Coupling Neutrino Oscillations and Simulations of Core-Collapse Supernovae

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Neutrinos in Supernova

- Neutrinos are incredibly important in core-collapse supernovae.
  - They are the transporters of energy/momentum and lepton number.

- In order to correctly compute how a supernova explodes, a great deal of attention and effort is directed to the neutrinos.
  - Neutrino transport is Hard: neutrinos are not everywhere in thermal equilibrium with the matter.

- Neutrinos are also the messengers which can tell us how the supernova explodes.
  - In 1987 we detected 20 neutrinos from a SN in the LMC which confirmed the basic paradigm of core-collapse supernovae.
  - If a SN occurs tomorrow in the Milky Way we will detect 10’s of thousands of neutrinos and be able to answer more detailed questions.
## The physics in supernova neutrinos

### Nuclear / Supernova
- Progenitor and structure,
- Neutrino opacities,
- Equation of State,
- Shock position / velocity,
- Standing Accretion Shock Instability,
- LESA
- Stalled shock duration,
- Nucleosynthesis conditions,
- ....

### Neutrinos
- Neutrino mass ordering
- Number of $\nu$ flavors
- Self-interaction effects,
- MSW effects,
- Turbulence effects
- Non-standard interactions,
- Magnetic moments,
- SUSY contribution,
- ....

- If you want to understand supernovae, you have to understand neutrinos.
Neutrino transport

- The generalized (6x6) neutrino density matrix $F$ at a given spacetime point for a given momentum evolves according to

$$i \frac{dF}{d\lambda} - \left[ H, F \right] = i C \left[ F \right]$$

- $H$ is the generalized Hamiltonian, $C$ the generalized collision term

  Sigl & Raffelt, Nuclear Physics B 406, 423 (1993)
  Volpe, Väänänen & Espinoza, PRD 87, 113010 (2013)
  Vlasenko, Fuller & Cirigliano, PRD 89 105004 (2014)
  Richers et al, PRD 99 123014 (2019)

- The diagonal elements of $F$ are the occupation numbers of the neutrino flavors, the off-diagonal are the coherences.
  - Classical supernova neutrino transport only treats the diagonal elements
The neutrino Hamiltonian is made up of several terms:

- the vacuum $H_V$ term,
- the matter potential $H_M$,
- the self-interaction $H_{SI}$.

The (3x3) vacuum term is

$$H_V = \frac{1}{2E} U_V \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U_V^\dagger$$

- $E$ is the neutrino energy, $m_1$, $m_2$ and $m_3$ are the neutrino masses.
- $U_V$ is the mixing matrix parameterized by three mixing angles $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$ and a CP phase $\delta$.

The antineutrino vacuum Hamiltonian is similar but $U_V \rightarrow U_V^\ast$. 
- In the presence of matter the neutrinos gain a potential energy.
- For mixing between active flavors we only need consider the Charged Current potential.
- For the neutrinos

\[
H_M = \begin{pmatrix}
V_{CC} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

\[
V_{CC} = \sqrt{2} G_F n_e
\]

- Not included is the small potential \( V_{\mu\tau} \).

- For the antineutrinos \( H_M \rightarrow - H_M^* \).

- Beyond the Standard Model physics can modify this matter term.

Stapleford et al, PRD 94 093007 (2016)
Esteban-Pretel et al, PRD 81 063003 (2010)
So many neutrinos are emitted in a supernova the Hamiltonian includes a term due to neutrino self-interactions.

At a given location and time, the self-interaction Hamiltonian due to the Standard Model V-A interaction is

\[ H_{SI}(q) = \sqrt{2} G_F \int \frac{d^3 q'}{(2\pi)^3} \left( 1 - \hat{q} \cdot \hat{q}' \right) \left( \rho(q') - \bar{\rho}^*(q') \right) \]

Beyond the Standard Model physics can modify this term too.

Blennow, Mirizzi & Serpico, PRD 78, 113004 (2008)

Das, Dighe & Sen, JCAP 5 051 (2017)

Yang & Kneller, PRD 97 103018 (2018)
The evolution of a single neutrino becomes dependent upon every other neutrino emitted even if they never meet!
The Bulb Model

- The current state of the art in neutrino flavor transformation calculations is based upon the Bulb Model.

  Duan et al PRL 97 241101 (2006)

  - The neutrinosphere is a hard surface with spherically symmetric neutrino emission (originally half isotropic).
  - No collisions or absorption/emission beyond the neutrinosphere.
  - The neutrino field is in steady state – the time derivative is zero.
  - The neutrino field has axial symmetry around the radial direction.

- This turns the neutrino transport into an initial-value problem.
- The imposed symmetries leave just two free variables:
  - The neutrino energy,
  - The angle of emission at the neutrinosphere.
Collective effects appear in the inverted mass ordering.

Wu et al, PRD 91, 065016 (2015)
Beyond ~1000 km, the flavor evolution is due to matter effects.

- MSW conversion plus shock effects and turbulence at later epochs.
  
  Kneller, McLaughlin & Brockman, PRD 77 045023 (2008)
  Kneller & Volpe, PRD 82 123 004-- (2010)
  Lund & Kneller, PRD 88 023008 (2013)
  Capozzi et al, JCAP 4 43 (2016)
  Patton, Kneller & McLaughlin, PRD 89 073022 (2014)

- MSW conversion and shock effects are easy to compute; turbulence effects require more work.
  - When the dynamic matter effects appear depends upon the progenitor: typically they appear after a few seconds postbounce.
In the Bulb Model oscillations do not occur during the accretion phase below the shock.

Any flavor transformation below the shock will have an effect upon the dynamics and the neutrino emission.
Beyond the Bulb model

- We need to go beyond the Bulb model:
  - Neutrinospheres are not hard surfaces
  - Collisions and emission are not negligible above the neutrinosphere (there are inwardly moving neutrinos even at large radii)
  - The symmetries can be spontaneously broken.
  - There is no feedback into the hydro.

- Hansen & Smirnov showed the finite thickness of the neutrino decoupling region suppresses the neutrino coherence.

  Hansen & Smirnov, JCAP, 10, 027 (2019)

  - The width of the neutrinosphere is 8-10 orders or magnitude larger than the oscillation length.
- Scattering can produce a diffuse neutrino halo.
  
  Cherry et al, PRL 108 261104 (2012)
  Zaizen et al, arXiv:1908.10594

- The self-interaction from unscattered neutrinos scales as $\sim 1/r^4$ because of the $1-\cos \theta$ term in the potential.

- But the same high density which creates the halo also suppress the flavor oscillations.

  Sarikas et al, PRD, 85 113007 (2012)
Fast Flavor Oscillations (FFO) are due to differences in the angular distribution of the neutrinos versus antineutrinos

Sawyer, PRD 72, 045003 (2005),
Mirizzi & Serpico, PRL 108, 231102 (2012)
Saviano et al, PRD 85, 113002 (2012),
Izaguirre, Raffelt & Tamborra, PRL 118, 021101 (2017)
Azari et al, PRD 99, 103011 (2019)

A study by Tamborra et al of a 1D simulation did not find the conditions for FFO.

A recent study by Abbar et al examined 2D and 3D simulations and found locations and times where FFO could occur.

Abbar et al, PRD, 100 043004 (2019)
See also Nagakura et al, arXiv:1910.04288

Even more recently the conditions for FFO were found in 1D.

Morinaga et al arXiv:1909.13131
- But also see Lucas et al, arXiv:1910.05682
Various arguments suggested that dense matter and collisions suppress flavor transformation below the shock.

Studies took snapshots from simulations computed without oscillations and then postprocessed them through flavor Bulb Model based transformation codes.

Chakraborty et al, PRL 107, 151101 (2011)
Dasgupta, O'Connor & Ott, PRD 85, 065008 (2012)
Efforts to solve the QKEs in supernovae are in the early stages.

Capozzi et al, PRL 122 091101 (2019)
Richers et al, PRD 99 123014 (2019)

Solving the QKEs in a supernova will be is super HARD
- The spatial resolution will need to be of order $\mu$m, not km.

As a first step we coupled a free-streaming neutrino oscillation code with the hydro.

Stapleford, Fröhlich & Kneller arXiv:1910.04172

The hydro is Agile-BOLTZTRAN, the oscillation code is SQA.
Agile-BOLTZTRAN

- Agile-BOLTZTRAN is a GR 1D Lagrangian hydro code with Boltzmann neutrino transport.


- The transport solves an implicitly finite differenced $O(v/c)$ Boltzmann equation for four neutrino flavors (e, x, e, x).

- We modified the transport so that it solves pairs of flavors (e and x, $\bar{e}$ and $\bar{x}$) together rather than separately.
SQA

- SQA is a free-streaming neutrino oscillation code for 6 flavors.
- It assumes:
  - spherically symmetric neutrino emission.
  - the neutrino field is in quasi steady-state.
  - the neutrino field has axial symmetry around the radial direction.

- We include the GR corrections.
  Yang & Kneller, PRD 96, 023009 (2017)

- We make use of the “Single Angle Approximation” to make the calculation feasible on a small cluster.
  - A multi-angle calculation would increase the run-time by ~1000.
• SQA computes the neutrino transition probabilities between two points in space.

• The density matrix $F$ at some spacetime location $\lambda$ is related to the initial state at $\lambda_0$ by a unitary matrix $S$.

$$F(\lambda) = S(\lambda, \lambda_0) F(\lambda_0) S^\dagger(\lambda, \lambda_0)$$

• The matrix $S$ evolves according to the Schrödinger equation

$$i \frac{dS}{d\lambda} = HS$$

• The probability that an initial state $j$ is detected as state $i$ at $\lambda$ is

$$P(\nu_j \rightarrow \nu_i) \equiv P_{ij} = \left| S_{ij} \right|^2$$
Agile-BOLTZTRAN-SQA

- The time-dependence of the simulation is controlled by Agile-BOLTZTRAN.

- SQA is invoked periodically: it’s inputs are
  - The density profile, electron fraction, enclosed gravitational mass,
  - The neutrino luminosities, mean energies, mean square energies
  - The neutrinosphere radius
  - The radii of the spatial grid

- SQA uses many more energies than Agile-BOLTZTRAN: SQA constructs a pinched spectrum for the neutrinos from the neutrino luminosities, mean and mean square energies.
• Flavor transformations begin 10 km above the neutrinosphere.
• SQA computes the transition probabilities across the spatial grid.
• The ‘initial’ density matrix for a given zone is the ‘final’ density matrix from the previous zone.
• The transition probabilities are fed back into BOLTZTRAN by converting to effective opacities for zone \( i \) and energy \( k \).

\[
\sigma_{i,k} = \frac{P_{i,k} c}{\Delta r_i}
\]

• The transport in BOLTZTRAN is modified by adding the terms

\[
\frac{df_{\alpha,i,k}}{dt} = -\sigma_{i,k} f_{\alpha,i,k} + \sigma_{i,k} f_{\beta,i,k}
\]

• The opacities are updated every 10 ms or when the zone dimensions due to regridding change by more than 10%.
The Effect of Oscillations

- We test the code on the 20 M⊙ Woosley & Heger progenitor.
  
  
  - This is the same progenitor used in the recent code 1D comparison paper by O’Connor et al.
  
  O’Connor et al, JPG **45**, 104001(2018)

- We use 20 energy groups and 8 angle bins for the neutrino transport, 192 spatial zones.

- In SQA the energy groups are subdivided so that we have ≤ 0.5 MeV resolution below 50 MeV, ≤ 1 MeV below 100 MeV

- We adopt an Inverted Mass Ordering (IMO) and the PDG values for the neutrino masses and mixing angles.
  
  - Single-angle calculations for the IMO are qualitatively similar to multi-angle: Normal Mass Ordering calculations are different
  
  Horiuchi & Kneller JPG **45**, 043002 (2018)
- We find ~2% changes in the mean energy and luminosities in the region just below the shock.
The net effect is slightly less heating ~1% at early times and ~4% extra heating in the gain layer after 300 ms.
The extra heating is not enough to cause an explosion.

The shock is slightly behind the simulation without oscillations.
  - The decrease heating at early times is not compensated later.
Summary

- A full quantum description of neutrinos in supernovae is now necessary and it will be a challenging problem.
- We have taken a first step in this direction.

- We saw \( \sim 4\% \) changes in the neutrino heating in a simulation of 20 \( M_\odot \) progenitor.
- This was not enough to change the outcome of a 1D simulation.
  - This matches the conclusion reached in previous, post-processed studies
  - The effect may be larger in multi-D simulations.