x^2(x^{-n} + H_m)^{\frac{2}{m}} - x^2 \tag{4.12}$$

is positive on a interval (t_1, t_2) with $0 < t_1 < t_2$ and $q(t_1) = q(t_2) = 0$. From (4.11), we know that $g(u)$ is periodic. And the period is the following

$$T = 2\int_{t_1}^{t_2} \frac{1}{\sqrt{C - t^2(t^{-n} + H_m)^{\frac{2}{m}} - t^2}} dt. \tag{4.13}$$

One can know from the following lemma the relation between hypersurfaces with two distinct principal curvature and rotational hypersurfaces.

Lemma 4.2 ([3, 15]). *Let M be a complete hypersurface in a unit sphere $S^{n+1}(1)$. Assume that the principal curvatures $\lambda_1, \ldots, \lambda_n$ of M satisfy $\lambda_1 = \lambda_2 = \cdots = \lambda_{n-1} = \lambda \neq 0$, $\lambda_n = \mu = \mu(\lambda)$ and $\lambda \neq \mu$, then M is a rotational hypersurface.*

On the other hand, if $S^k(a)$ denote a sphere with radius a, then the sectional curvature of $S^k(a)$ is equal to $\frac{1}{a^2}$. From (4.1), Theorem 3.1 and Lemma 4.2, we have

$$\frac{1}{r^2} = [(\log|\lambda^m - H_m|^{1/n})']^2 + \lambda^2 + 1.$$

Then we know from (3.23), (4.11) that

$$r(u) = \frac{g(u)}{\sqrt{C}}, \quad g(u) = (\lambda^m - H_m)^{-\frac{1}{n}}. \tag{4.14}$$

From (4.14), one can have that the period of $r(u)$ is also T, that is $2\int_{t_1}^{t_2} \frac{1}{\sqrt{C - t^2(t^{-n}+H_m)^{\frac{2}{m}} - t^2}} dt$, therefore we obtain from (4.8), (4.9), (4.11) that the period $P(H_m, n, c)$ of hypersurfaces is

$$P(H_m, n, C) = \theta(T) = \int_0^T \frac{\sqrt{1 - r^2 - \dot{r}^2}}{1 - r^2} ds$$

$$= \int_0^T \frac{r(s)\lambda(s)}{1 - r^2(s)} ds = 2 \int_0^{\frac{T}{2}} \frac{r(s)\lambda(s)}{1 - r^2(s)} ds. \tag{4.15}$$

It is clear that the profile curve gives rise to an immersed hypersurface if and only if the period $P(H_m, n, C)$ is a rational multiple of 2π, and to an embedded hypersurface if and only if the period $P(H_m, n, C) = \frac{2\pi}{k}$ for some integer k (also see [6, 11, 12]).

5. Embedded Hypersurfaces with Constant $H_m > 0$

From Sec. 4, one knows that embedded problem is equivalent to find some constant C and some integer k such that $P(H_m, n, C) = \frac{2\pi}{k}$. In order to estimate the period $P(H_m, n, C)$, we will give the following lemma ([11, 12]).

Lemma 5.1. *Let ϵ and δ be positive numbers and $f : (t_0 - \epsilon, t_0 + \epsilon) \to R$ and $y : (-\delta, \delta) \times (t_0 - \epsilon, t_0 + \epsilon) \to R$ be two smooth functions such that $f(t_0) = f'(t_0) = 0$ and $f''(t_0) = -2a < 0$. If for any small $c > 0$, $t_1(c) < t_0 < t_2(c)$ are such that $f(t_1(c)) + c = 0 = f(t_2(c)) + c$, then*

$$\lim_{c \to 0^+} \int_{t_1(c)}^{t_2(c)} \frac{y(c, t)dt}{\sqrt{f(t) + c}} = \frac{y(0, t_0)\pi}{\sqrt{a}}.$$

We next state our main theorem.

Theorem 5.1. *For any $n \geq 5$ and any integer $k \geq 3$, if 4th mean curvature H_4 takes value between $\frac{1}{(\tan \frac{\pi}{k})^4}$ and $\frac{k^4 - 4}{n(n-4)}$, then there exists an n-dimensional compact nontrivial embedded hypersurface with constant $H_4 > 0$ in $S^{n+1}(1)$.*

Proof. Firstly, we consider the general case: $1 \leq m \leq (n - 1)$ and $H_m \geq 0$.

From (4.11), one has

$$(g')^2 = q(g), \quad \text{where } q(v) = C - v^2(v^{-n} + H_m)^{\frac{2}{m}} - v^2. \tag{5.1}$$

A direct calculation shows that

$$\begin{aligned} q'(v) &= 2v \left\{ -(v^{-n} + H_m)^{\frac{2}{m}} + \frac{n}{m} v^{-n} (v^{-n} + H_m)^{\frac{2-m}{m}} - 1 \right\} \\ &= -2v \left\{ (v^{-n} + H_m)^{\frac{2-m}{m}} \left[\frac{m - n}{m} v^{-n} + H_m \right] + 1 \right\} \end{aligned} \tag{5.2}$$

$$\begin{aligned} q''(v) = &-\frac{2(v^{-n} + H_m)^{\frac{2-2m}{m}}}{m^2} \{(2n^2 - 3nm + m^2)v^{-2n} \\ &+ m(n^2 - 3n + 2m)H_m v^{-n} + m^2 H_m^2\} - 2 < -2. \end{aligned} \tag{5.3}$$

From (5.3), one obtains that $q'(v)$ is a decreasing function of v in $[0, +\infty)$. From (5.2), one has $q'(v) > 0$ if $v \to 0$; $q'(v) < 0$ if $v \to \infty$. Hence there exists

$0 < v_0 < \infty$ such that $q'(v_0) = 0$. Moreover, the function $q(v)$ is a monotone increasing function of v in $(0, v_0]$ and decreasing function of v in $[v_0, +\infty)$. Hence, for some value of C, the function q has two positive roots t_1 and t_2, such that $t_1 \leq t_2,\ q(t_1) = q(t_2) = 0$ and $q(t) > 0$ if $t \in (t_1, t_2)$.

In particular, if $m = 4$ and $H_4 = 1$, we have form (5.2) that the only positive root of $q'(v)$ is

$$v_0 = \left(\frac{(n-4)^2}{8n-16}\right)^{\frac{1}{n}}, \tag{5.4}$$

and the maximum of $q(v)$ is $q(v_0) = C - c_0$, where

$$c_0 = v_0^2((v_0^{-n} + 1)^{\frac{1}{2}} + 1) = \left(\frac{(n-4)^2}{8n-16}\right)^{\frac{2}{n}} \times \left(\frac{n}{n-4} + 1\right). \tag{5.5}$$

Therefore, whenever $C > c_0$, the function $q(v)$ has two positive roots denoted by $t_1(C)$ and $t_2(C)$. In this special case, (5.3) reduces to

$$q''(v_0) = -\frac{4(n-2)^2}{n} :\triangleq -2a. \tag{5.6}$$

Hence, we get from (4.15) that

$$P(H_4, n, C) = 2\int_0^{\frac{T}{2}} \frac{r(s)\lambda(s)}{1 - r^2(s)} ds, \tag{5.7}$$

it follows from $r(s) = \frac{g(s)}{\sqrt{C}}$ and $\lambda(s) = (g^{-n} + 1)^{\frac{1}{4}}$ that

$$P(H_4 = 1, n, C) = 2\int_0^{\frac{T}{2}} \frac{\sqrt{C}g(s)(g^{-n}(s) + 1)^{\frac{1}{4}}}{C - g^2(s)} ds. \tag{5.8}$$

Substituting $t = g(s)$, one derives from $g(0) = t_1(C)$, $g(\frac{T}{2}) = t_2(C)$ and (5.8) that

$$P(H_4 = 1, n, C) = 2\int_{t_1(C)}^{t_2(C)} \frac{\sqrt{C}t(t^{-n} + 1)^{\frac{1}{4}}}{C - t^2} \frac{1}{\sqrt{q(t)}} ds. \tag{5.9}$$

From (5.4)–(5.6), we know that the function $q(v) - (C - c_0)$ satisfies the conditions in Lemma 5.1, hence we apply Lemma 5.1 to function $q(v) - (C - c_0)$, then we obtain

$$\lim_{C \to c_0^+} P(H_4 = 1, n, C) = \frac{2\pi}{\sqrt{a}} \frac{\sqrt{c_0}v_0(v_0^{-n} + 1)^{\frac{1}{4}}}{c_0 - v_0^2} = \frac{2\pi\sqrt{n-2}}{n-2}. \tag{5.10}$$

On the other hand, we will estimate $P(H_m, n, C)$ when $C \to \infty$, we make the substitution $t = r(s)$ and obtain

$$P(H_m, n, C) = 2\int_{\frac{t_1(C)}{\sqrt{C}}}^{\frac{t_2(C)}{\sqrt{C}}} \frac{t((\sqrt{C}t)^{-n} + H_m)^{\frac{1}{m}}}{(1 - t^2)\sqrt{1 - t^2(1 + (H_m + (\sqrt{C}t)^{-n})^{\frac{2}{m}})}} dt. \tag{5.11}$$

Putting

$$\tilde{q} = 1 - t^2(1 + (H_m + (\sqrt{C}t)^{-n})^{\frac{2}{m}}), \tag{5.12}$$

we deduce from (5.1) and (5.12) that

$$\tilde{q}\left(\frac{v}{\sqrt{C}}\right) = \frac{q(v)}{C},$$

since the function q has two positive roots t_1 and t_2, one has from the relationship between q and $\tilde{q}$ that $\tilde{q}$ has two positive roots $\frac{t_1}{\sqrt{C}}$, $\frac{t_2}{\sqrt{C}}$ and $\frac{t_1}{\sqrt{C}} \leq \frac{t_2}{\sqrt{C}}$. Moreover, if C converges to $+\infty$, then $\frac{t_1}{\sqrt{C}}$ converges to 0, $\frac{t_2}{\sqrt{C}}$ converge to $\frac{1}{\sqrt{1+H_m^{\frac{2}{m}}}}$. (When $m = 1$, also see [12].) Hence we obtain from (5.11) that

$$\lim_{C\to\infty} P(H_m, n, C) = 2\int_0^{\frac{1}{\sqrt{1+H_m^{\frac{2}{m}}}}} \frac{tH_m^{\frac{1}{m}}}{(1-t^2)\sqrt{1-t^2(1+H_m^{\frac{2}{m}})}}dt = 2\arctan\frac{1}{H_m^{\frac{1}{m}}}. \tag{5.13}$$

In particular, if $m = 4$ and $H_4 = 1$, we have from (5.13) that

$$\lim_{C\to\infty} P(H_4 = 1, n, C) = 2\arctan\frac{1}{(H_4)^{\frac{1}{4}}} = \frac{\pi}{2}. \tag{5.14}$$

Next, we consider the case $m = 4$ and $0 < H_4 \neq 1$.

In this case, we have from (5.2) that the only positive root of q' is

$$v_0 = \left(\frac{\sqrt{n(n-4)H_4+4} - nH_4 + 4H_4 - 2}{4H_4(1-H_4)}\right)^{\frac{1}{n}}. \tag{5.15}$$

A direct calculation shows that $q(v_0) = C - c_0$, where

$$\begin{aligned} c_0 &= v_0^2(v_0^{-n} + H_4)^{\frac{1}{2}} + v_0^2 \\ &= \left(\frac{\sqrt{n(n-4)H_4+4} - nH_4 + 4H_4 - 2}{4H_4(1-H_4)}\right)^{\frac{2}{n}} \\ &\quad \times \left[\left(\frac{H_4(\sqrt{n(n-4)H_4+4} - nH_4 + 2)}{\sqrt{n(n-4)H_4+4} - nH_4 + 4H_4 - 2}\right)^{\frac{1}{2}} + 1\right]. \end{aligned} \tag{5.16}$$

$$\begin{aligned} q''(v_0) &= \frac{-2H_4^{\frac{1}{2}}}{(|\sqrt{n(n-4)H_4+4} - nH_4 + 2|)^{\frac{3}{2}}(|\sqrt{n(n-4)H_4+4} - nH_4 + 4H_4 - 2|)^{\frac{1}{2}}} \\ &\quad \times \{n^2(n-4)H_4^2 + n(-n^2+4n+4)H_4 - 4n \\ &\quad + [n^2 - 2n + (-n^2+2n)H_4]\sqrt{n(n-4)H_4+4}\} :\triangleq -2a. \end{aligned} \tag{5.17}$$

Therefore, whenever $C > c_0$, $q(v)$ has two positive roots denoted by t_1 and t_2.

Using the results of Sec. 4, we have from (4.15) that

$$P(H_4, n, C) = 2\int_0^{\frac{T}{2}} \frac{r(s)\lambda(s)}{1-r^2(s)}ds. \tag{5.18}$$

In this special case, (4.14) is equivalent to $r(s) = \frac{g(s)}{\sqrt{C}}$ and $\lambda(s) = (g(s)^{-n} + H_4)^{\frac{1}{4}}$, then it follows from (5.18) that

$$P(H_4, n, C) = 2\int_0^{\frac{T}{2}} \frac{\sqrt{C}g(s)(g(s)^{-n} + H_4)^{\frac{1}{4}}}{C - g^2(s)}ds. \tag{5.19}$$

Doing the substitution $t = g(s)$ and applying Lemma 5.1, one concludes from $g(0) = t_1$ and $g(\frac{T}{2}) = t_2$ that

$$\begin{aligned}
\lim_{C\to c_0^+} P(H_4, n, C) &= \frac{2\pi\sqrt{c_0}}{\sqrt{a}\sqrt{c_0 - v_0^2}} \\
&= 2\pi\frac{|(n-2)(n-nH_4) + (nH_4 - n)\sqrt{n(n-4)H_4+4}|^{\frac{1}{2}}}{\begin{aligned}|n^2(n-4)H_4^2 + n(-n^2+4n+4)H_4 - 4n \\ + [n^2 - 2n + (-n^2+2n)H_4]\sqrt{n(n-4)H_4+4}|^{\frac{1}{2}}\end{aligned}} \\
&= 2\pi\frac{|(n-2) - \sqrt{n(n-4)H_4+4}|^{\frac{1}{2}}}{|(n(4-n)H_4 - 4) + (n-2)\sqrt{n(n-4)H_4+4}|^{\frac{1}{2}}} \\
&= \frac{2\pi}{[n(n-4)H_4+4]^{\frac{1}{4}}}.
\end{aligned} \tag{5.20}$$

On the other hand, we know from (5.13) that

$$\lim_{C\to\infty} P(H_4, n, C) = 2\arctan\frac{1}{H_4^{\frac{1}{4}}}. \tag{5.21}$$

Therefore, for any fixed $H_4 > 0$, the function $P(H_4, n, C)$ takes all the values between

$$A(H_4) = 2\arctan\frac{1}{H_4^{\frac{1}{4}}}, \quad B(H_4) = \frac{2\pi}{[n(n-4)H_4+4]^{\frac{1}{4}}}. \tag{5.22}$$

It is not so difficult to know that $A(H_4)$ and $B(H_4)$ are decreasing functions of H_4,

$$A\left(\frac{1}{(\tan\frac{\pi}{k})^4}\right) = B\left(\frac{k^4-4}{n(n-4)}\right) = \frac{2\pi}{k}, \tag{5.23}$$

where $k \geq 3$ is any integer, then we deduce that the number $\frac{2\pi}{k}$ lies between $A(H_4)$ and $B(H_4)$. Hence, by the continuity of $P(H_4, n, C)$, there exists some constant C_1 such that $P(H_4, n, C_1) = \frac{2\pi}{k}$. If the period is $\frac{2\pi}{k}$, then there exists a compact embedded hypersurface with constnat H_4 which is not isometric to a round sphere or a Clifford hypersurface. We complete the proof of Theorem 5.1. □

For constant $H_m > 0$, we can prove the following

Theorem 5.2. *For any integer $1 \le m \le n-1$, there exist many nontrivial embedded hypersurfaces with constant $H_m > 0$ in $S^{n+1}(1)$.*

Proof. On one hand, we consider the case $H_m = 0$. By using the similar arguments with the proof of Theorem 5.1, we have that

$$v_0 = \left(\frac{n-m}{m}\right)^{\frac{m}{2n}}, \quad c_0 = \left(\frac{n-m}{m}\right)^{\frac{m}{n}} \times \frac{n}{n-m}, \quad q''(v_0) = -\frac{4n}{m} : \triangleq -2a. \tag{5.24}$$

From (4.14), (4.15) and Lemma 5.1, we obtain

$$\lim_{C \to c_0^+} P(H_m = 0, n, C) = \frac{2\pi\sqrt{c_0}}{\sqrt{a}\sqrt{c_0 - v_0^2}} = \sqrt{2}\pi, \tag{5.25}$$

by continuity arguments, we can fix H_m sufficiently small such that

$$\lim_{C \to c_0^+} P(H_m, n, C) > \pi. \tag{5.26}$$

On the other hand, we deduce from (5.13) and $H_m > 0$ that

$$\lim_{C \to \infty} P(H_m, n, C) = 2 \arctan \frac{1}{H_m^{\frac{1}{m}}} < \pi. \tag{5.27}$$

By (5.26), (5.27) and the continuity of $P(H_m, n, C)$, there exists $c_0^+ < C_2 < \infty$, such that $P(H_m, n, C_2) = \pi$. We complete the proof of Theorem 5.2. □

Remark 5.1. When $m = 1$, Theorem 5.2 reduces to the results of Brito and Leite ([2]).

Using the similar arguments as above, we can obtain the following:
When $m = 2$, we have:

Proposition 5.1. *For any $n \ge 3$ and any integer $k \ge 2$, if $H_2 = \frac{R-n(n-1)}{n(n-1)}$ takes value between $\frac{1}{(\tan\frac{\pi}{k})^2}$ and $\frac{k^2-2}{n}$, then there exists an n-dimensional compact nontrivial embedded hypersurface M with constant 2nd mean curvature $H_2 > 0$ (i.e. scalar curvature $R > n(n-1)$) in $S^{n+1}(1)$, where R is the scalar curvature of M.*

Proof. In this case, $m = 2$. By using the similar arguments with the proof of Theorem 5.1, one has that

$$v_0 = \left(\frac{n-2}{2(H_2+1)}\right)^{\frac{1}{n}}, \quad c_0 = \left(\frac{n-2}{2(H_2+1)}\right)^{\frac{2}{n}} \times \frac{n(H_2+1)}{n-2}, \tag{5.28}$$

$$q''(v_0) = -2n(H_2+1) :\triangleq -2a, \tag{5.29}$$

$$\lim_{C \to c_0^+} P(H_2, n, C) = \frac{2\pi\sqrt{c_0}}{\sqrt{a}\sqrt{c_0 - v_0^2}} = \frac{2\pi}{\sqrt{nH_2+2}}. \tag{5.30}$$

On the other hand, one sees from (5.13) that

$$\lim_{C \to \infty} P(H_2, n, C) = 2\arctan \frac{1}{H_2^{\frac{1}{2}}}. \tag{5.31}$$

For any fixed $H_2 > 0$, the function $P(H_2, n, C)$ takes all the values between

$$E(H_2) = 2\arctan \frac{1}{H_2^{\frac{1}{2}}}, \quad F(H_2) = \frac{2\pi}{\sqrt{nH_2+2}}$$

since $P(H_2, n, C)$ is a continuous function. By a direct calculation, one obtains that $E(H_2)$ and $F(H_2)$ are decreasing functions and

$$E\left(\frac{1}{(\tan\frac{\pi}{k})^2}\right) = F\left(\frac{k^2-2}{n}\right) = \frac{2\pi}{k},$$

where $k \geq 2$ is any integer, then it follows that the number $\frac{2\pi}{k}$ lies between $E(H_4)$ and $F(H_4)$. Hence, there exists some constant C_3 such that $P(H_2, n, C_3) = \frac{2\pi}{k}$, that is, there exists a compact embedded hypersurface with constnat H_2 (i.e. constant scalar curvature) which is not isometric to a round sphere or a Clifford hypersurface. We complete the proof of Proposition 5.1. □

Remark 5.2. Since $H_2 = \frac{R-n(n-1)}{n(n-1)}$, by a direct calculation, one concludes that when $3 \leq n \leq 6$, Proposition 5.2 reduces to Theorems 1.1 and 1.2 due to Li–Wei ([8]); when $n > 6$ and $k = 2$, Proposition 5.2 reduces to Theorem 1.3 due to Li–Wei ([8]). In Proposition 5.2, we find there exist a lot of new examples satisfying $R > n(n-1)$. Hence, Proposition 5.2 is the generalization of Li–Wei's results ([8]).

Remark 5.3. For some special $4 \neq m > 3$, we can also obtain some nontrivial embedded hypersurfaces with $H_m =$ constant in $S^{n+1}(1)$ using the same methods.

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