Mass and Radius of Neutron Stars and Supernova Neutrinos

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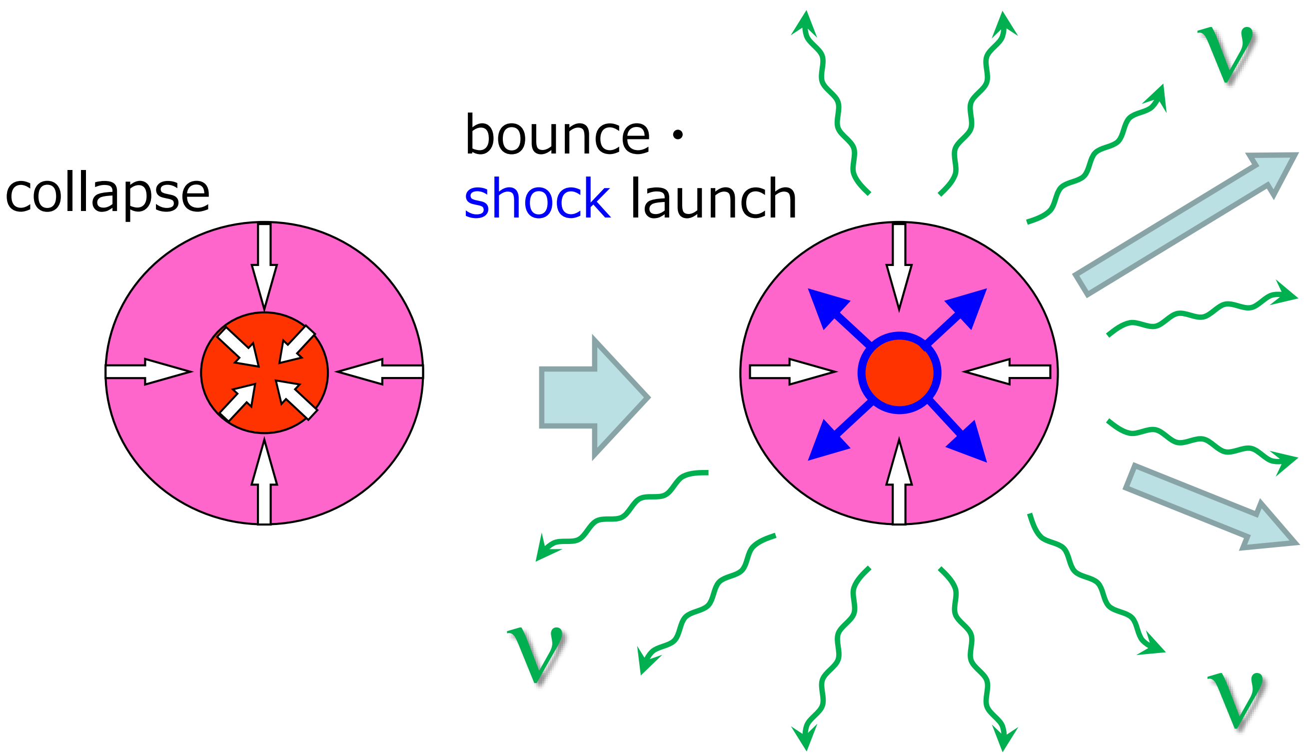
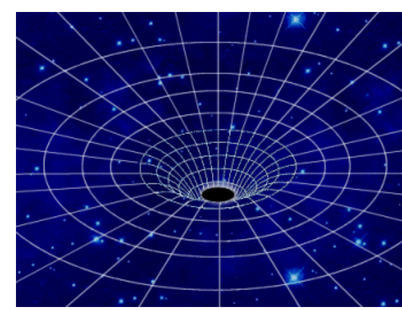
Core-collapse supernova

- Explosion caused by the death of massive star with $\gtrsim 10M_{\odot}$.
  - a large amount of ν emission
  - formation of NS or BH

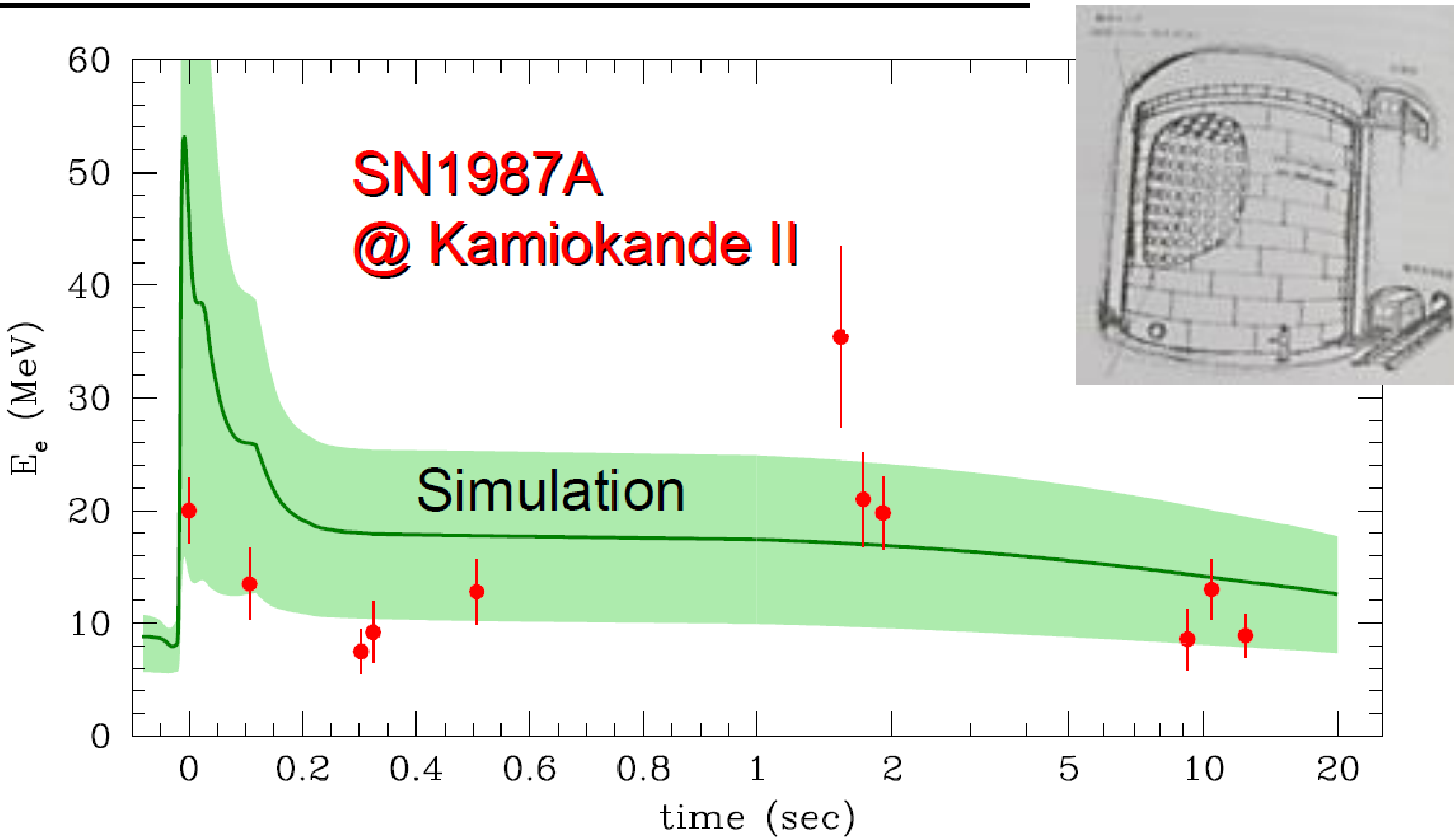


SN explosion → neutron star (NS)



black hole (BH)

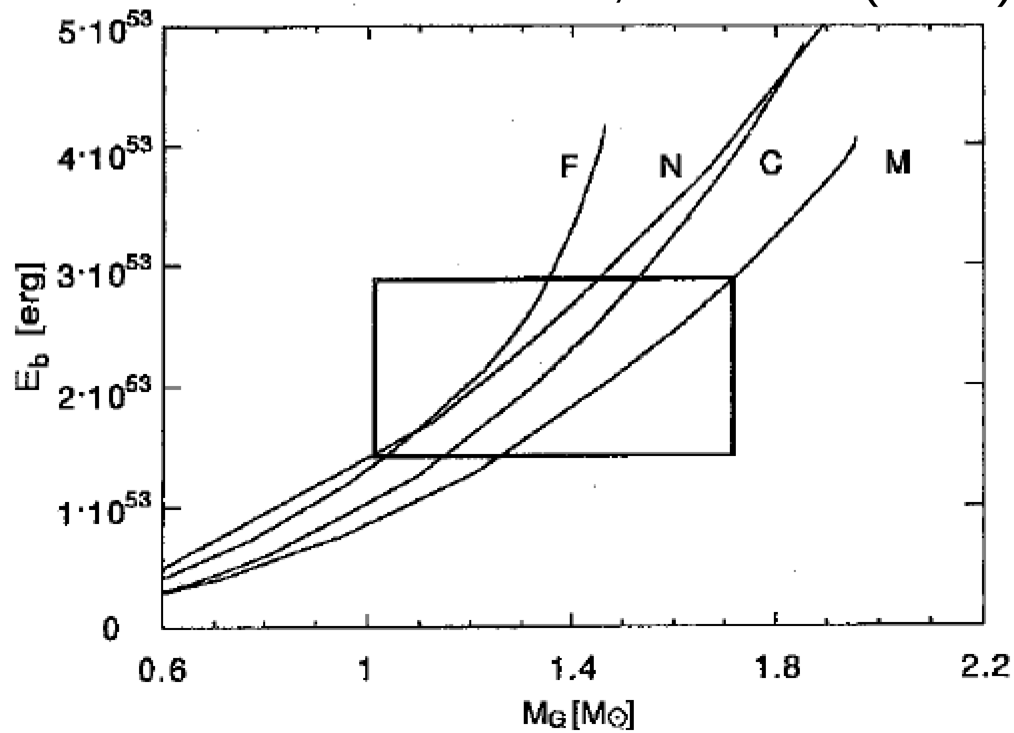
Neutrinos from SN1987A



- The standard scenario is confirmed from event number, energy and duration.

Diagnosing the NS mass from SN neutrinos

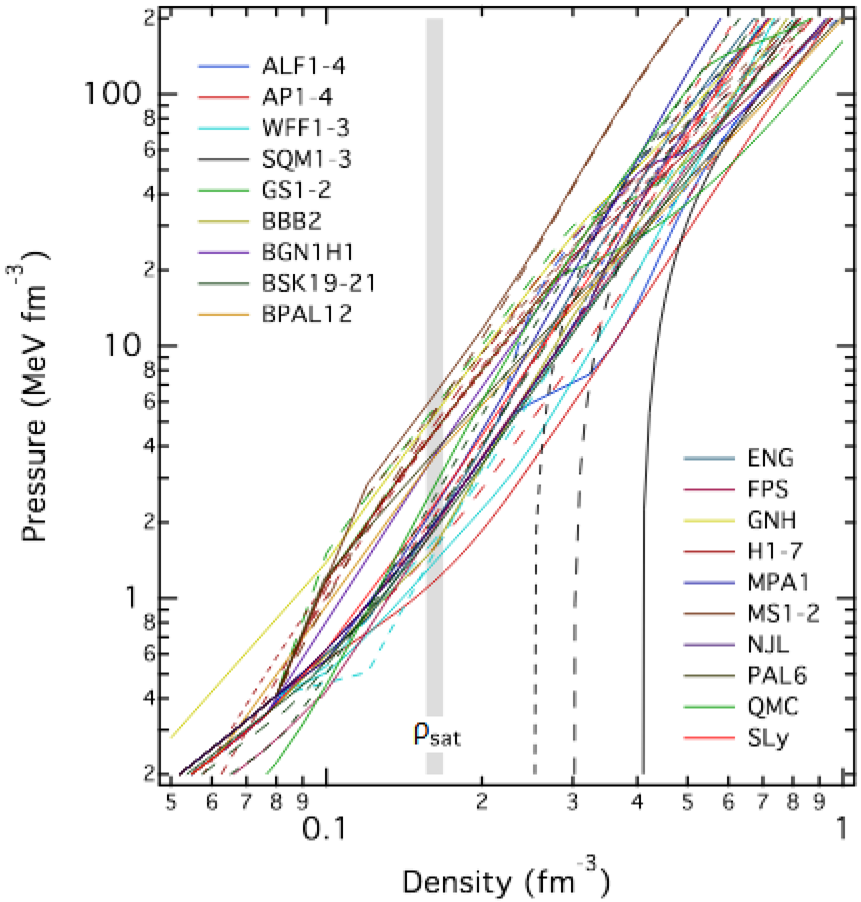
Sato & Suzuki, PLB **196** (1987)

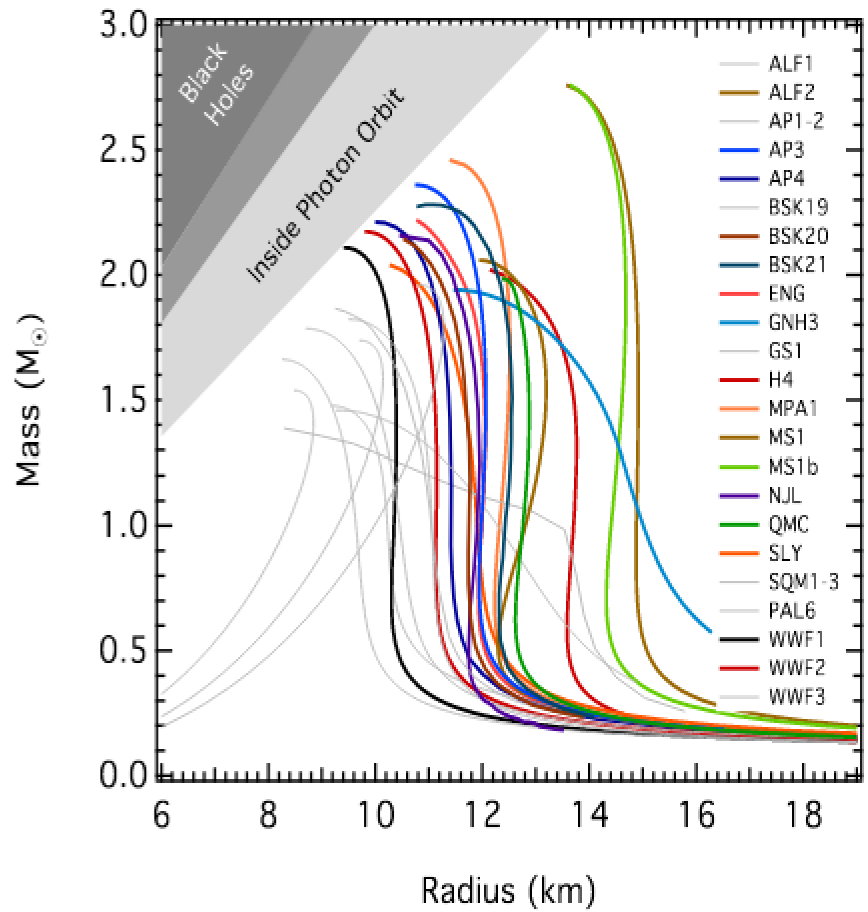


- total energy of emitted neutrinos
  = binding energy of NS
- Estimation of NS mass is possible using the neutrino observations of SN 1987A.
- For this, the equation of state (EOS) of nuclear matter is needed.

Mass-radius relation of NSs

Özel & Freire, ARAA **54** (2016)

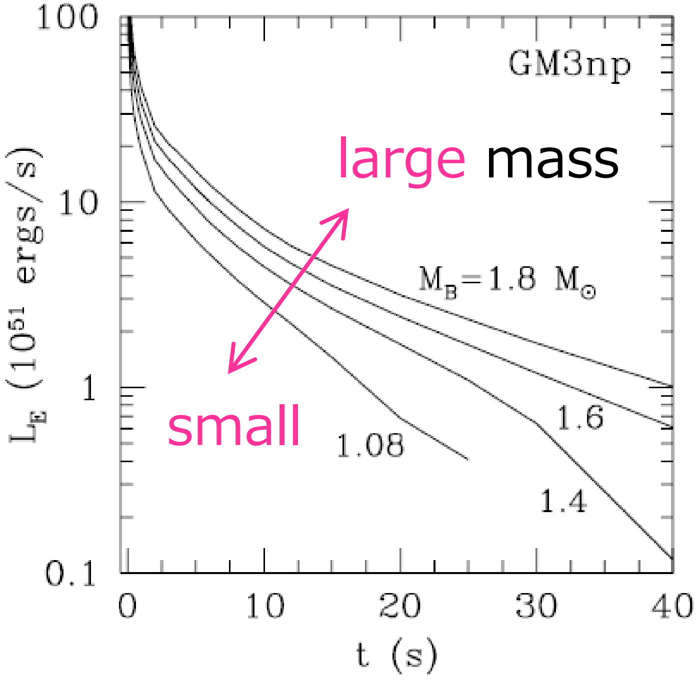




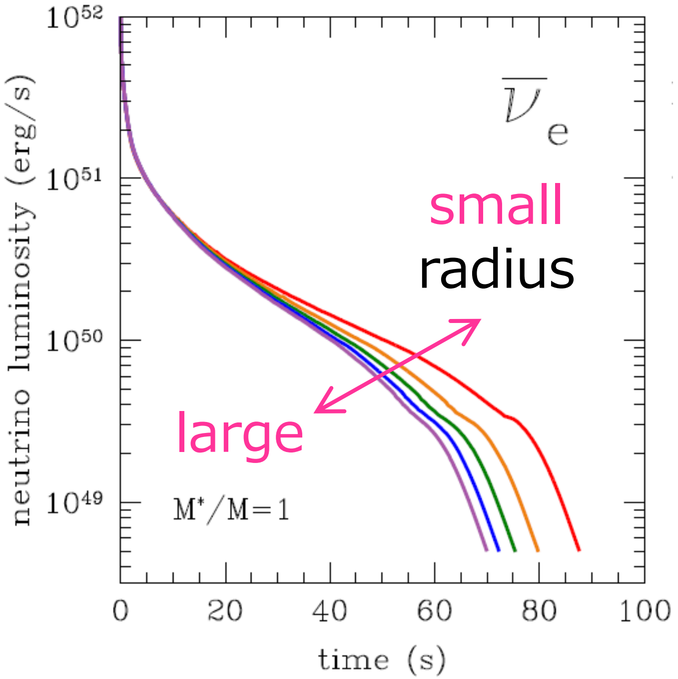
- The EOS of nuclear matter determines the mass and radius of NSs.

Cooling of proto-neutron stars

Pons et al., ApJ **513** (1999)

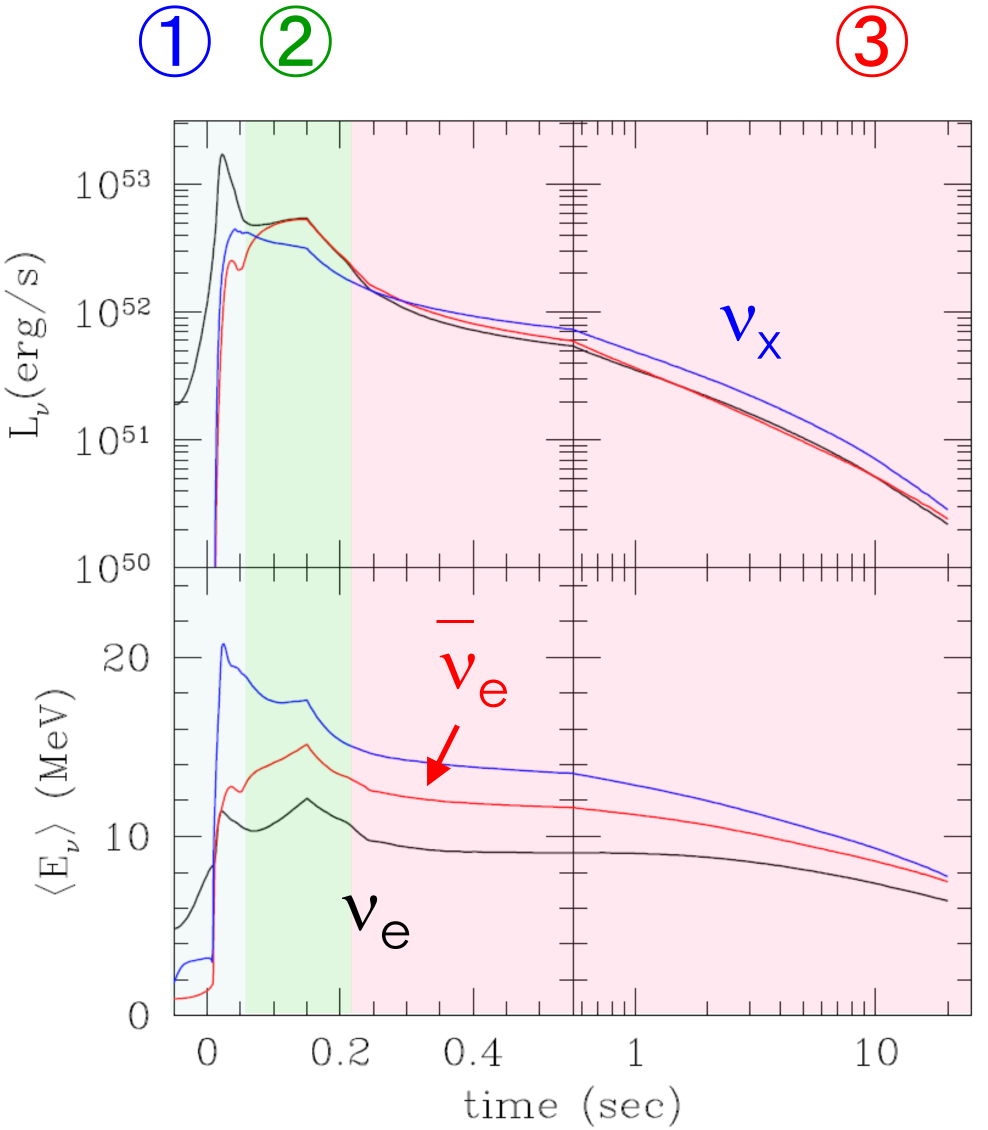


Nakazato & Suzuki, ApJ **878** (2019)



- Proto-neutron star (PNS) is a hot compact object born just after the SN explosion.
- The cooling time scale of PNS depends on the mass and radius.

Three phases of neutrino emission



Nakazato et al., ApJS **205** (2013)

① neutronization burst

~ *O* (10 ms)

② accretion phase

~ *O* (100 ms)

③ cooling phase

~ *O* (10 sec)

target of this study

Formula of PNS cooling timescale

- Kelvin-Hermholtz timescale

cooling timescale → $\tau_{\mathrm{KH}} = \dfrac{|E_g|}{L}$ ← gravitational energy, ← luminosity

- For NS mass m and radius r, we assume:
  1. luminosity scales with surface area : $L \propto r^2$
  2. time dilation in general relativity
  3. $|E_g| \to E_b$ (binding energy of NSs)

$$\tau_{\mathrm{cool}} \propto \frac{E_b}{r^2\sqrt{1 - {}^{2Gm}/_{rc^2}}}$$

Binding energy of NS as a function of mass & radius

Lattimer & Prakash, ApJ **550** (2001)

- For a large class of EOSs, the following is approximately satisfied:

$$\frac{E_b}{mc^2} = \frac{0.6 \times {Gm}/{rc^2}}{1 - 0.5 \times {Gm}/{rc^2}}$$

m: NS mass

r: NS radius

E_b: Binding energy of NS

$$\Rightarrow \tau_{\text{cool}} \propto \left(\frac{m}{1.4 M_\odot}\right)^2 \left(\frac{r}{10\ \text{km}}\right)^{-3} \frac{1}{(1-0.5\beta)\sqrt{1-2\beta}}, \quad \beta = \frac{Gm}{rc^2}$$

Setup of numerical simulations

- Initial conditions are taken form the results of core-collapse simulations.
  - PNSs with baryon mass of 1.47, 1.62, 1.78$M_{\odot}$.

- Quasi-static evolutionary calculation of PNS
  - transfer of ν_e, $\bar{\nu}_e$, ν_μ ($= \nu_\tau = \bar{\nu}_\mu = \bar{\nu}_\tau$) is treated in Multigroup Flux Limited Diffusion scheme
  - $e^- + p \leftrightarrow n + \nu_e$, $e^+ + n \leftrightarrow p + \bar{\nu}_e$, $\nu + N \leftrightarrow \nu + N$, $\nu + e \leftrightarrow \nu + e$, $\nu_e + A \leftrightarrow A' + e^-$, $\nu + A \leftrightarrow \nu + A$, $e^- + e^+ \leftrightarrow \nu + \bar{\nu}$, $\gamma^* \leftrightarrow \nu + \bar{\nu}$, $N + N' \leftrightarrow N + N' + \nu + \bar{\nu}$

- A series of phenomenological EOSs is used.

Phenomenological EOS model

- Zero temperature EOS.

Nakazato & Suzuki, ApJ **878** (2019)

$$w(n_b, Y_p) = w_0 + \frac{K_0}{18n_0^2}(n_b - n_0)^2 + S(n_b)(1 - 2Y_p)^2,$$

proton fraction

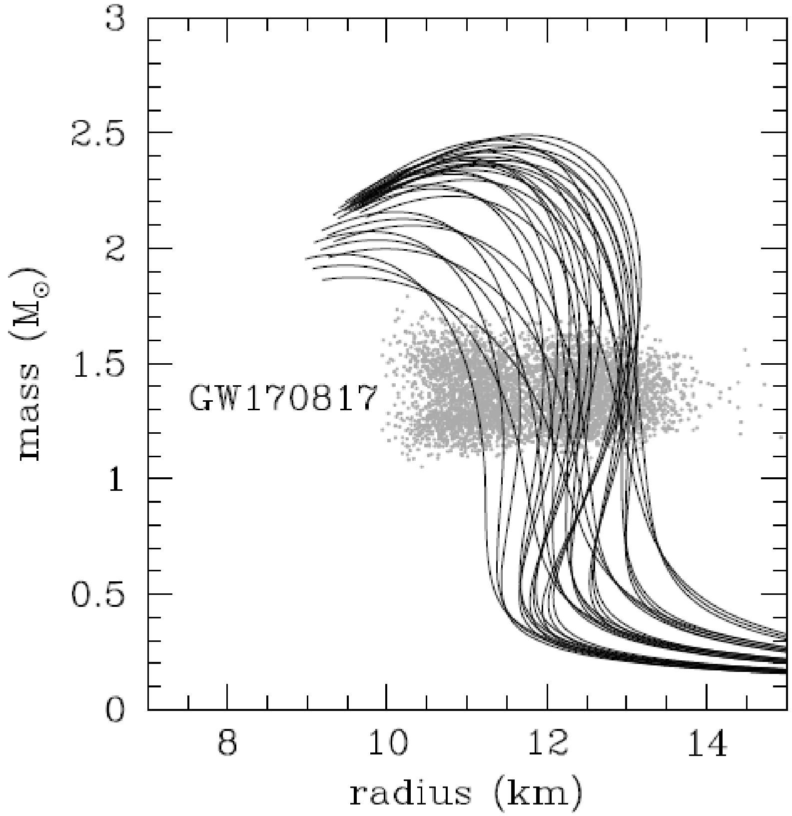
symmetry energy

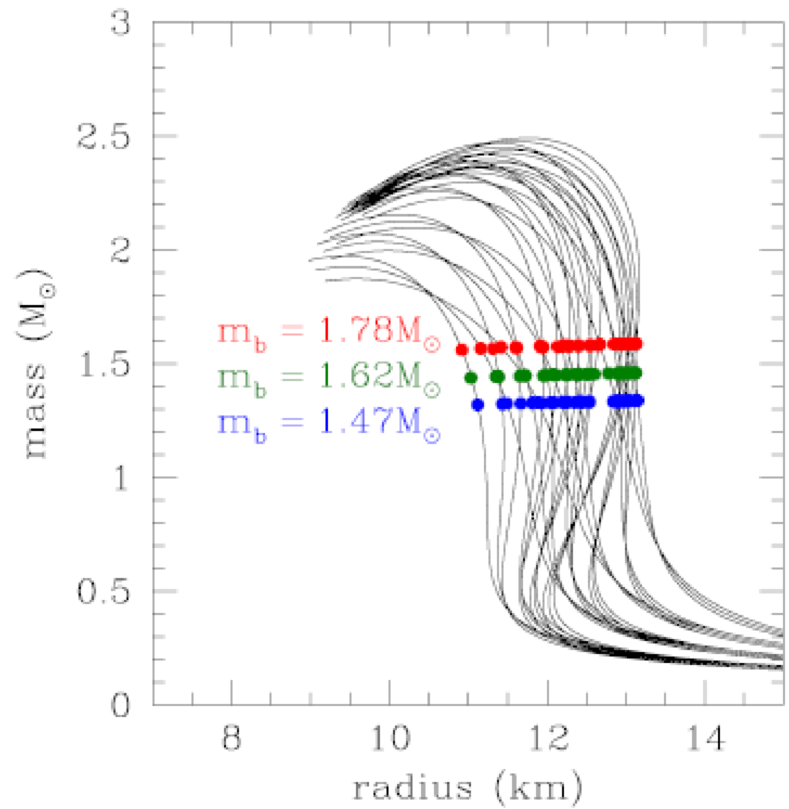
$$S(n_b) = S_0 + \frac{L}{3n_0}(n_b - n_0) + \frac{1}{n_0^2}\left(S_{00} - S_0 - \frac{L}{3}\right)(n_b - n_0)^2,$$

symmetry energy at $n_b = 2n_0$

- We choose
  - $K_0 = 220, 245, 270$ MeV
  - $(S_0, L, S_{00}) = (30, 35, 35), (30, 35, 40), (30, 35, 45), (30, 35, 55), (31, 50, 40), (31, 50, 45), (31, 50, 55), (32, 65, 45), (32, 65, 55), (33, 80, 55)$ in MeV unit.

Mass-radius relations of our EOS





- Consistent with GW170817.
- Cooling simulations of PNS with gravitational mass of 1.3－1.6$M_\odot$ and radii of 11－13 km.

Finite temperature effects

- Utilizing expressions of ideal Fermi gas.

$$F_b(n_b, Y_p, T) = \frac{1}{n_b}\Big[\varepsilon_b^{(0)}(n_b, Y_p) + \varepsilon_n^{\rm F}(n_n, T; M_n^*) - \varepsilon_n^{\rm F}(n_n, 0; M_n^*)$$
$$+ \varepsilon_p^{\rm F}(n_p, T; M_p^*) - \varepsilon_p^{\rm F}(n_p, 0; M_p^*)\Big] - T s_b(n_b, Y_p, T),$$

$$\varepsilon_b^{(0)} = n_b w$$

internal energy of ideal Fermi gas

$$s_b(n_b, Y_p, T) = (1 - Y_p) s_n^{\rm F}(n_n, T; M_n^*) + Y_p s_p^{\rm F}(n_p, T; M_p^*),$$

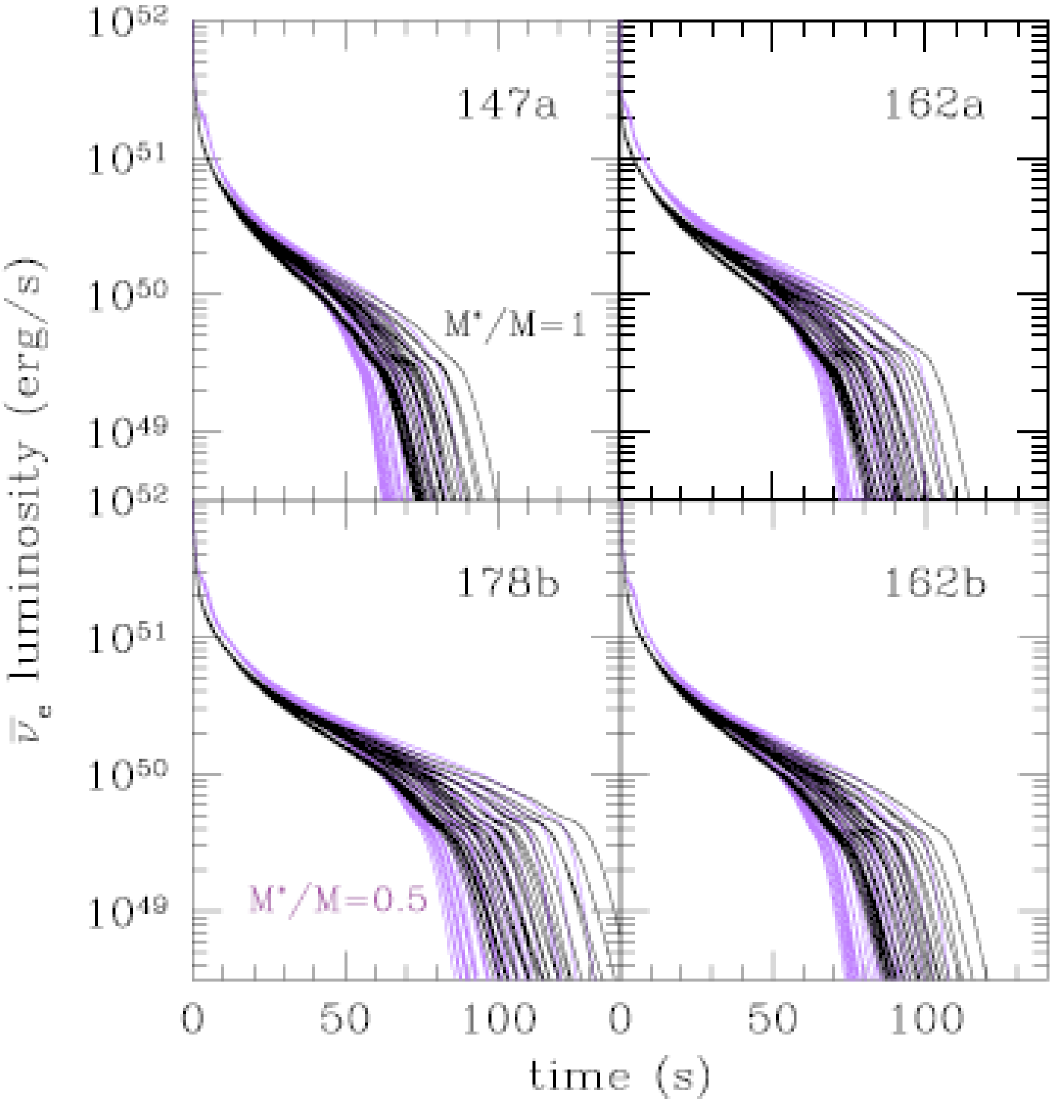
entropy of ideal Fermi gas

effective mass $M_n^*/M_n = M_p^*/M_p \equiv u$

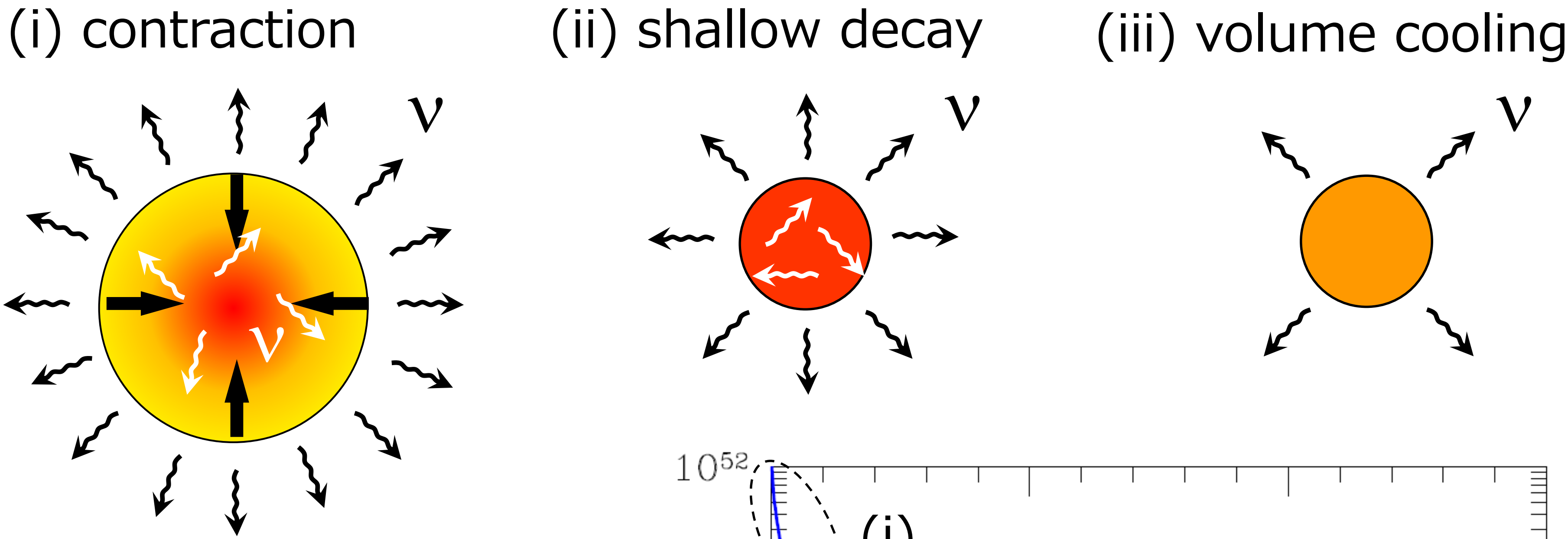
- We choose $u = 0.5, 1.$

Summary of models and results

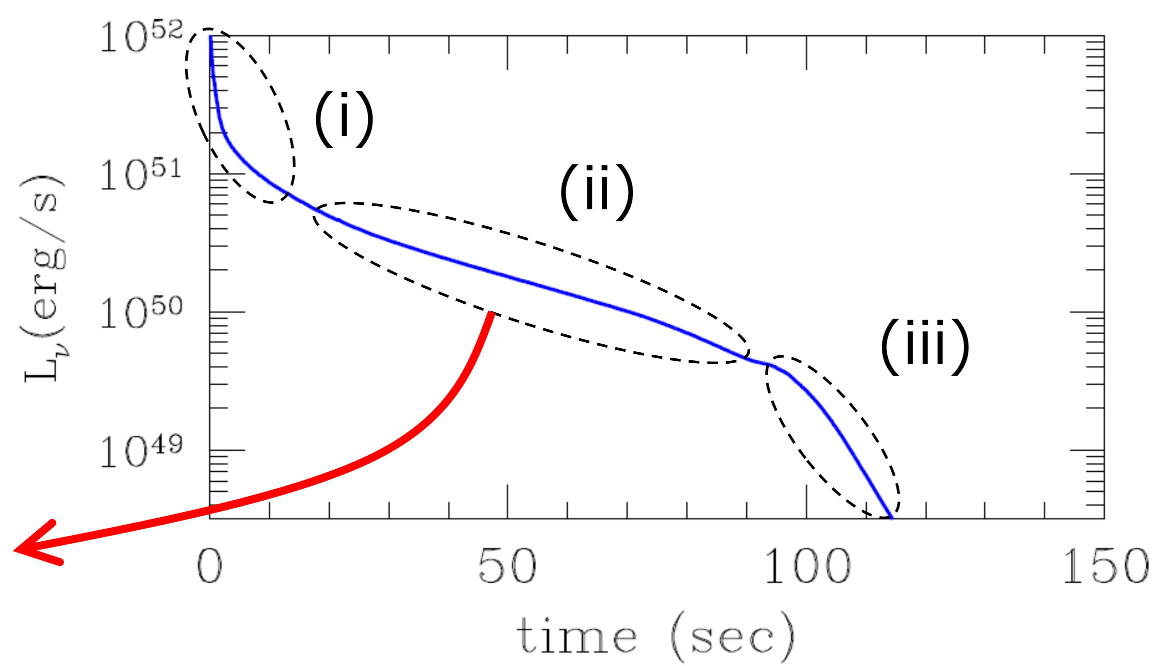
- 4 models of PNS.
  - 2 cases of the initial conditions for baryon mass $1.62M_{\odot}$ model.
- 30 models of zero temperature EOS.
- 2 choices of the effective mass.
- 240 runs in total.



Schematic picture of PNS cooling



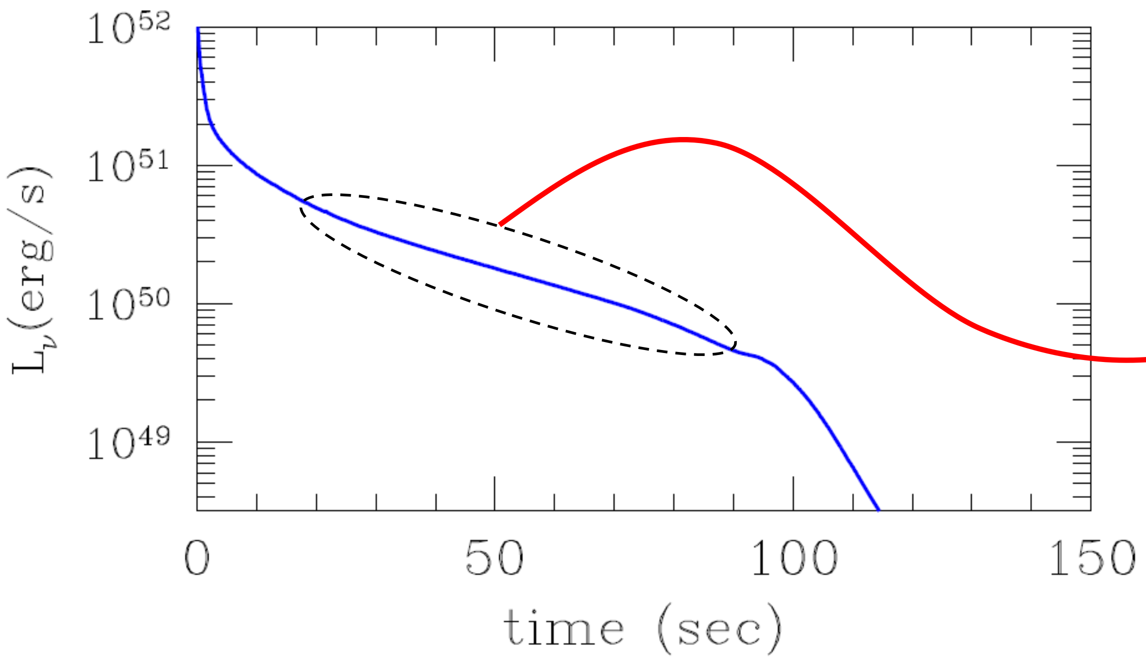
- Decay time of neutrino light curve has the maximum here.



Evaluation of cooling timescale

- Cooling timescale of PNS is defined by the maximum e-folding time of the neutrino light curve for each model.

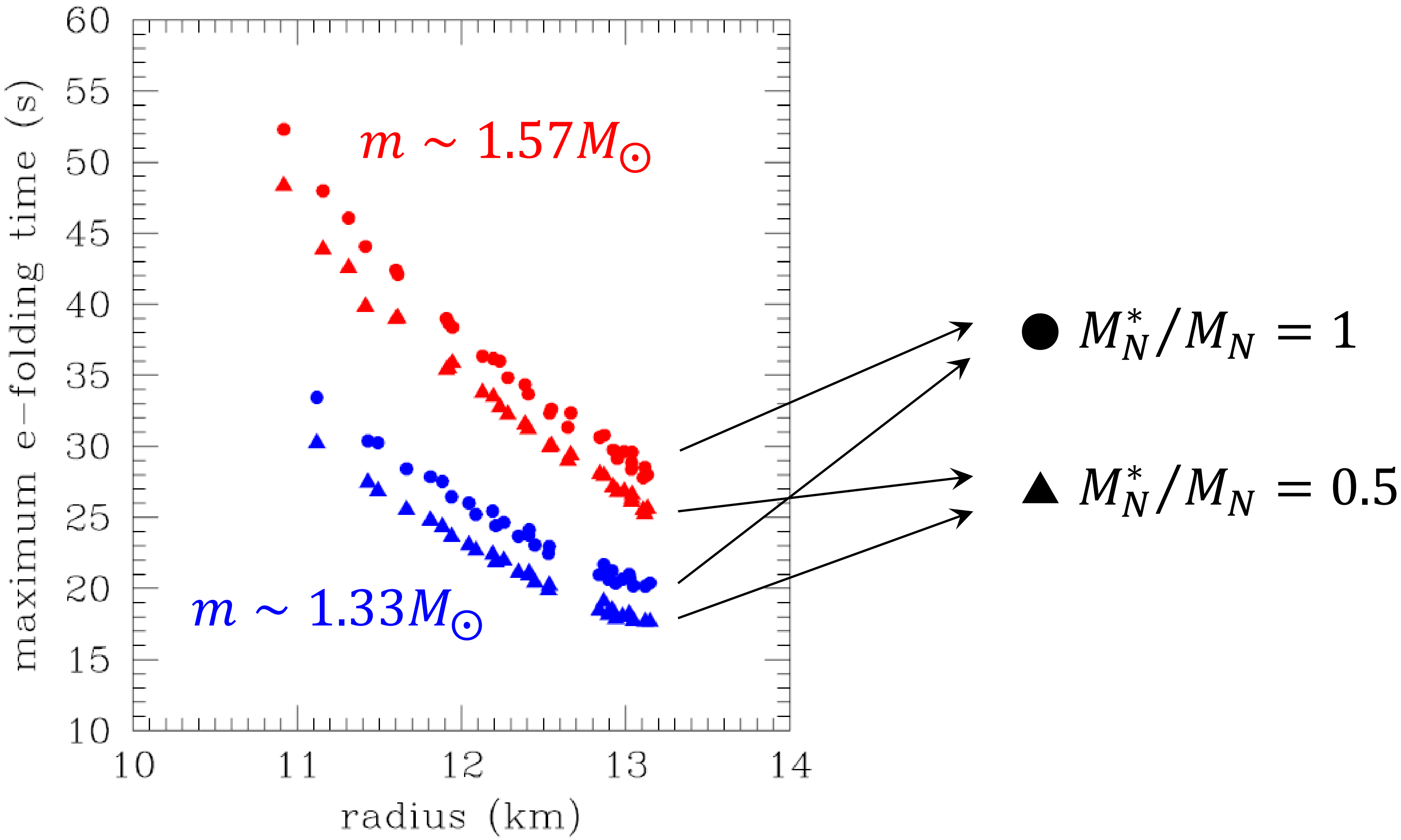
$$L_{\bar{\nu}_e}(t+\tau_{\bar{\nu}_e}) = \frac{L_{\bar{\nu}_e}(t)}{e}, \qquad \tau_{\rm cool} = \max_t \tau_{\bar{\nu}_e}$$



$$L_{\bar{\nu}_e}(t) \sim L_0 \exp\left(-\frac{t}{\tau_{\rm cool}}\right)$$

Dependence of cooling timescale

- Cooling timescale of PNS depends on mass-radius of NS and effective mass of EOS.

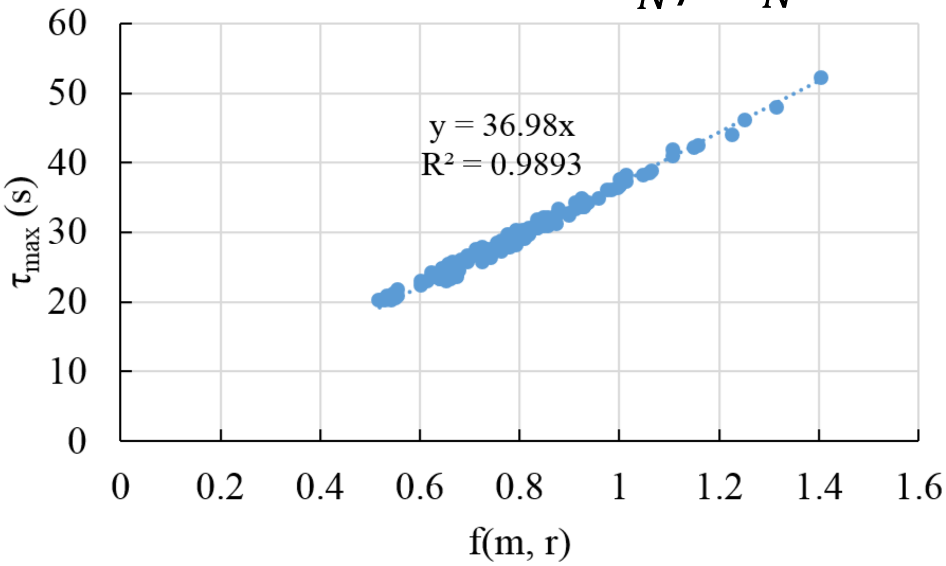


Formula vs. simulation results

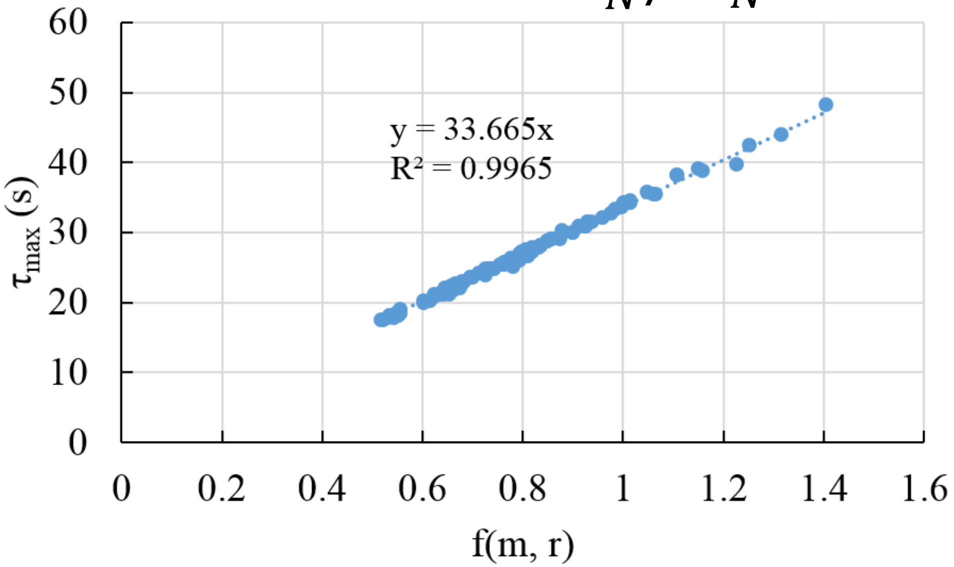
$$\tau_{\rm cool} = \tau^* \underbrace{\left(\frac{m}{1.4M_\odot}\right)^2 \left(\frac{r}{10\ \rm km}\right)^{-3} \frac{1}{(1-0.5\beta)\sqrt{1-2\beta}}}_{f(m,r)}, \quad \beta = \frac{Gm}{rc^2}$$

- ✓ Theory describes simulation results faithfully.
- ✓ $33.7\ s \le \tau^* \le 37.0\ s$ (depends on effective mass)

effective mass: $M_N^*/M_N = 1$



effective mass: $M_N^*/M_N = 0.5$



Estimation of NS mass & radius

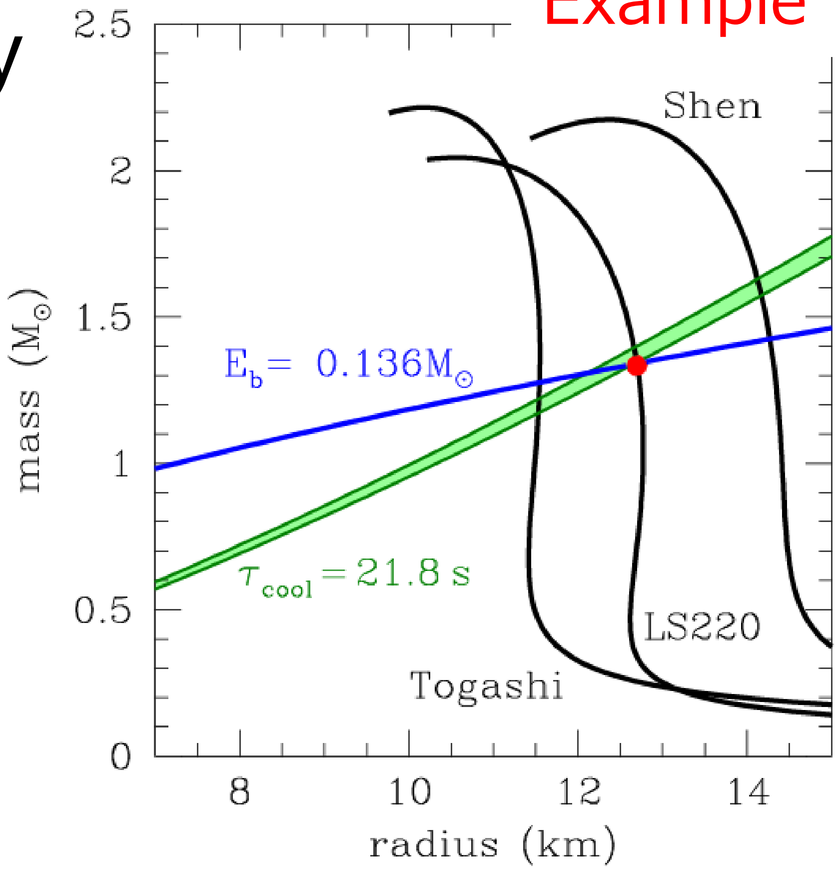
- Crossing point of neutrino cooling timescale

$$\tau_{\rm cool} = \tau^* \left(\frac{m}{1.4 M_\odot}\right)^2 \left(\frac{r}{10\ {\rm km}}\right)^{-3} \frac{1}{(1-0.5\beta)\sqrt{1-2\beta}}$$

and total emission energy

$$\frac{E_b}{mc^2} = \frac{0.6\beta}{1-0.5\beta} \qquad \left(\beta = \frac{Gm}{rc^2}\right)$$

Example



- Numerical results with realistic EOSs also follow these trends.

→ future EOS constraints

Summary

- The cooling timescale of PNS follows

$$\tau_{\rm cool} = \tau^* \left(\frac{m}{1.4 M_\odot}\right)^2 \left(\frac{r}{10\ \rm km}\right)^{-3} \frac{1}{(1-0.5\beta)\sqrt{1-2\beta}}, \quad \beta = \frac{Gm}{rc^2}$$

$$33.7\ s \le \tau^* \le 37.0\ s$$

from neutrino light curve.

- Combining the above and empirical relation of NS binding energy, we can estimate the NS mass & radius with neutrino observation.

→ Future neutrino observations will provide constraints on the nuclear EOS.

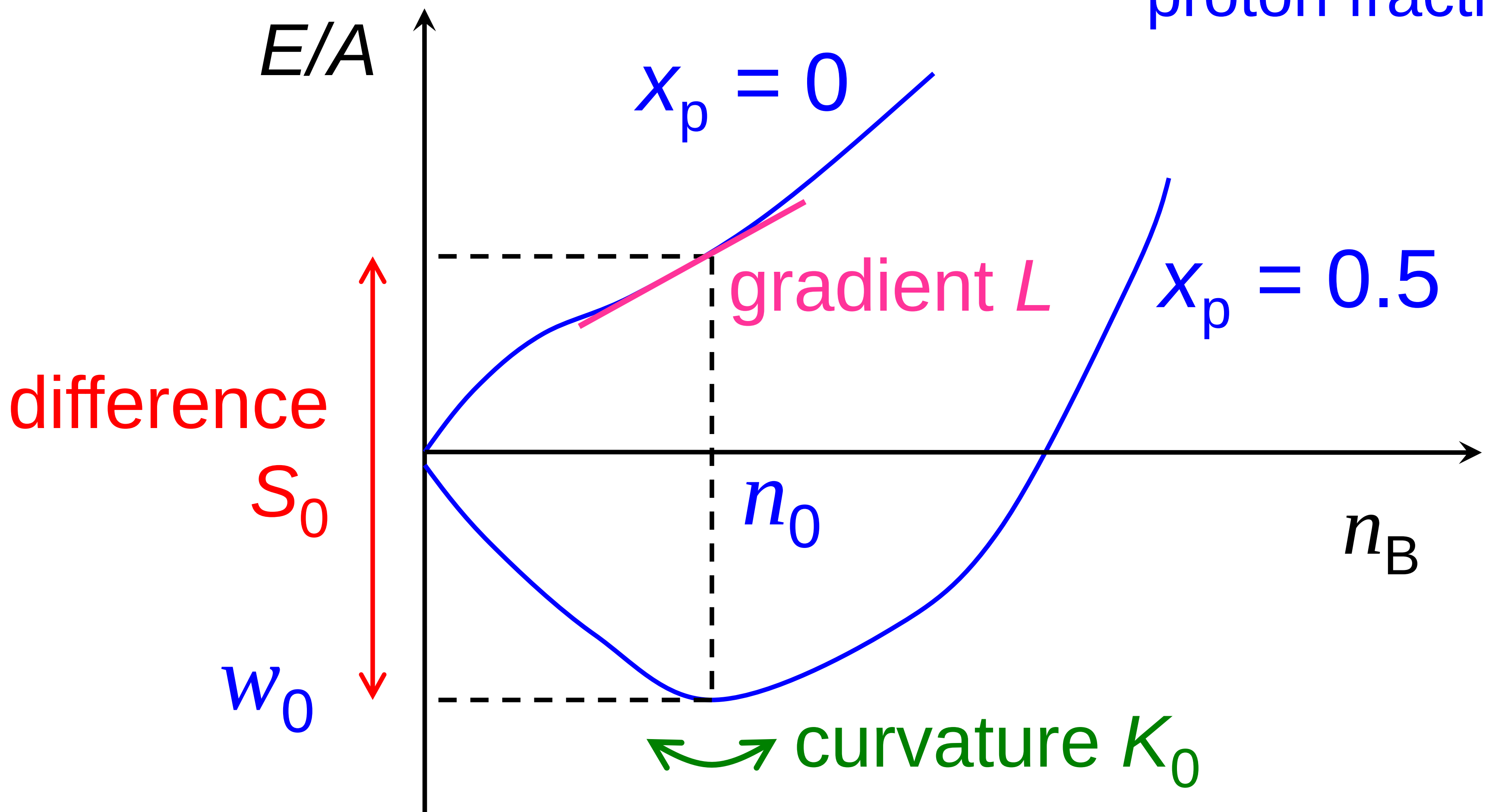
Thank you for your attention

EOS and saturation parameters

- Saturation parameters characterize EOS.

$$w = w_0 + \frac{K_0}{18n_0^2}(n - n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n - n_0)\right](1 - 2x_p)^2$$

proton fraction

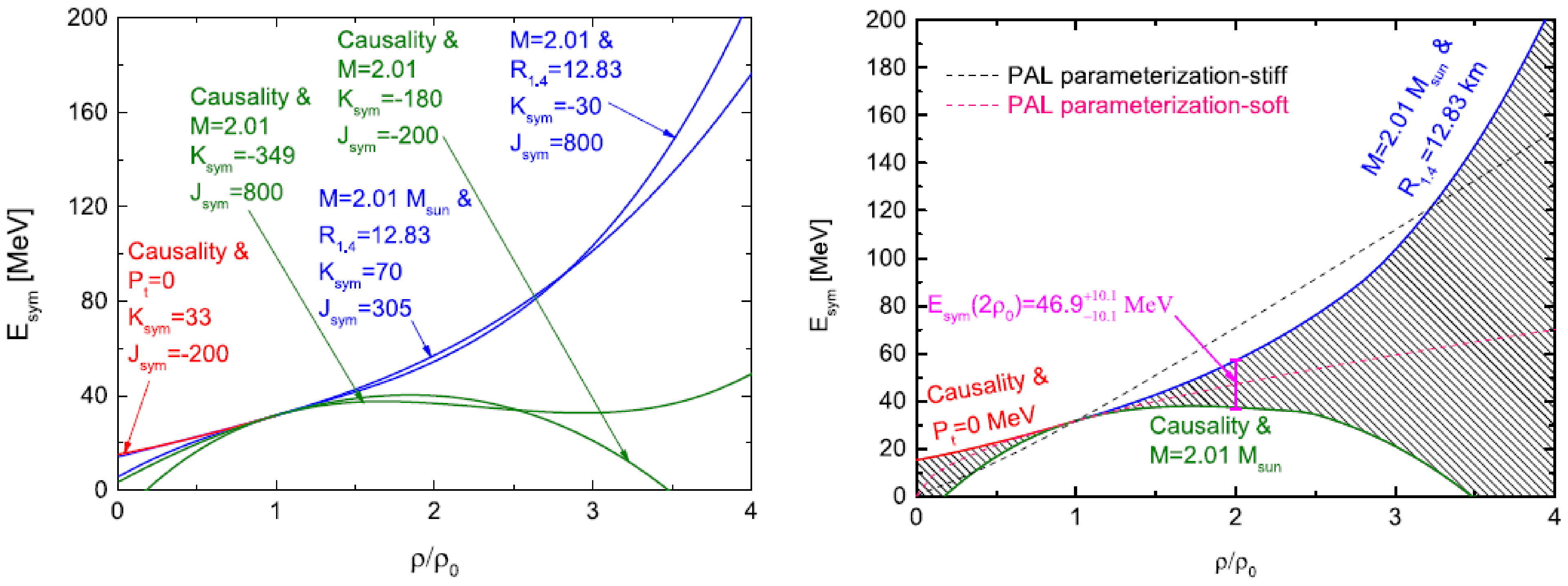


Symmetry energy at high densities

Zhang & Li, EPJA **55** (2019)

- Symmetry energy at $n_b = 2n_0$ estimated from GW170817 is

$$S_{00} = 46.9 \pm 10.1\ \text{MeV}$$

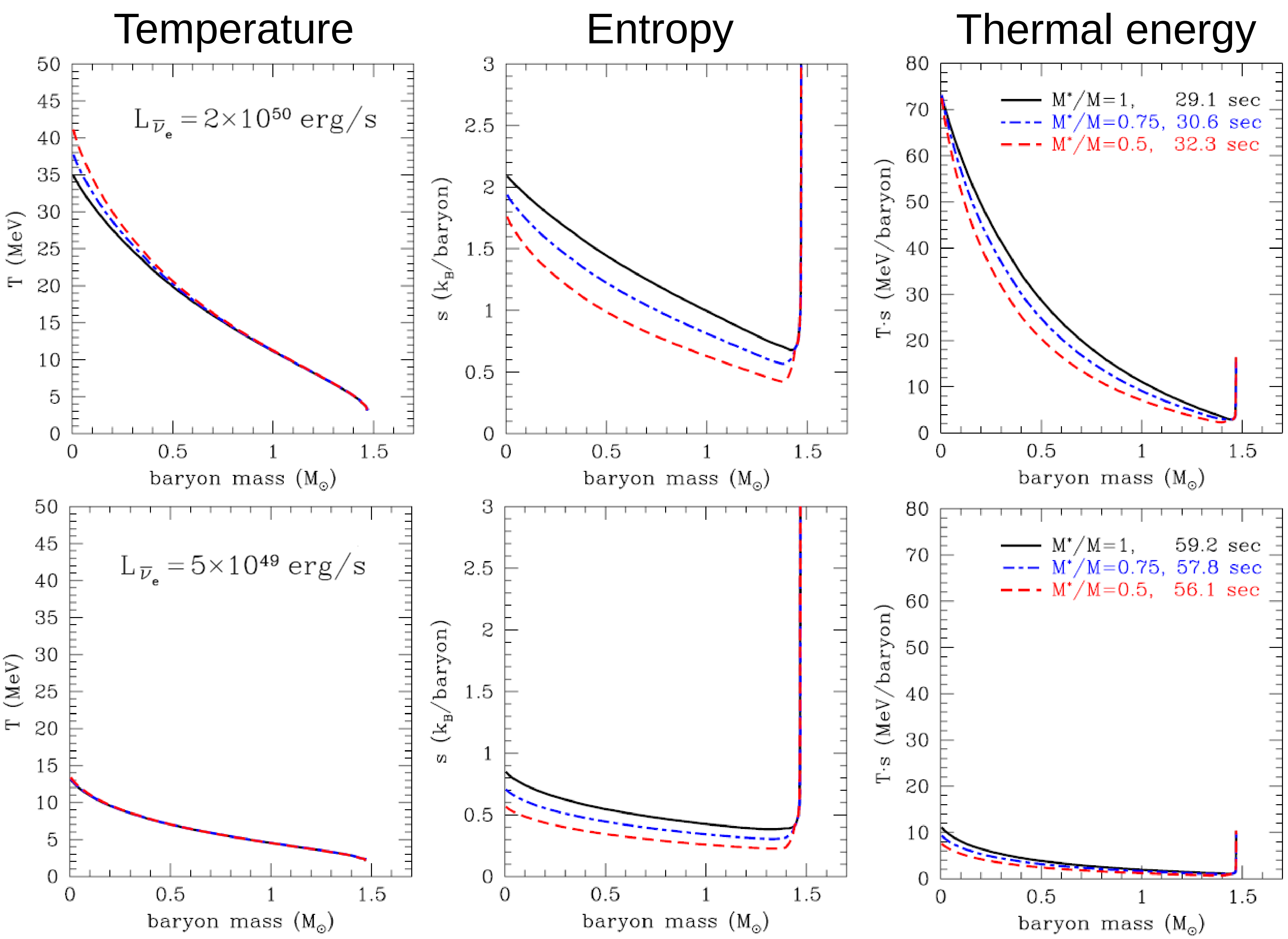


Dependences on effective mass

Nakazato & Suzuki, ApJ **878** (2019)

- cooling timescale: $\tau \sim \dfrac{E^{\mathrm{th}}}{L_\nu}$ — E^{th}: thermal energy, L_ν: ν luminosity
- Thermal energy of nucleons: $E_b^{\mathrm{th}} \propto uT^2$
- The neutrino luminosity is determined by the temperature.

  → The cooling timescale is shorter for models with smaller effective mass $u = M_N^*/M_N$.
- Estimating matter entropy with $E^{\mathrm{th}} \sim Ts$.
  - Entropy of nucleons: $s_b \propto uT$


Temperature
Entropy
Thermal energy
$L_{\bar{\nu}_e} = 2\times10^{50}$ erg/s
$L_{\bar{\nu}_e} = 5\times10^{49}$ erg/s
T (MeV)
s (k_B/baryon)
T·s (MeV/baryon)
baryon mass ($M_\odot$)
M*/M=1, 29.1 sec
M*/M=0.75, 30.6 sec
M*/M=0.5, 32.3 sec
M*/M=1, 59.2 sec
M*/M=0.75, 57.8 sec
M*/M=0.5, 56.1 sec