Microscopic description of inclusive neutrino-nucleus reactions

Multi-dimensional Modeling and Multi-Messenger observation from Core-Collapse Supernovae (4M-COCOS)
Fukuoka University, Fukuoka, Japan
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Contents

- A microscopic description of electro-weak nuclear responses for light nuclei
  - *Ab initio* few-body calculations: Correlated Gaussian method
  - Application to nuclear photoabsorption reaction
  - Application to neutrino-nucleus reactions

- Progress report

  Toward unified description of neutrino-nucleus reactions for a wide range of mass number and energy and momentum transfer

Electroweak nuclear reactions

- Nuclear reaction involving light elements
  - Fuel of a star
  - Origin of heavier elements

- Electromagnetic response (photon)
  - Radiative capture X(a,γ)Y
    - ⇔ Photoabsorption(dissociation) reaction X(γ,b)Y
  - Electron scattering X(e,e')Y
- "Weak" response (weak bosons)
  - Beta decay X→Y+e+ν, Electron capture X+e→Y+ν
  - Neutrino-nucleus reaction X(ν,ν')Y, X(ν,eν')Y

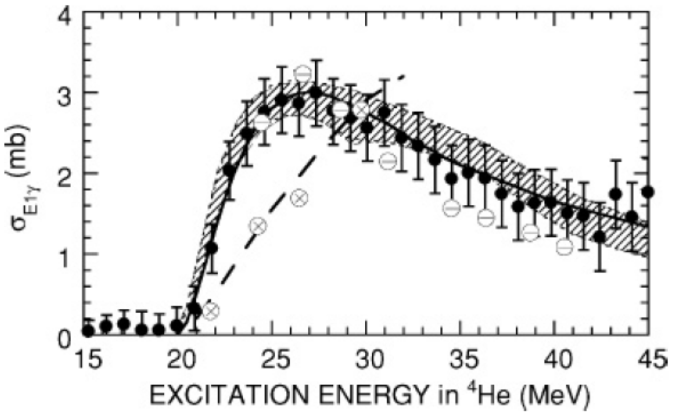
Electroweak excitations of light nuclei

- Photoabsorption reaction of ^{4}He
  - Electric dipole excitation
  - Recent measurements
    - Peak　～27MeV

      S. Nakayama et al., PRC 76, 021305 (2007).
    - Peak　～30 MeV

      T. Shima et al., PRC 72, 044004 (2005).

Taken from S. Nakayama et al.
PRC 76, 021305 (2007).



- Excitation of light nuclei induced by the weak interaction
  - Neutrino-nucleus reaction (Gamow-Teller, Spin-dipole, etc.)

    → important for the supernova explosion scenario

    neutrino heating, shock-wave revival, neutrino oscillations, etc.

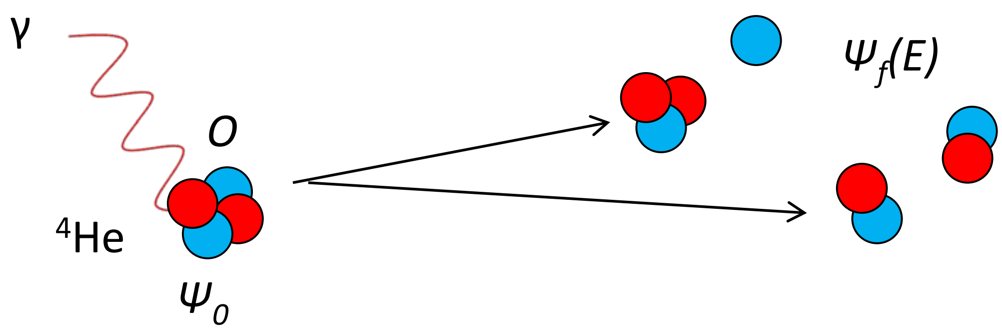
Neutrino-nucleus cross section is too small to measure

Reliable theoretical model is desired

Electroweak response functions

- Response (strength) function
  - Resonant and continuum structure
  - The ground state properties and interactions

$$S(E) = \mathcal{S}_{f\mu} |\langle \Psi_f | \mathcal{O}_{\lambda\mu} | \Psi_0 \rangle|^2 \delta(E_f - E_0 - E)$$



  - Evaluate *S(E)* with *ab initio* theoretical model
    - Nucleon (proton and neutron) degrees of freedom
    - Realistic nuclear force (NN scattering, ^{2}H properties)
    - No specific model assumption

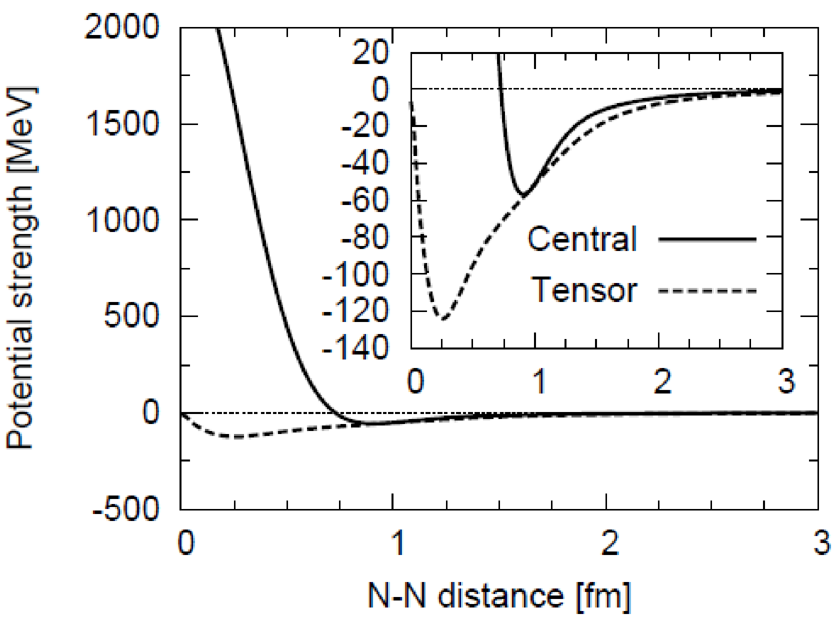
Hamiltonian and nuclear forces

Hamiltonian

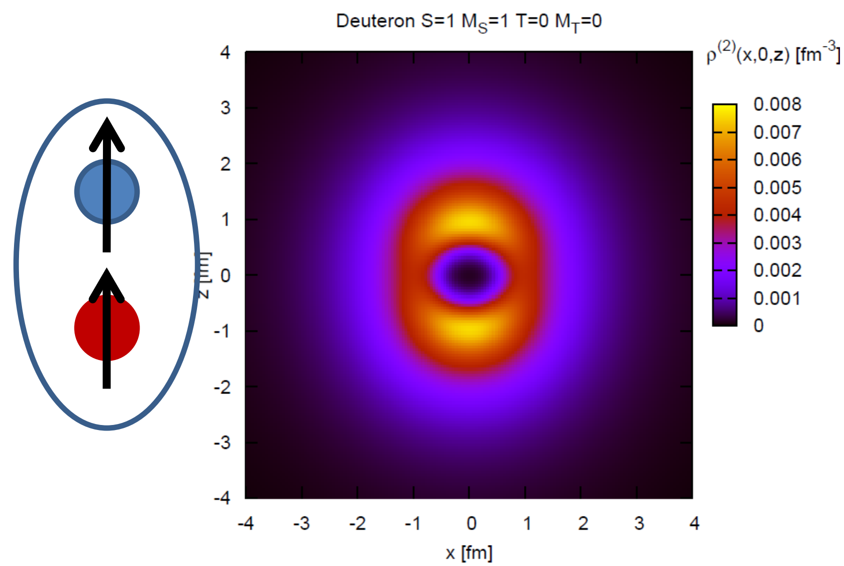
$$H = \sum_{i=1}^{A} T_i - T_{\rm cm} + \sum_{i<j}^{A} v_{ij} + \sum_{i<j<k}^{A} v_{ijk}$$

$$v_{12} = V_c(r) + V_{\rm Coul.}(r) P_{1\pi} P_{2\pi} + V_t(r) S_{12} + V_b(r) \boldsymbol{L} \cdot \boldsymbol{S}$$

- Argonne v8 type interactions (AV8', G3RS); "bare" interaction
  central, tensor, spin-orbit
- Three-nucleon force (3NF) E. Hiyama et al. PRC70, 031001(R) (2002)
  → reproduce binding energies of ^{3}H, ^{4}He



Deuteron (pn) density



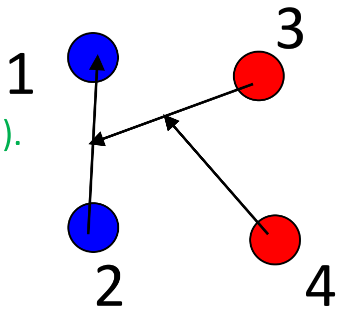
Variational calculation for many-body system

- Many-body wave function Ψ has all information of the system
- Solve many-body Schoedinger equation
  ⇔ Eigenvalue problem with Hamiltonian matrix
  $$H\Psi = E\Psi$$
- Variational principle $\langle\Psi|H|\Psi\rangle = E \geqq E_0$ （“Exact” energy）
  （Equal holds if Ψ is the “exact” solution）
- Expand the wave function in the explicitly correlated Gaussian functions
  $$\Psi = \Sigma_k c_k \exp\{- \Sigma_{i,j} \beta^k_{ij}(r_i-r_j)^2\}$$
  - Parameter β_{ij} explicitly describe correlations among particles
  - Optimal parameters are selected stochastically

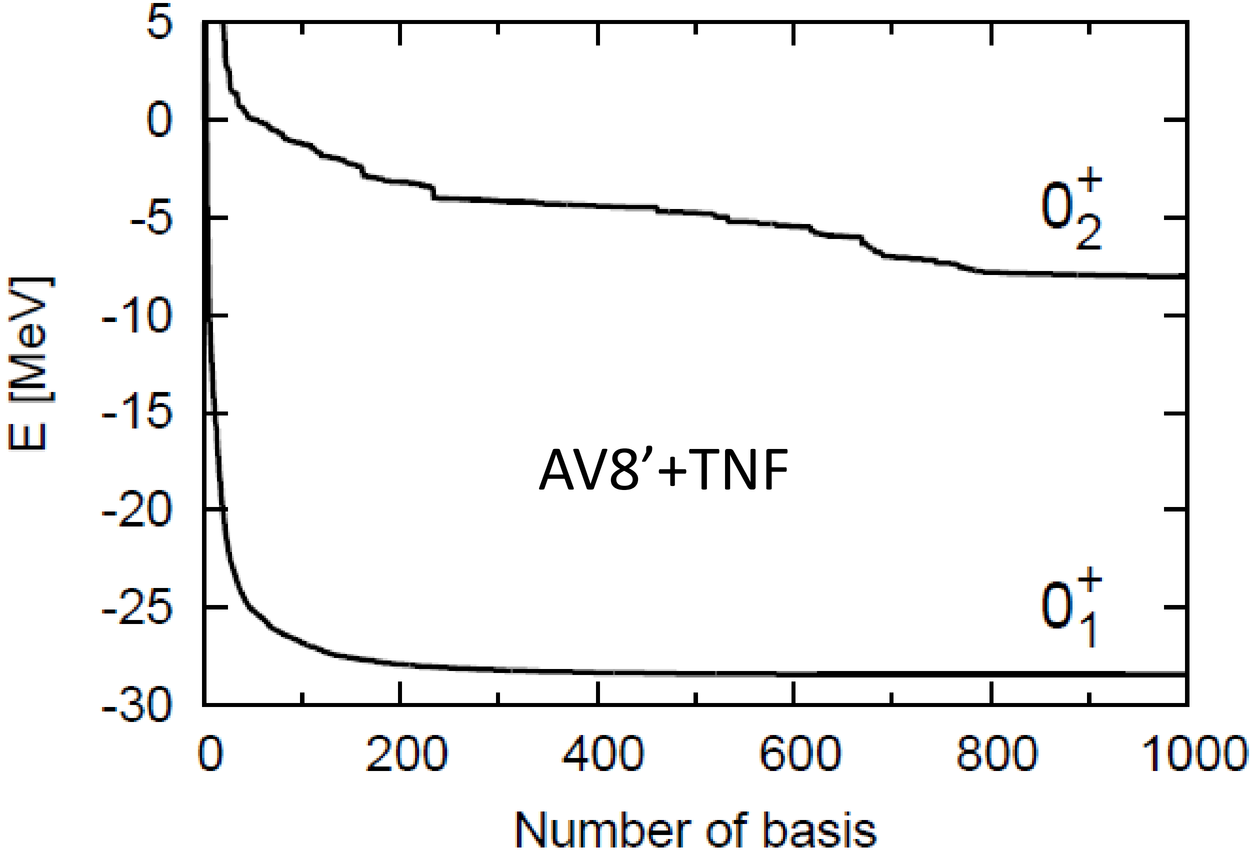
  Stochastic Variational Method K. Varga and Y. Suzuki, PRC52, 2885 (1995).

  1. Randomly generate candidates
  2. Calculate energy for each candidate
  3. Select the basis which gives the lowest energy among them
  4. Increase the basis size
  5. Return to 1. and repeat the procedure until energy is converged

  → accurate solution can be obtained with a small basis size



Energy convergence of ^{4}He



Ground state energy agrees with the other accurate methods (FY, GFMC, NCSM,…) within 60 keV

H. Kamada et al., PRC64, 044001 (2001)

Electro-weak operators (Long-wave-length approximation)

- Photoabsorption reaction (operator: r)

$$\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$

Electric dipole strength function

- Neutrino-nucleus reaction
  - Charged current ^{4}He → ^{4}Li or ^{4}H

    Isovector transition, (IV±; $\tau_\pm$)

    - Gamow-Teller $\sigma\tau_\pm$
    - Spin-dipole $[r \times \sigma]_{\lambda\mu}\,\tau_\pm$
    - $(\nabla \cdot \sigma)\,\tau_\pm$
  - Neutral current ^{4}He → ^{4}He*
    - Isovector (IV0; τ_0) and Isoscalar (IS) transition

Total photoabsorption cross section

Photoabsorption cross section

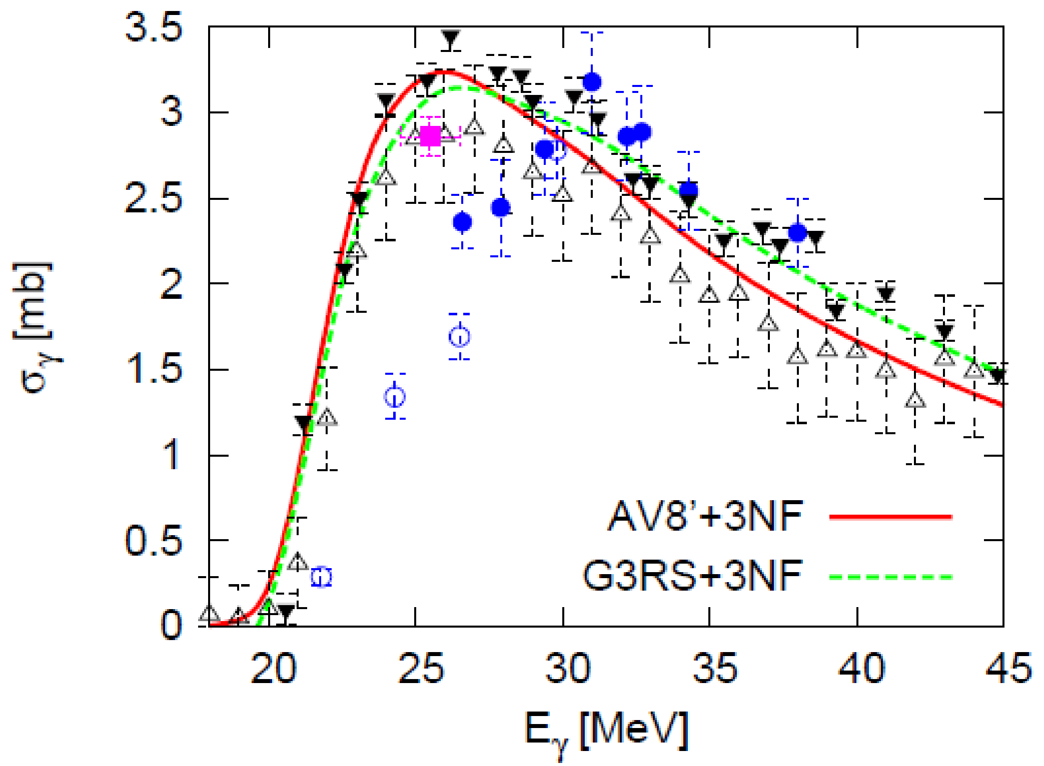
$$\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$

Interaction: AV8'+3NF, G3RS+3NF

3NF: E. Hiyama et al., PRC70, 031001(2004).

The continuum $J^\pi T=1^-1$ state is expanded in several thousand of basis states including explicit decay to two and three-body channels.

Comparison with the measurements
→ good agreement above 30 MeV
Disagree at the low energy with the data by Shima et al.

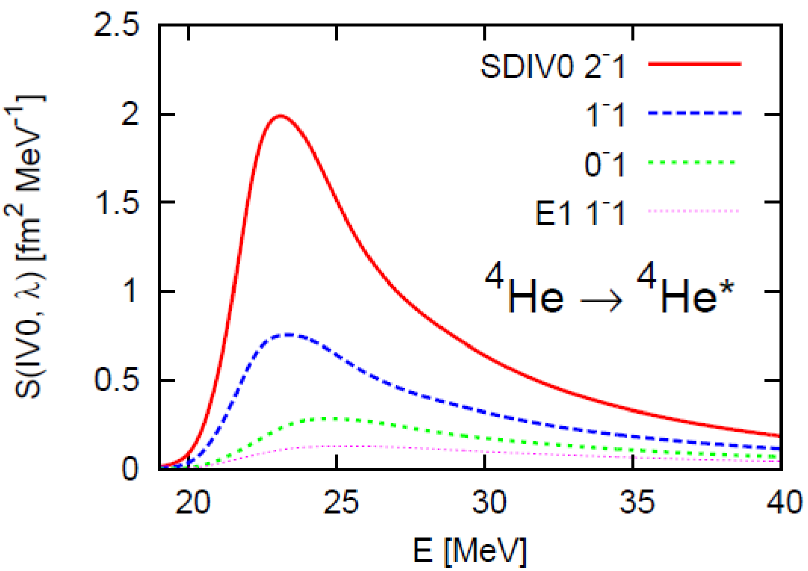


△ S. Nakayama et al., (2007)
■ D.P. Wells et al. (1992)
▼ Y. M. Arkatov et al.,(1974).
○ T. Shima et al., (2005).
● T. Shima et al., new measurement

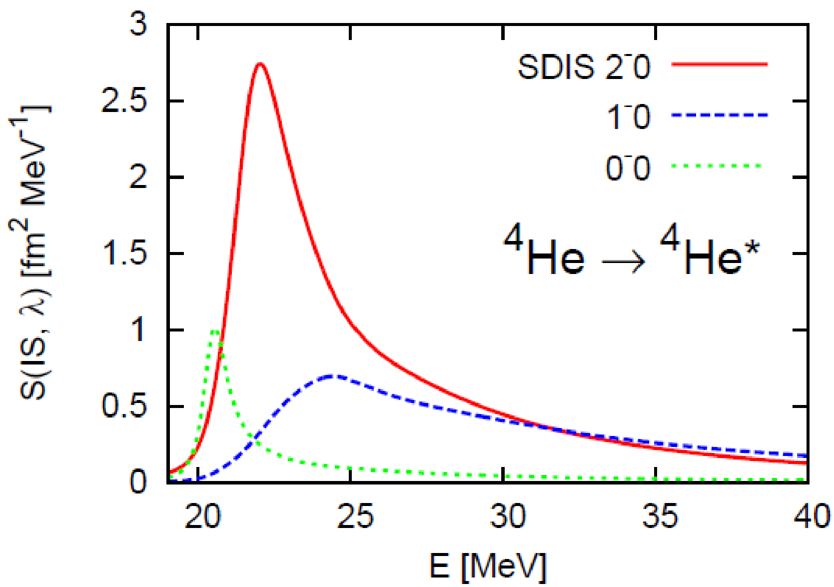
Spin-dipole strength function

Spin-dipole operator $\mathcal{O}^p_{\lambda\mu} = \sum_{i=1}^{4} [(\boldsymbol{r}_i - \boldsymbol{x}_4) \times \boldsymbol{\sigma}_i]_{\lambda\mu} \, \tau^p_{3i}$

Isovector (IV0)



Isoscalar (IS)



Sum rule ～100%

IV0	
$m_0(p,\lambda)$	SR
4.59	4.59
9.35	9.36
18.36	18.38

- Relatively small decay widths of 0^-0, 2^-0 (0.84, 2.01 MeV)
- Resonant structure ⇔ Strength function

Neutrino-nucleus cross sections

Neutrino-nucleus reaction (Gamow-Teller, Dipole, Spin-dipole, etc.)

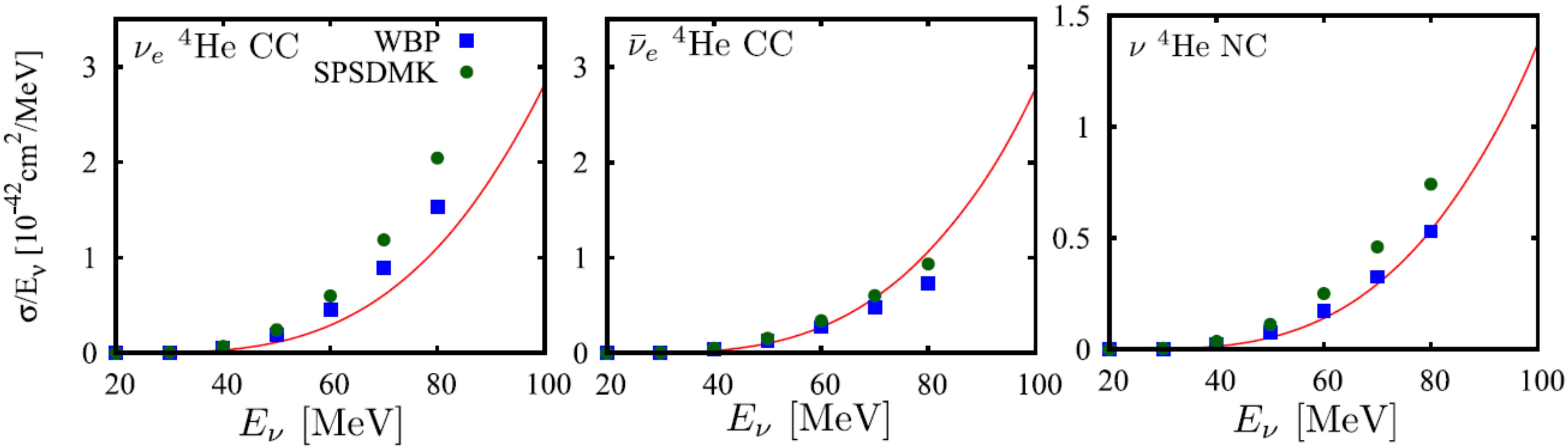
→ important for the supernova explosion scenario

Neutrino-nucleus cross section is too small to measure

Reliable theoretical evaluation is desired

Inclusive neutrino-^{4}He reactions

$$\nu_e + {}^4\text{He} \to e^- + 3p + n,\ \ e^- + 2p + d,\ \ e^- + p + {}^3\text{He}$$
$$\bar{\nu}_e + {}^4\text{He} \to e^+ + p + 3n,\ \ e^+ + 2n + d,\ \ e^+ + n + {}^3\text{H}$$
$$\nu + {}^4\text{He} \to \nu + 2p + 2n,\ \ \nu + p + n + d,\ \ \nu + n + {}^3\text{He},\ \ \nu + p + {}^3\text{H},\ \ \nu + 2d$$



S.X. Nakamura et al., Rep. Prog. Phys. 80, 056301 (2017)

Shell model cal. taken from T. Yoshida et al. Astro. J 686, 448 (2008)

Summary and outlook

- Four-body calculations for electroweak strength functions of ^{4}He
  - Explicitly correlated basis reinforced with complex scaling method
    - Based on the realistic nuclear force
    - No speciffic model assumption
    - Unified description of bound and unbound states

- Dipole type (E1, SD) electroweak strength functions
  - Non-energy-weighted sum rules are fully satisfied
  - Consistent agreement with experimental values
    - Photoabsorption reaction WH, Y. Suzuki, K. Arai, Phys. Rev. C 85, 054002(2012)
    - SD strength function and spectrum WH, Y. Suzuki, Phys. Rev. C 87, 034001 (2013)
    - Neutrino-nucleus reaction S.X. Nakamura et al., Rep. Prog. Phys. 80, 056301 (2017)

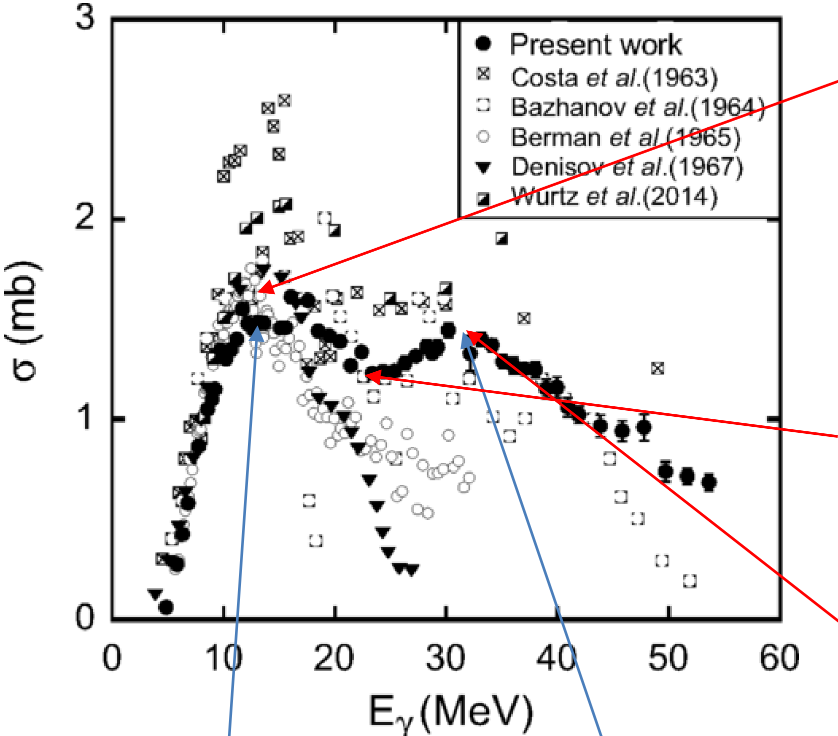
- Work in progress towards

  Unified description of inclusive neutrino-nucleus reactions for a wide range of mass number, energy and momentum transfer

Progress report

Photoabsorption of ^{6}Li

S. Satsuka and WH, Phys. Rev. C 100, 024334 (2019)
published on 29 August

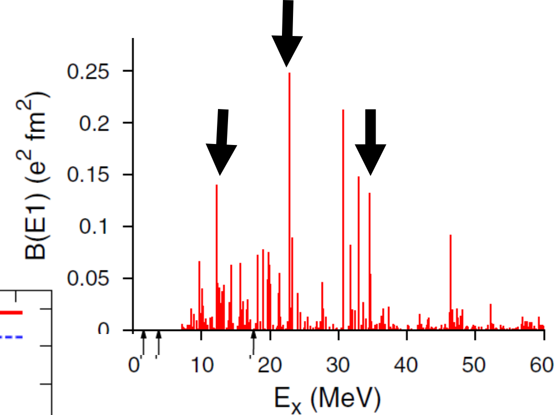


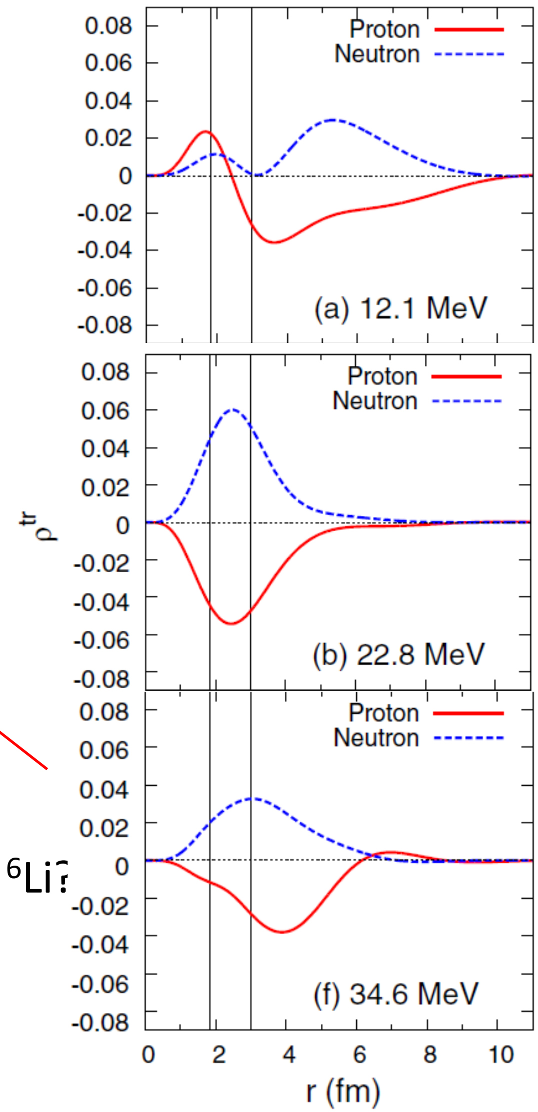
First peak: Giant dipole resonance (GDR) of ^{6}Li?

Second peak: GDR of alpha cluster in ^{6}Li?

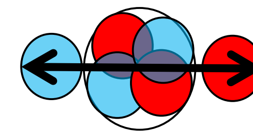
T. Yamagata, S. Nakayama, H. Akimune, S. Miyamoto, Phys. Rev. C 95, 044307 (2017)

Note: Only a one-peak structure is found in S. Bacca et al., PRL89, 052502 (2002)



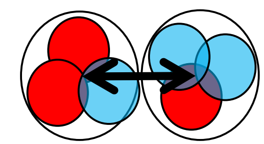


Soft GT excitation



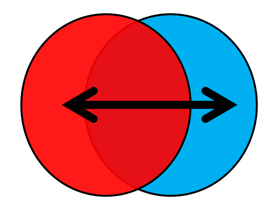
$S_{\alpha pn}^2 \sim 1$

Cluster (^{3}He+^{3}H) exc.



$S_{ht}^2 \sim 0.85$

Typical GT excitation



$S_{\alpha pn}^2 \sim 0.2$

$S_{ht}^2 \sim 0$

Extension to lepton-nucleus scattering

^{4}He (e,e') ^{4}He* ($J^\pi T=0^+0$, $E_R=20.1\pm0.05$ MeV, $\Gamma=270\pm50$ keV)

→ isoscalar monopole transition

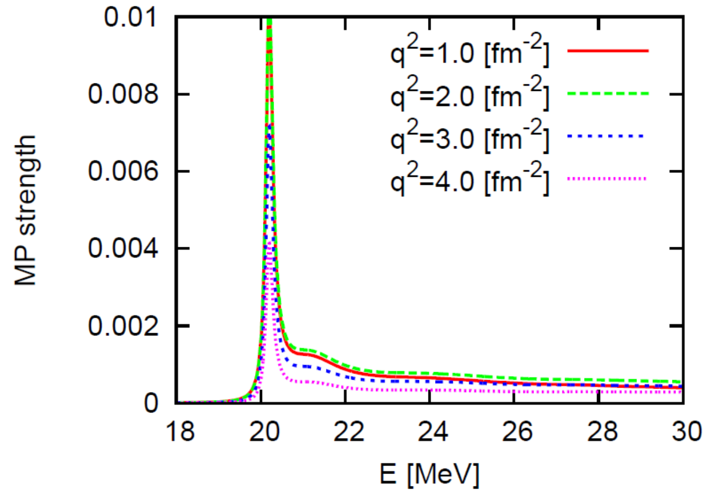
$$\mathcal{M}(q) = \frac{1}{2}\sum_{i=1}^{A} j_0(qr_i)$$ q: momentum transfer [fm^{-1}]

$$S(q,E) = S_{\rm res}(q,E) + S_{\rm bg}(q,E)$$

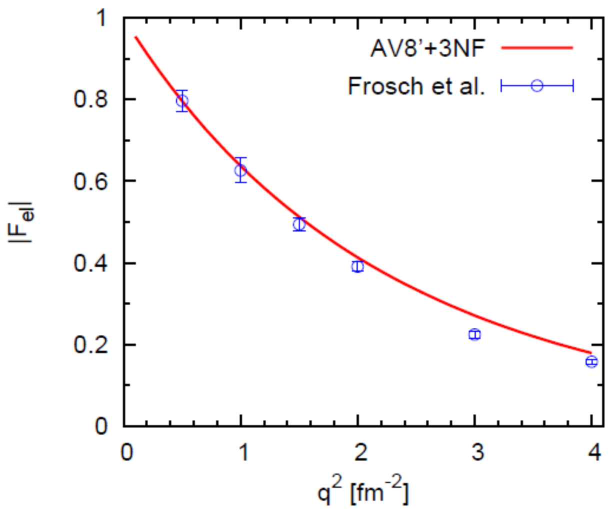
$$F_{\rm el}(q) = \frac{G_E^p(q)}{Z}\langle\Psi_0|\,\mathcal{M}(q)\,|\Psi_0\rangle$$

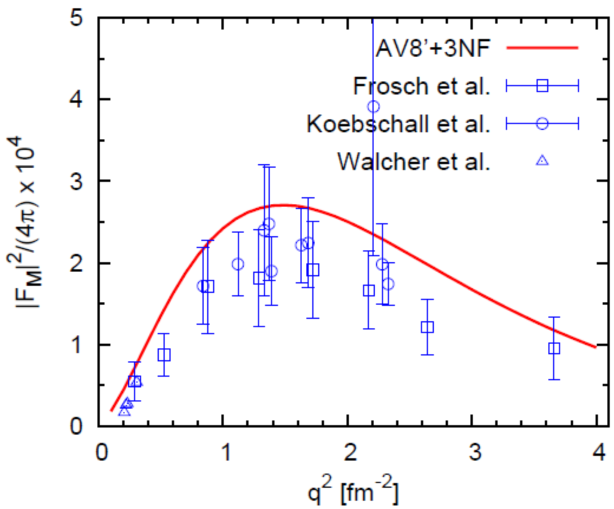
$$|F_{\rm inel}(q)|^2 = \left(\frac{G_E^p(q)}{Z}\right)^2 \int dE\, S_{\rm res}(q,E)$$

Isoscalar monopole strength functions



Theory reproduces E. Hiyama et al. PRC70, 031001(R) (2002)





Description with high-q

Extension to various energy and momentum transfer regions desired

However, for describing high-momentum transfer reaction, high-angular momentum states contribute

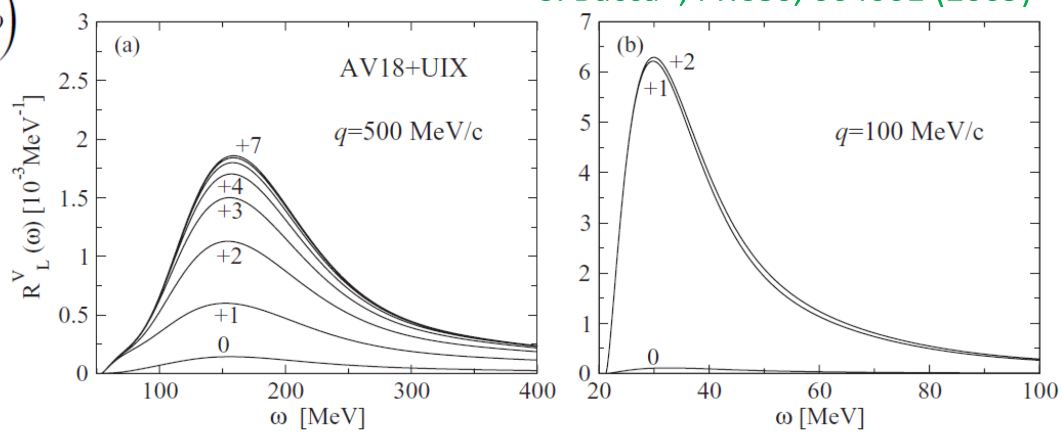
S. Bacca+, PRC80, 064001 (2009)

$$R_L(\omega, q) = \sum_f |\langle\Psi_f|\hat{\rho}(\mathbf{q})|\Psi_0\rangle|^2 \delta\left(E_f + \frac{q^2}{2M} - E_0 - \omega\right)$$

$$\hat{\rho}(\mathbf{q}) = \frac{e}{2}\sum_k \exp[i\mathbf{q}\cdot\mathbf{r}_k] + \frac{e}{2}\sum_k \tau_k^3 \exp[i\mathbf{q}\cdot\mathbf{r}_k] \equiv \hat{\rho}_S(\mathbf{q}) + \hat{\rho}_V(\mathbf{q}).$$

$$\hat{\rho}_X(\mathbf{q}) = 4\pi \sum_{J\mu} \hat{C}_\mu^{J,X}(q)\, Y_\mu^J(\hat{q})^*$$

$$\hat{C}_\mu^{J,X}(q) \equiv \frac{1}{4\pi}\int d\hat{q}'\, \hat{\rho}_X(\mathbf{q}') Y_\mu^J(\hat{q}'),$$



- **Develop** an efficient method and operators for neutrino-nucleus reactions without explicit angular momentum decomposition

Space-grid representation of the correlated Gaussian basis

$$g(\boldsymbol{s}; A, \boldsymbol{x}) = \exp\left(-\tfrac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x} + \tilde{\boldsymbol{s}}\boldsymbol{x}\right)$$

***s* is represented on a space-grid**

"Standard" angular momentum projection

$$F_{(L_1L_2)LM}(u_1, u_2, A, \boldsymbol{x}) = \frac{B_{L_1}B_{L_2}}{L_1!L_2!}\iint d\boldsymbol{e}_1\, d\boldsymbol{e}_2\, [Y_{L_1}(\boldsymbol{e}_1)Y_{L_2}(\boldsymbol{e}_2)]_{LM} \times \frac{\partial^{L_1+L_2}}{\partial\lambda_1^{L_1}\,\partial\lambda_2^{L_2}} g(\lambda_1\boldsymbol{e}_1u_1 + \lambda_2\boldsymbol{e}_2u_2; A, \boldsymbol{x})\Big|_{\lambda_1=\lambda_2=0},$$

- **Applications** to neutrino-nucleus reactions in wide momentum and energy transfers at quasi-elastic regions

Progress of our project in collaboration with T. Sato and K. Yoshida

- Develop electro-weak operators for neutrino-nucleus scattering (T. Sato)
  - Expression without angular momentum expansion for electron- and neutrino-nucleus reactions (completed)
  - Correct treatment of the electron Coulomb wave function (completed)
    - Behrens formula have been revised

H. Behrens and W. Bühring, *Electron Radial Wave Functions and Nuclear Beta-Decay* (Clarendon, Oxford, 1982).

- *Ab initio* calculations for light nuclei (WH)
  - Photoabsorption of ^{6}Li (completed) S. Satsuka and WH, Phys. Rev. C 100, 024334 (2019)
  - Formulation for new CG basis (completed)
  - Test calculation with electron-nucleus scattering (in progress)
  - Develop a code for neutrino-nucleus scattering (in progress)

- Weak transitions in medium- to heavy-mass nuclei with nuclear density functional theory (K. Yoshida)
  - Weak transitions for medium mass nuclei (completed)

    K. Yoshida, Phys. Rev. C 100, 024316 (2019)
  - Beta decay with the exact electron wave function (in progress)
  - Develop a code for neutrino-nucleus scattering (in progress)

Correlated basis approach: global vector representation

Correlated Gaussian combined with two global vectors Y. Suzuki, W.H., M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$\phi^{\pi}_{(L_1L_2)LM_L}(A, u_1, u_2) = \exp(-\tilde{x}Ax)[\mathcal{Y}_{L_1}(\tilde{u}_1 x)\mathcal{Y}_{L_2}(\tilde{u}_2 x)]_{LM_L}$$

x: any relative coordinates (cf. Jacobi) $\mathcal{Y}_\ell(\boldsymbol{r}) = r^\ell Y_\ell(\hat{\boldsymbol{r}})$

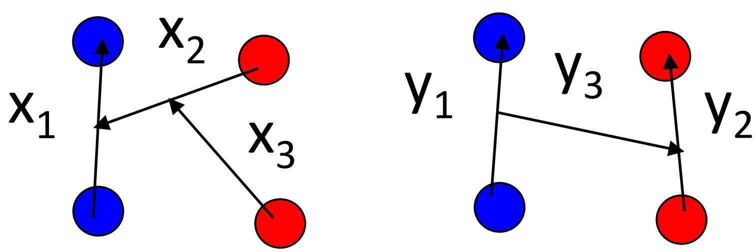
$$\tilde{x}Ax = \sum_{i,j=1}^{N-1} A_{ij} x_i \cdot x_j$$

$$\tilde{u}_i x = \sum_{k=1}^{N-1} (u_i)_k x_k$$

"bare" interaction can be used

Some advantages

- Formulation for *N* particle system
- Analytical expression for matrix elements
- The functional form does not change under any coordinate transformations



$$y = Tx \implies \tilde{y}By = \tilde{x}\tilde{T}BTx$$

$$\tilde{v}y = \widetilde{\tilde{T}v}x$$

Variational parameters *A*, *u* → Stochastic variational method

K. Varga and Y. Suzuki, PRC52, 2885 (1995).

Dipole type operators and spectrum

Electric dipole (E1) $J^{\pi}T=0^{+}0 \rightarrow 1^{-}1$

Photo nuclear reaction, radiative capture, etc

$$\mathcal{M}_{1\mu} = \sum_{i=1}^{4} \frac{e}{2}(1-\tau_{3_i})(\boldsymbol{r}_i - \boldsymbol{x}_4)_{\mu}$$

Spin-dipole (SD) $J^{\pi}T=0^{+}0 \rightarrow \lambda^{-}0, \lambda^{-}1$

Beta decay, Neutrino nucleus reaction, etc.

$$\mathcal{O}^{p}_{\lambda\mu} = \sum_{i=1}^{4} [(\boldsymbol{r}_i - \boldsymbol{x}_4) \times \boldsymbol{\sigma}_i]_{\lambda\mu}\, T_i^{p}$$

$$T_i^{\mathrm{IS}} = 1, \quad T_i^{\mathrm{IV0}} = \tau_z(i), \quad T_i^{\mathrm{IV}\pm} = t_{\pm}(i)$$

- ^{4}He: seven negative parity states

  J^{π} T= $0^{-}1$, $1^{-}1$, $2^{-}1$, $0^{-}0$, $1^{-}0$, $2^{-}0$

- Degenerate levels only with central force

  → Non-central force is necessary