STUDYING STANDING ACCRETION SHOCK INSTABILITY (SASI) FLUCTUATIONS FROM NEUTRINOS AND GRAVITATIONAL WAVE OSCILLATIONS

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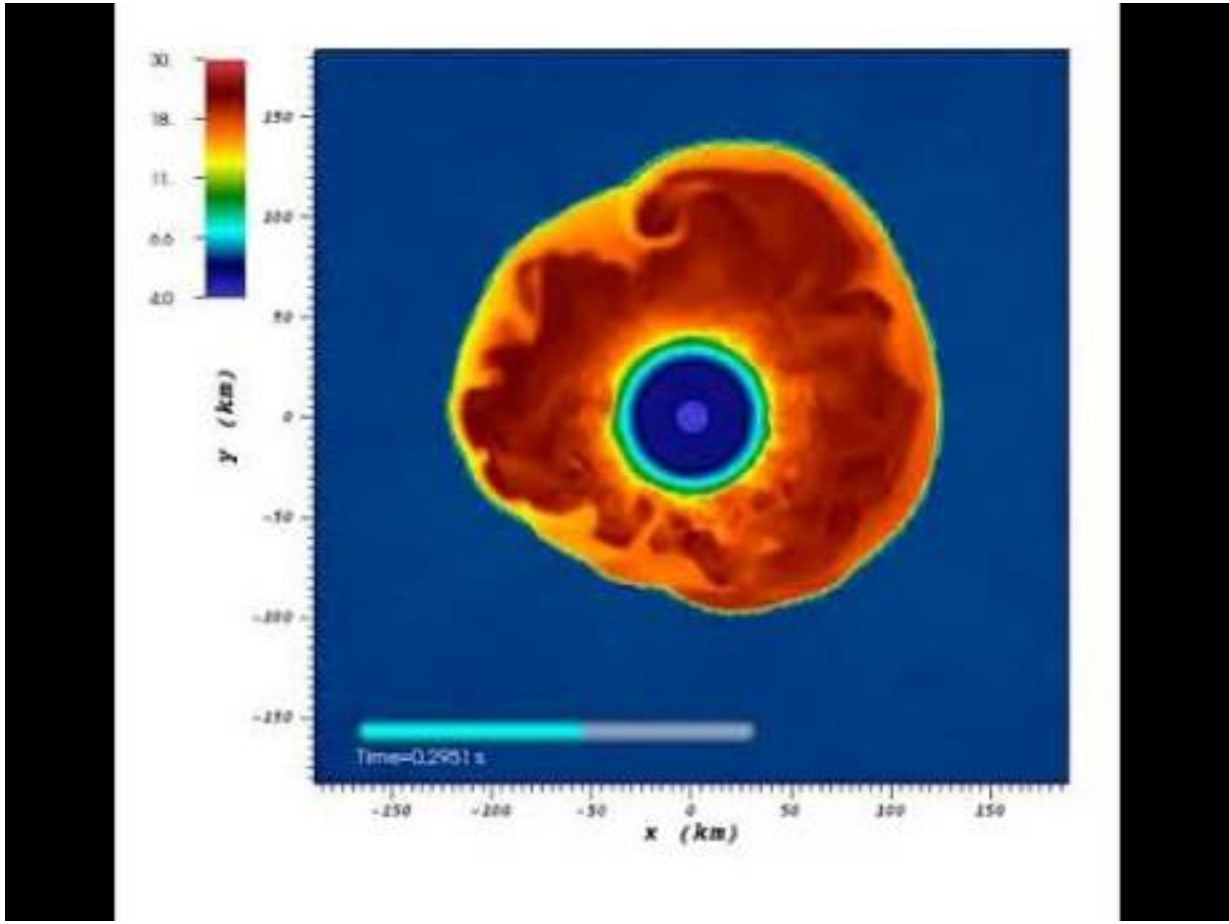
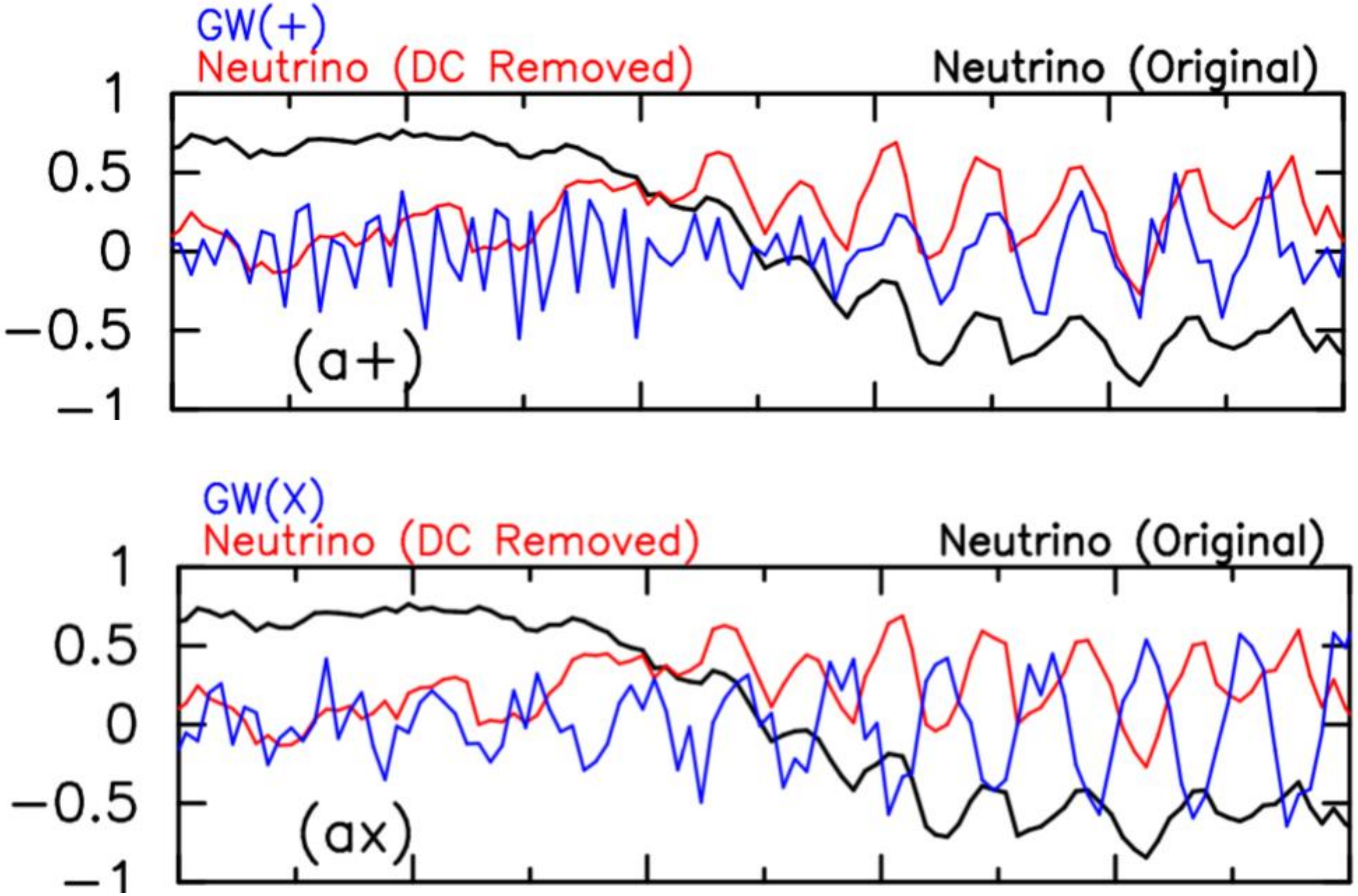
Standing Accretion Shock Instability

- ✦ An acoustic, convective hydrodynamic instability may develop between the proto-neutron star and the shock wave (SASI)
- ✦ SASI is a harmonic precession of the shock wave front around the proto-neutron star with a frequency between 80-150 Hz
- ✦ SASI produces Gravitational Waves and has an oscillatory effect on the observed neutrino flux from the Supernova



Gravitational Waves and Neutrinos



Kuroda, et al (2017)

Detection and Parameter Estimation

Neutrino

- How to establish the presence of SASI in neutrino luminosity (SASImeter), and with which confidence?
- How precisely can we measure the deterministic parameters?
- An example of an estimate for these parameters.

Gravitational Waves

- Not included in this presentation

Modeling Neutrino Fluctuations

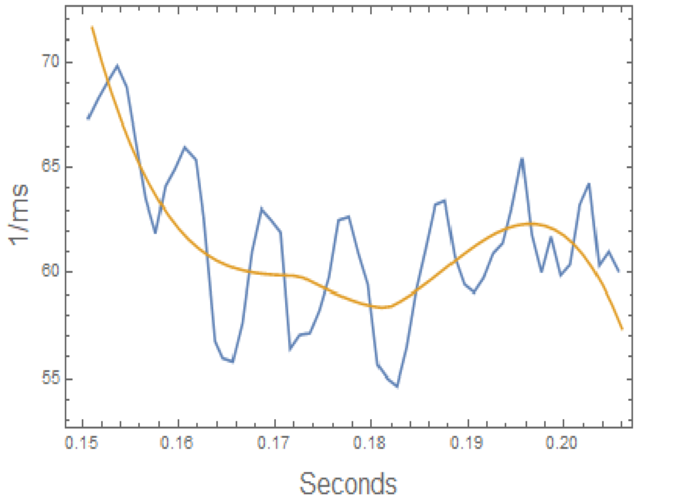
The recorded number of neutrinos detected (IceCube, Super-K, Hyper-K…) is subject to Poisson fluctuations. This means that the luminosity is a stochastic process with a variance proportional to $\sqrt{N}$ where the mean is given by:

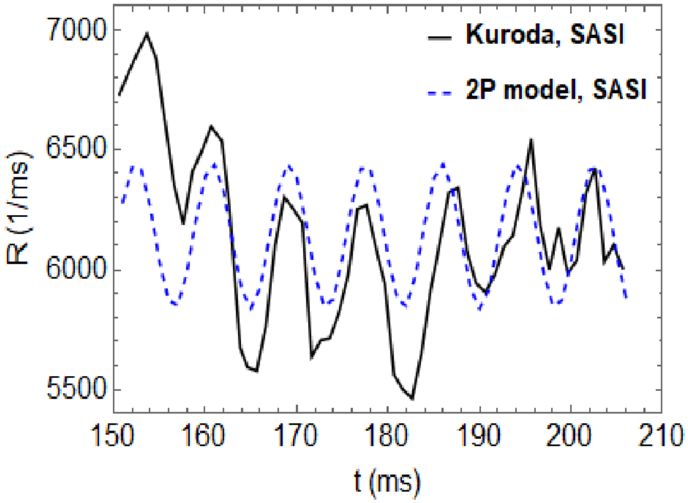
$$\boldsymbol{R(t) = (A - n)\big(1 + a\, Sin(2\,\pi\, f_a\, t)\big) + n}$$

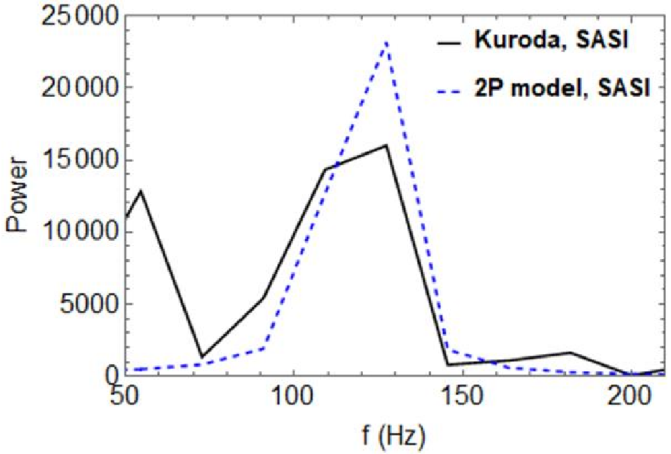
- A is the DC of the signal
- a is the relative amplitude of the signal
- f is the frequency of the signal
- n is the background of the detector

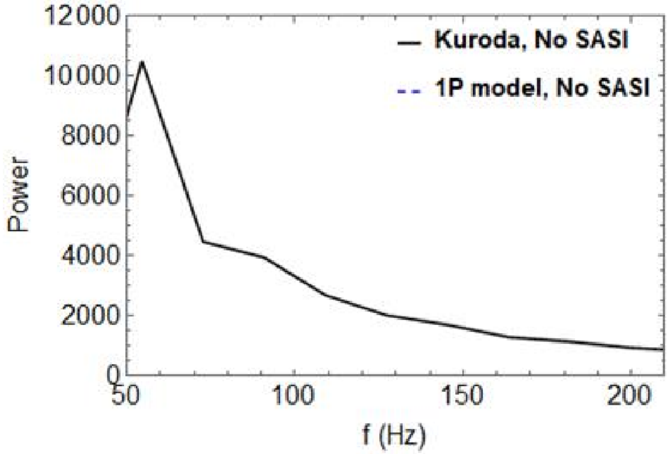
Modeling the SASI

Blue is the original
Yellow is a Low-Pass filter

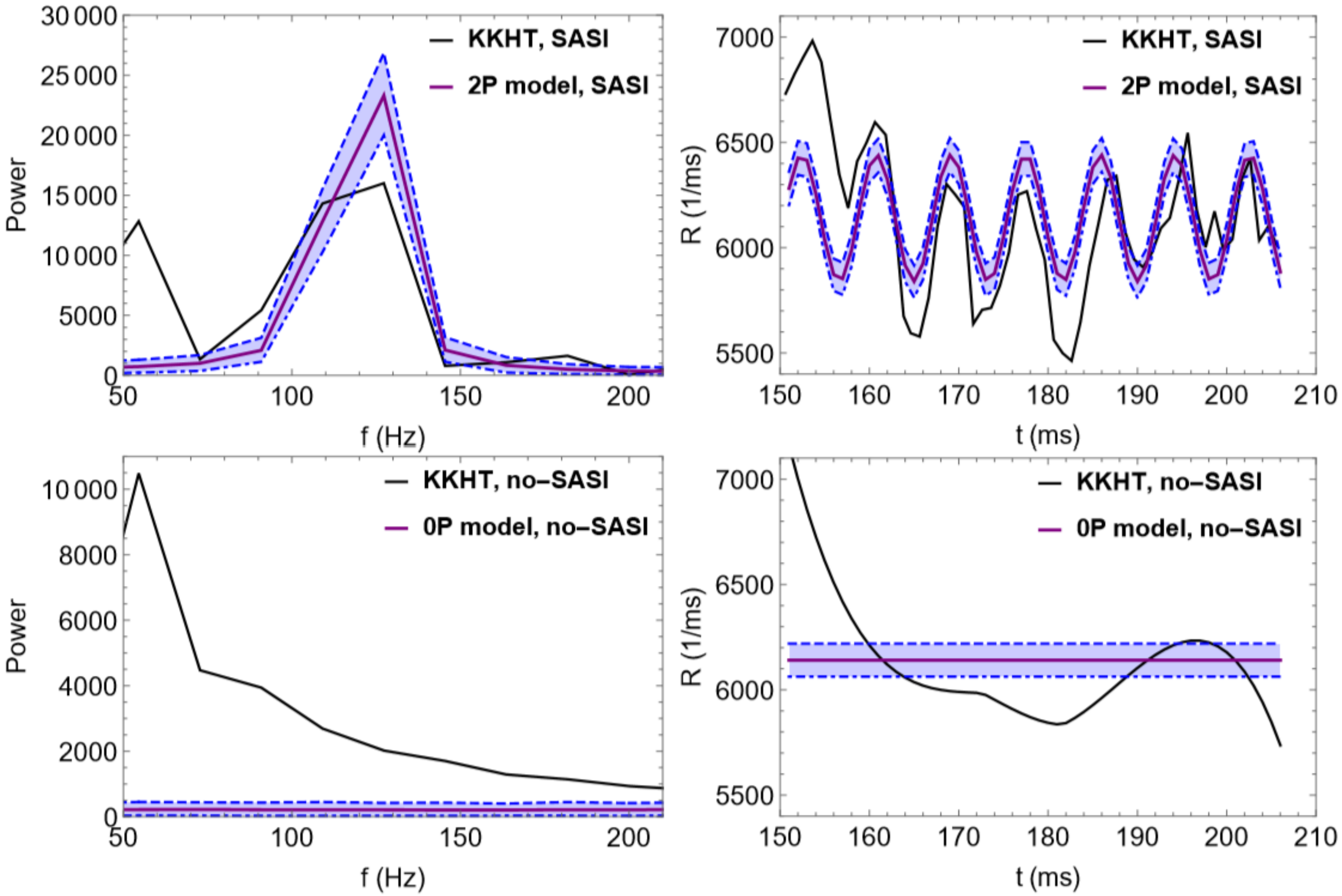








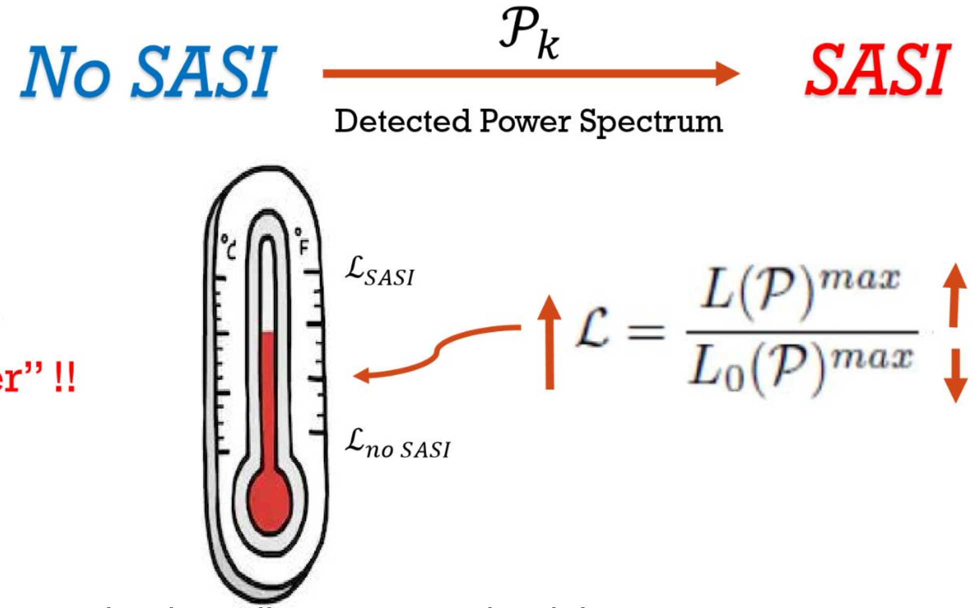
Modeling the SASI



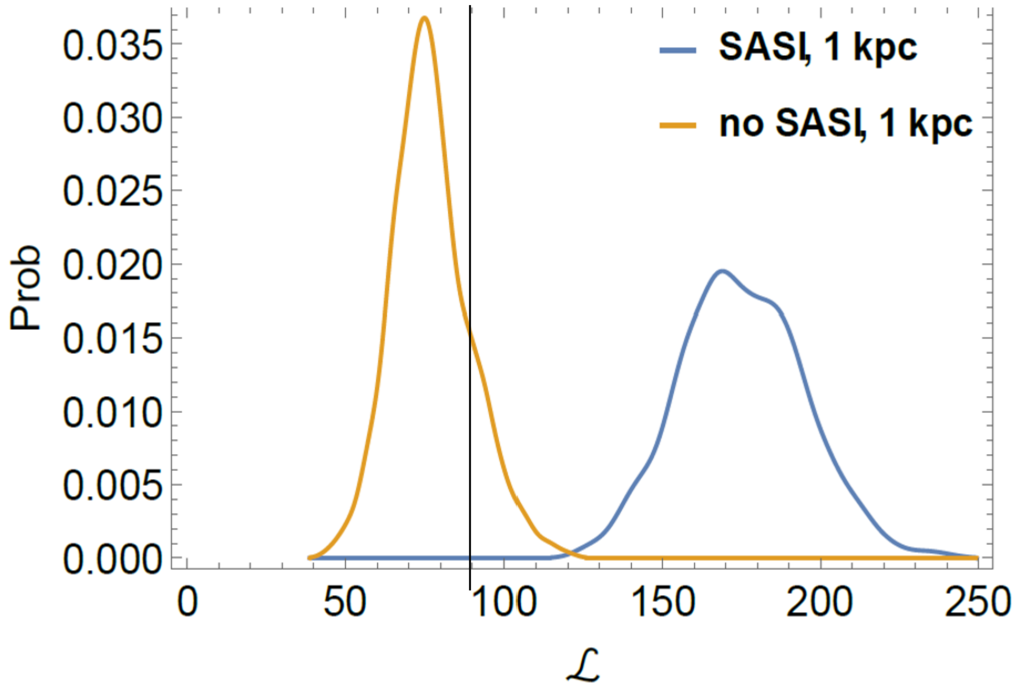
Statistics

The Poisson fluctuations allow us to determine the probability of obtaining a specific total power in the neutrino luminosity in both scenarios [SASI ($L(P)$), No-SASI($L_o(P)$)].

SASImeter

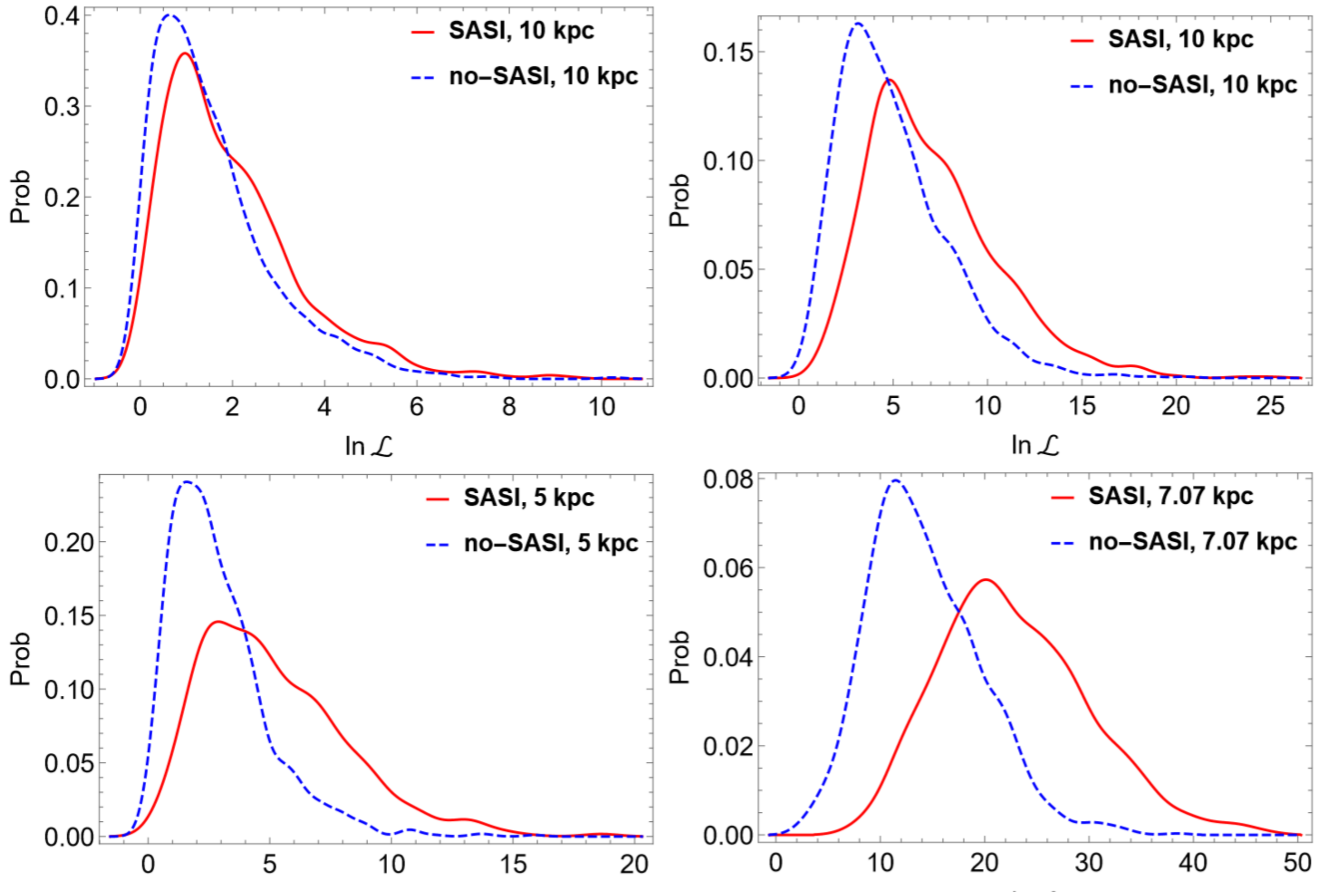


This is our "SASImeter" !!



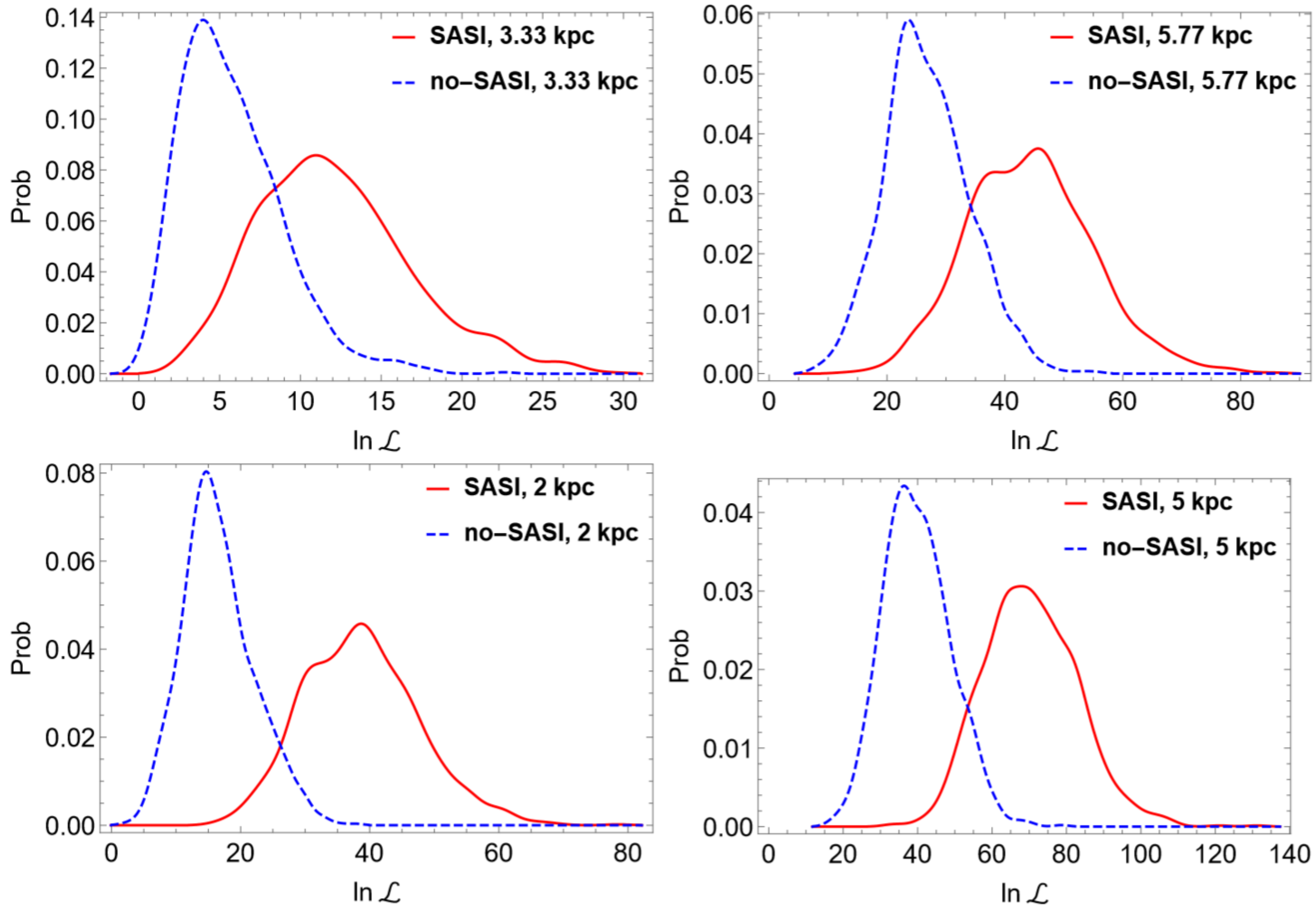
The area under the yellow curve to the right of the line is the false alarm probability.
The area under the blue curve to the right of the line is the detection probability.

SASImeter



SASI, 10 kpc
no–SASI, 10 kpc
Prob
ln $\mathcal{L}$
SASI, 10 kpc
no–SASI, 10 kpc
Prob
ln $\mathcal{L}$
SASI, 5 kpc
no–SASI, 5 kpc
Prob
SASI, 7.07 kpc
no–SASI, 7.07 kpc
Prob

SASImeter

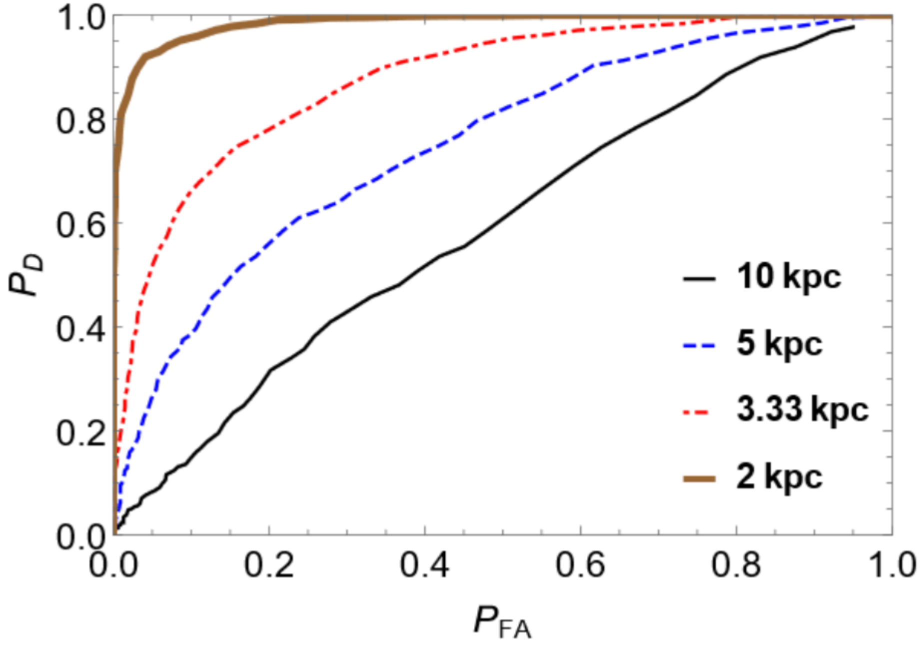


SASI, 3.33 kpc
no–SASI, 3.33 kpc
SASI, 5.77 kpc
no–SASI, 5.77 kpc
SASI, 2 kpc
no–SASI, 2 kpc
SASI, 5 kpc
no–SASI, 5 kpc
Prob
ln $\mathcal{L}$

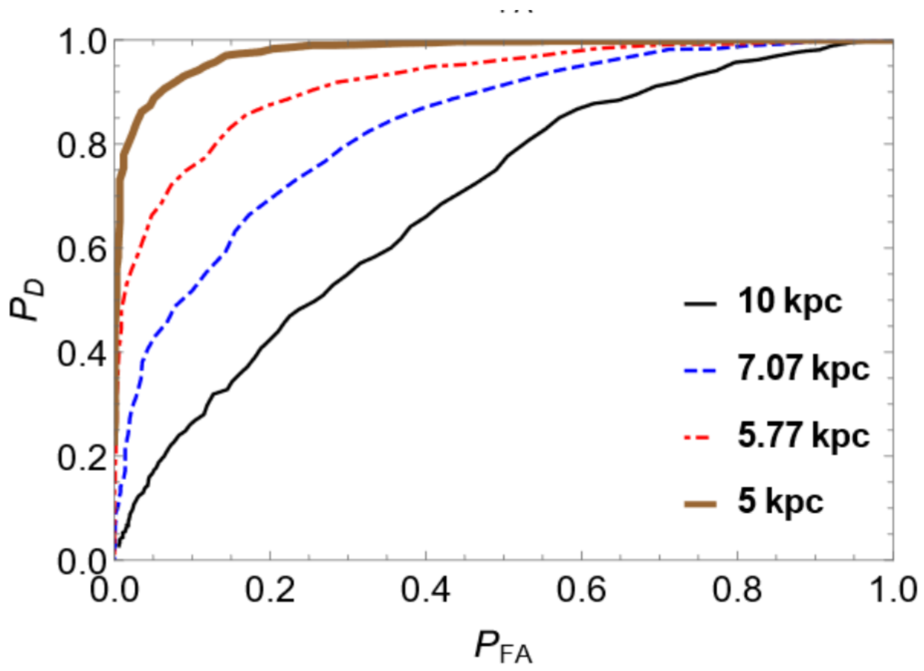
Receiver Operating Curves

For any point where the line is located, we get a value for both the false alarm and the detection probability.

Receiver Operating Curves HK



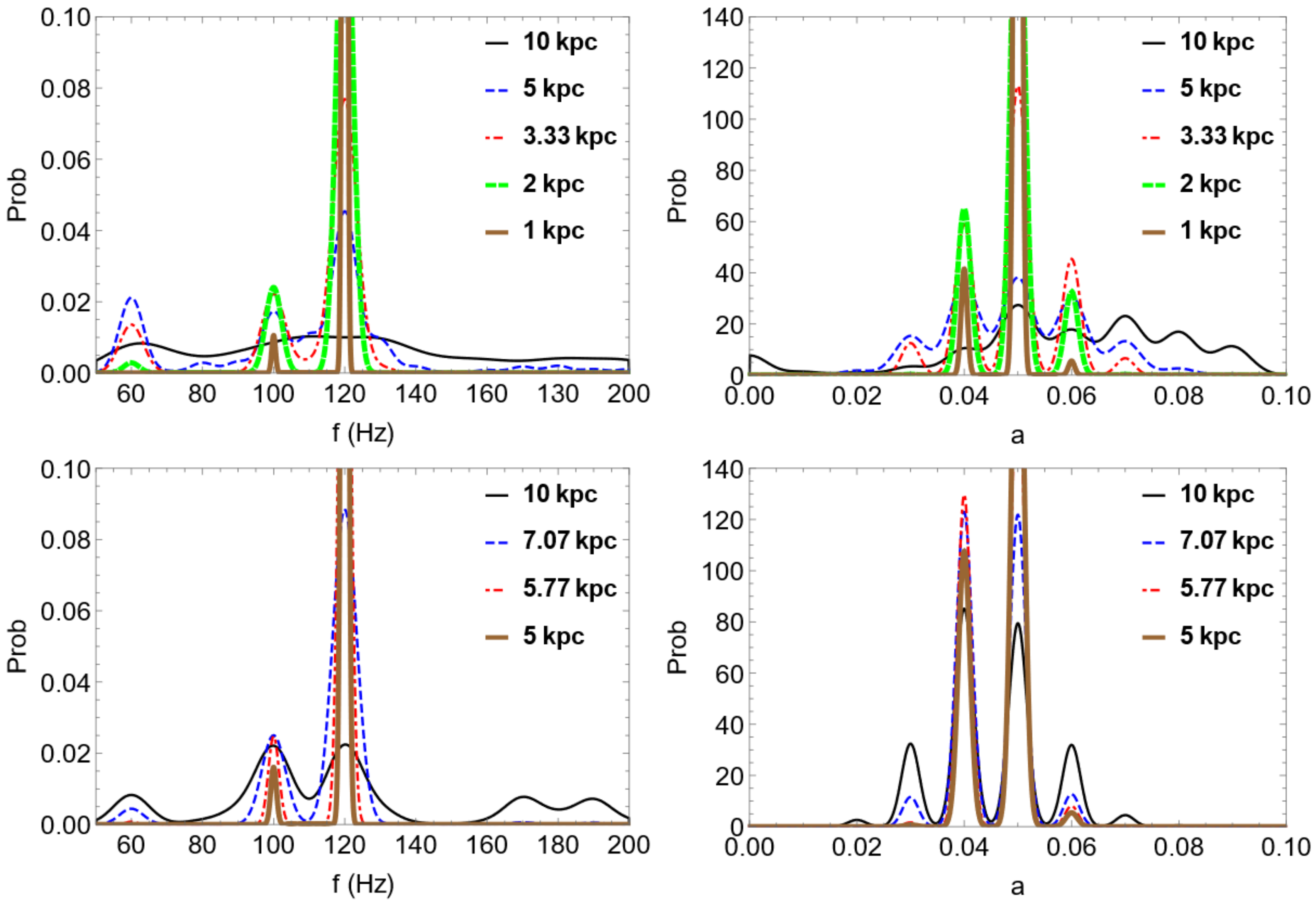
Receiver Operating Curves IC



Parameter Estimation

We can also evaluate how much the probability of the observed power changes assuming different values of the parameters.

Parameter Estimation



Fisher Information and The Cramer Rao Bound

Forms a matrix which shows how much a data set is influenced by a certain parameter.

Amplitude of the SASI Oscillations + Frequency of the SASI Oscillations + DC component of the Neutrino Rate

$$Minimum\ Error\ (\alpha) = \sqrt{(I_{\alpha\alpha})^{-1}}$$

Using the information in the Fisher Matrix, you can calculate the Cramer Lower Bound

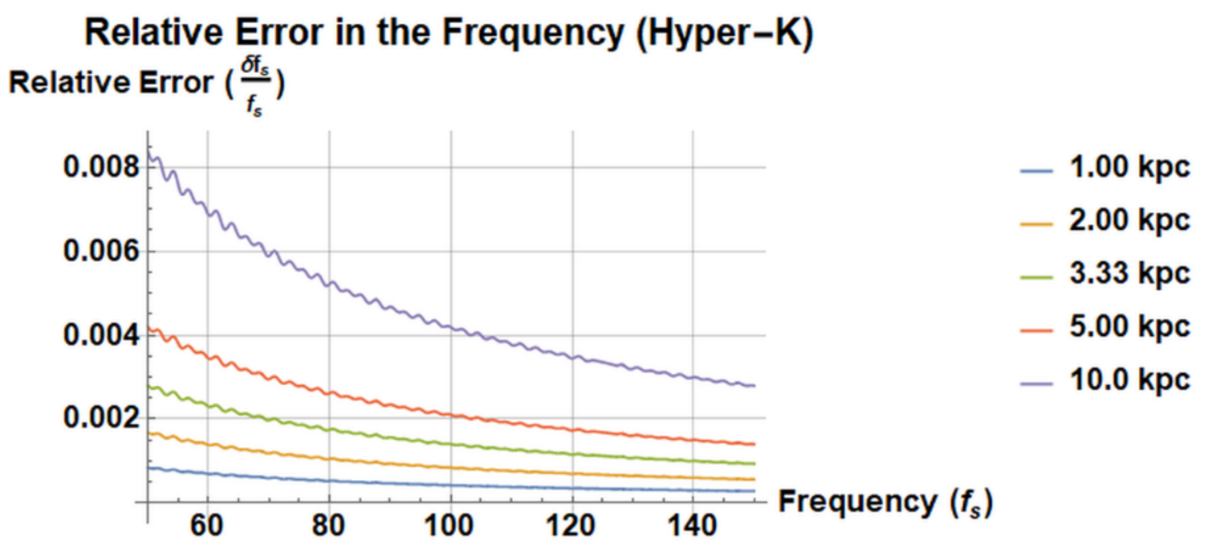
Estimated Parameters

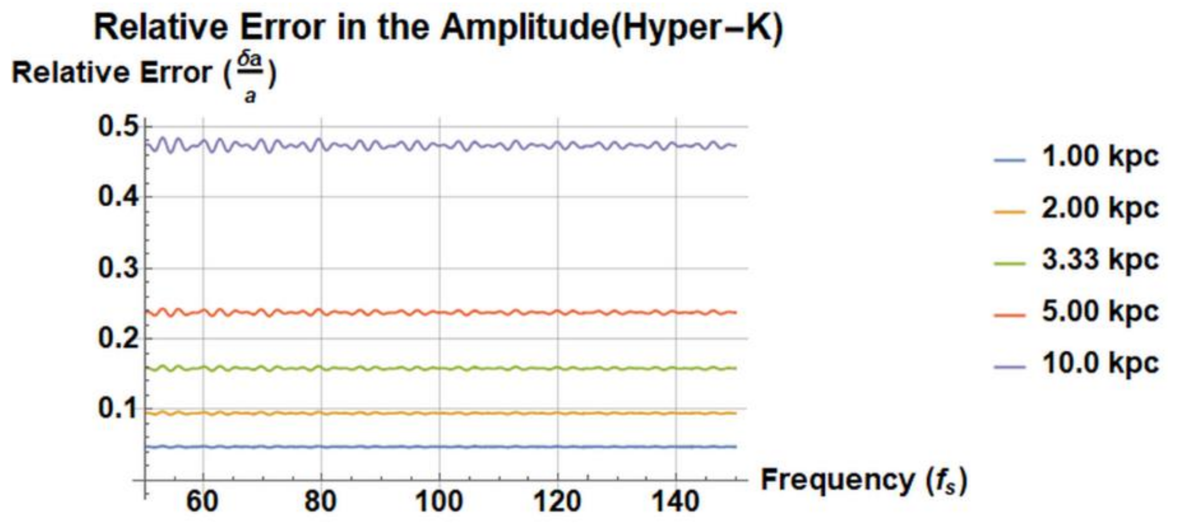
TABLE I: Estimated parameters (which maximized the likelihood) and their correspond uncertainties (standard deviation) when there is SASI at 10, 7.07, 5.77 and 5 kpc, for Ice Cube. Numbers in the parentheses are calculated using Fisher matrix.

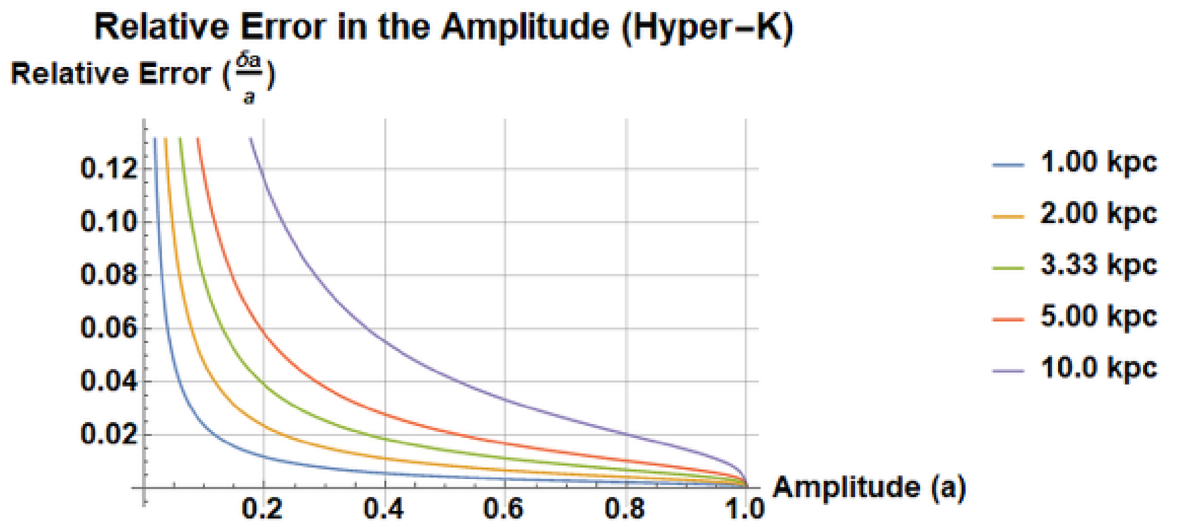
SASI	$10Kpc$	$7.07Kpc$	$5.77Kpc$	$5Kpc$
A(1/ms)	2312.4	3284.9	4257.3	5229.7
f(Hz)	120.27	114.18	117.95	119.36
δf(Hz)	37.27 (0.17)	14.81 (0.11)	6.62 (0.078)	3.52 (0.063)
a	0.045	0.045	0.046	0.047
δa	0.01 (0.0091)	0.0066(0.0053)	0.0055(0.0041)	0.0049 (0.0034)
n (1/ms)	1340	1340	1340	1340

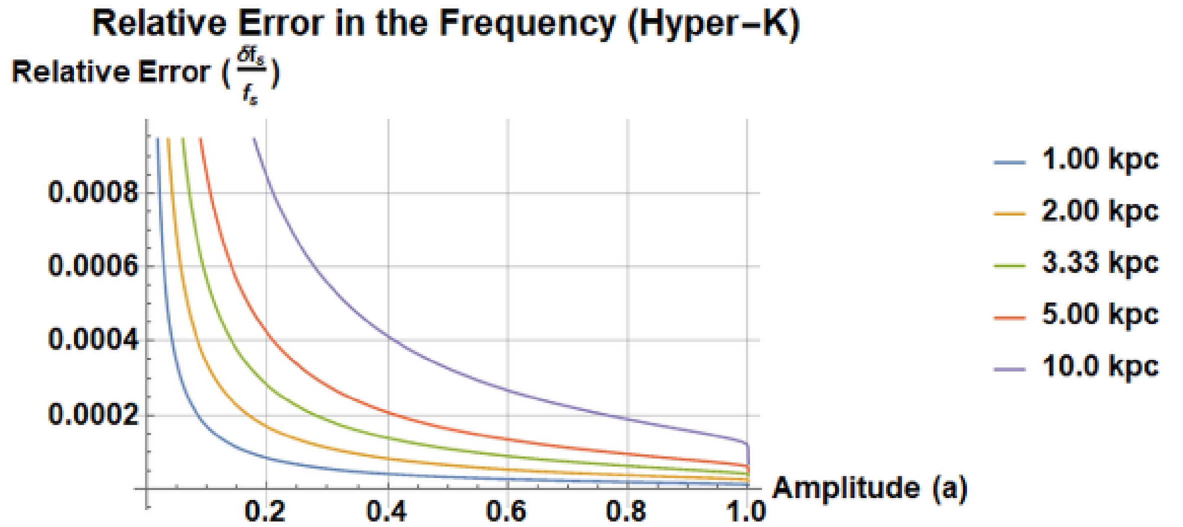
TABLE III: Estimated parameters (which maximized the likelihood) and their correspond uncertainties (standard deviation) when there is SASI at 10, 5, 3.33, 2 and 1 kpc, for Hyper-K. Numbers in the parentheses are calculated using Fisher matrix in *the time domain, in the power domain.*

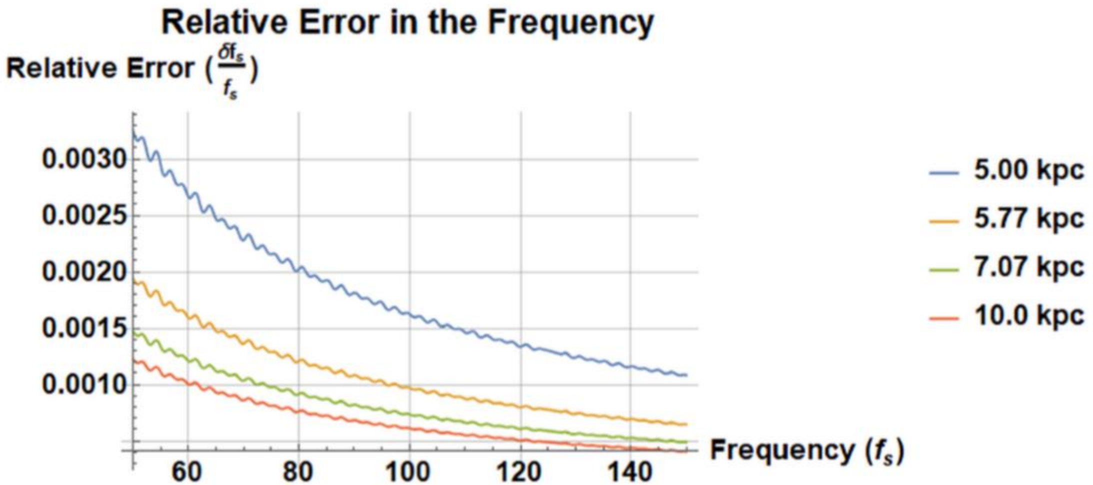
SASI	$10Kpc$	$5Kpc$	$3.33Kpc$	$2Kpc$	$1Kpc$
A(1/ms)	61.41	245.7	552.7	1535.38	6141
f(Hz)	119.18	108	109.33	116.34	119.72
δf(Hz)	41.25(0.35)	29.48(0.21)	20.02(0.14)	9.91(0.08)	2.35(0.04, *0.23009*)
a	0.0579	0.0496	0.049	0.049	0.049
δa	0.0227(0.023)	0.0128(0.012)	0.0087(0.0077)	0.0055(0.0047)	0.0026(0.0023, *0.00264*)

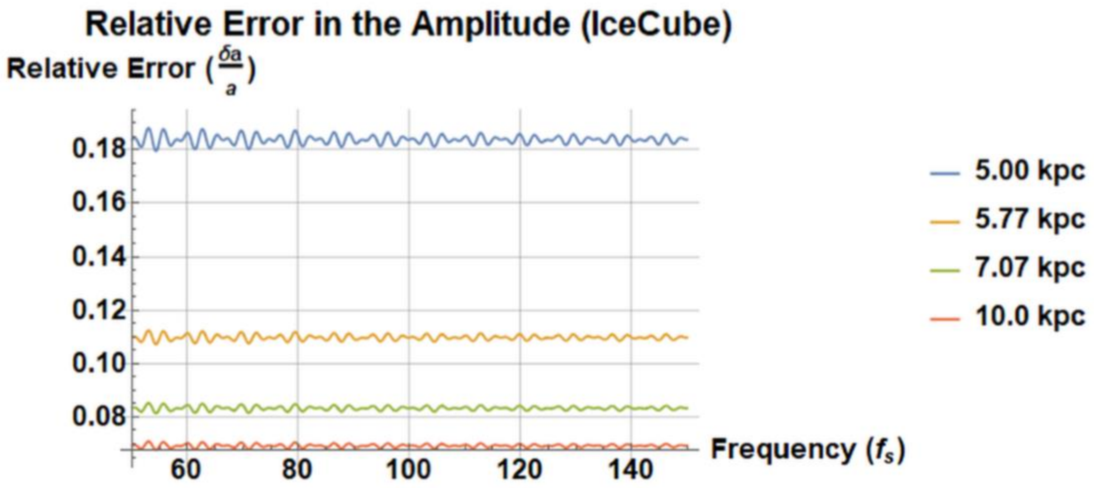

Relative Error in the Frequency (Hyper−K)
Relative Error ($\frac{\delta f_s}{f_s}$)
0.008
0.006
0.004
0.002
60
80
100
120
140
Frequency (f_s)
1.00 kpc
2.00 kpc
3.33 kpc
5.00 kpc
10.0 kpc

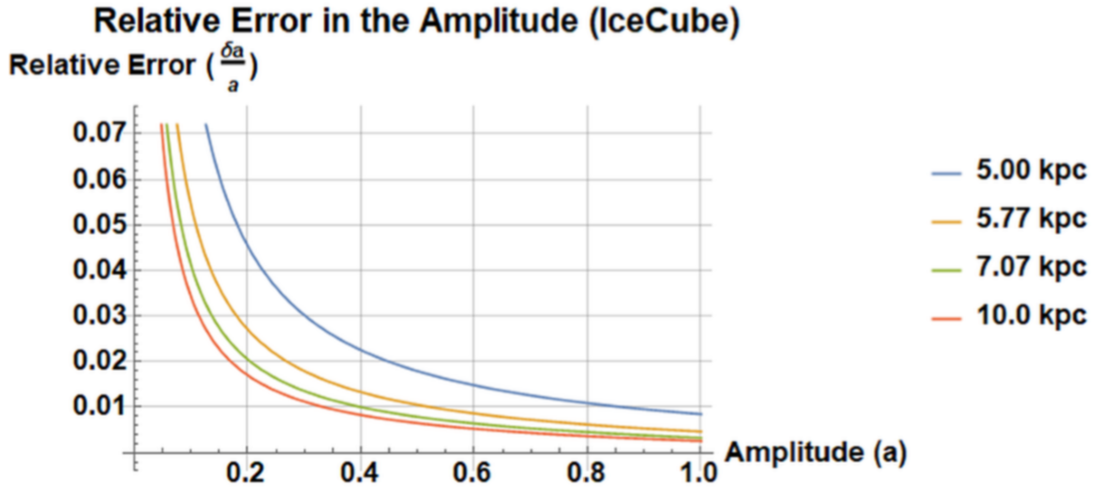

Relative Error in the Amplitude(Hyper−K)
Relative Error ($\frac{\delta a}{a}$)
0.5
0.4
0.3
0.2
0.1
60
80
100
120
140
Frequency (f_s)
1.00 kpc
2.00 kpc
3.33 kpc
5.00 kpc
10.0 kpc

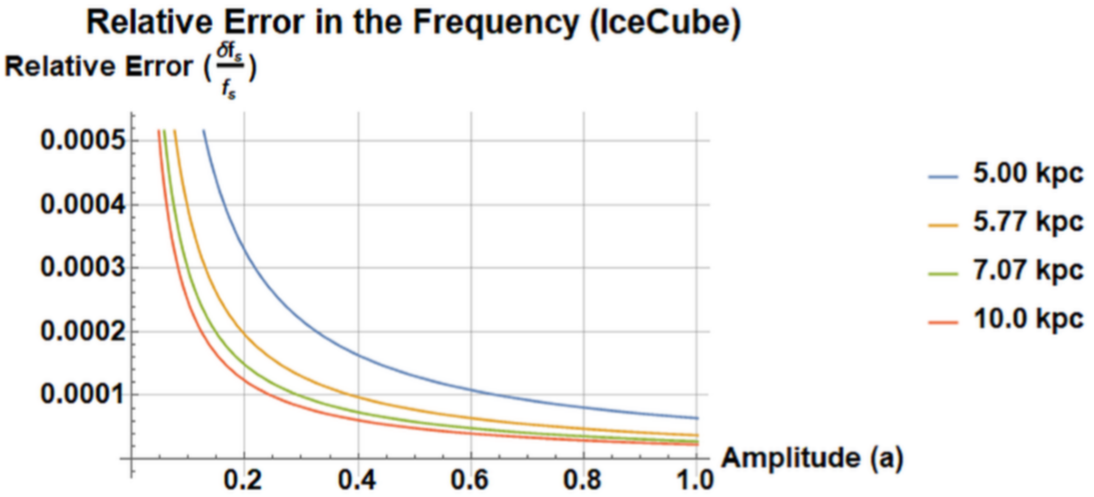

Relative Error in the Amplitude (Hyper−K)
Relative Error ($\frac{\delta a}{a}$)
0.12
0.10
0.08
0.06
0.04
0.02
0.2
0.4
0.6
0.8
1.0
Amplitude (a)
1.00 kpc
2.00 kpc
3.33 kpc
5.00 kpc
10.0 kpc


Relative Error in the Frequency (Hyper−K)
Relative Error ($\frac{\delta f_s}{f_s}$)
0.0008
0.0006
0.0004
0.0002
0.2
0.4
0.6
0.8
1.0
Amplitude (a)
1.00 kpc
2.00 kpc
3.33 kpc
5.00 kpc
10.0 kpc


Relative Error in the Frequency
Relative Error ($\frac{\delta f_s}{f_s}$)
0.0030
0.0025
0.0020
0.0015
0.0010
60
80
100
120
140
Frequency (f_s)
5.00 kpc
5.77 kpc
7.07 kpc
10.0 kpc


Relative Error in the Amplitude (IceCube)
Relative Error ($\frac{\delta a}{a}$)
0.18
0.16
0.14
0.12
0.10
0.08
60
80
100
120
140
Frequency (f_s)
5.00 kpc
5.77 kpc
7.07 kpc
10.0 kpc


Relative Error in the Amplitude (IceCube)
Relative Error ($\frac{\delta a}{a}$)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.2
0.4
0.6
0.8
1.0
Amplitude (a)
5.00 kpc
5.77 kpc
7.07 kpc
10.0 kpc


Relative Error in the Frequency (IceCube)
Relative Error ($\frac{\delta f_s}{f_s}$)
0.0005
0.0004
0.0003
0.0002
0.0001
0.2
0.4
0.6
0.8
1.0
Amplitude (a)
5.00 kpc
5.77 kpc
7.07 kpc
10.0 kpc

Acknowledgements

- Kuroda, K., Kotake, K., Hayama, K., and Takiwaki, T. (2017). *Correlated Signatures of Gravitational-Waves and Neutrino Emission in Three-Dimentional General-Relativistic Coer-Collapse Supernova Simulations*.
- Lund, T., Marek, A., Lunardini, C., Janka, H.T., and Raffelt, G. (2010). *Fast time variations of supernova neutrino fluxes and their detectability.*
- Zanolin, M., Naftali, E., and Markis, N. *Second Order Bias of a Multivariate Gaussian Maximum Likelihood Estimate with a Chain Rule for Higher Moments.*