Effects of nuclear saturation properties on the equation of state for hot nuclear matter in core-collapse supernovae

H. Togashi (Kyushu Univ.)

The aim of this study

To investigate systematically the effects of

nuclear saturation properties

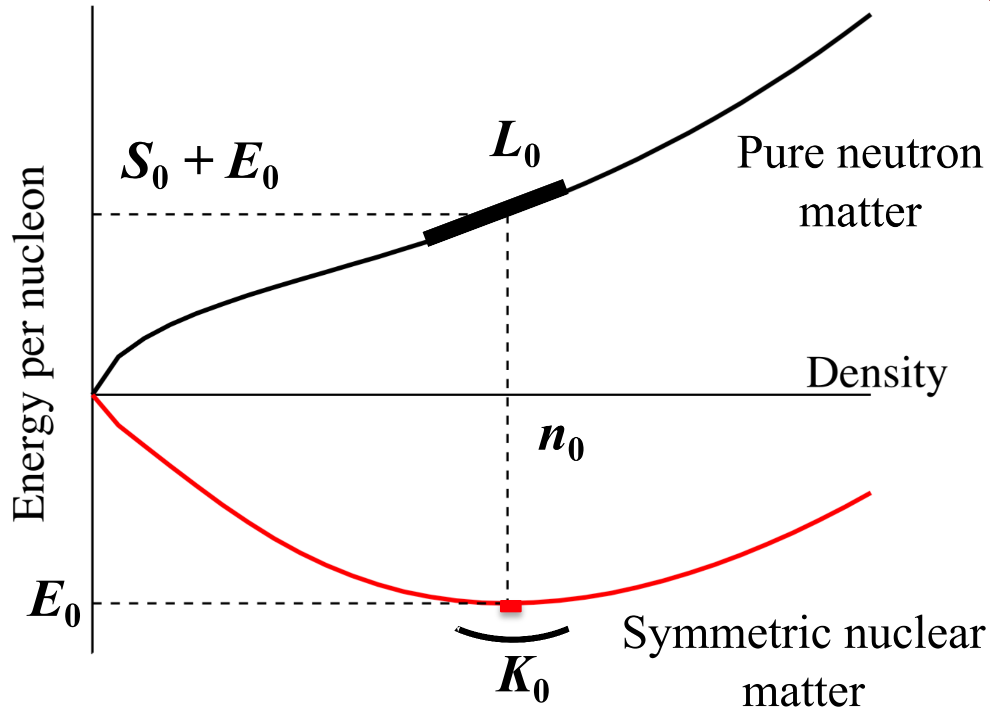
on the mechanism of ***core-collapse supernovae***

Multi-dimensional Modeling and Multi-Messenger observation from CCSNe @ Fukuoka, Oct. 21, 2019

1. Introduction

Nuclear equation of state (EOS) :
characterized by the empirical saturation parameters

- n_0: **saturation density**
- E_0: **saturation energy**
- K_0: **incompressibility**
- S_0: **symmetry energy**
- L_0: **density derivative coefficient of** S_0



The effects of saturation properties on the neutron-star structures has been studied with various nuclear theories.

(e.g. K. Oyamatsu & K. Iida PRC 75 (2007) 015801, S. Gandolfi et al., PRC 85 (2012) 032801)

Nuclear EOSs for Core-Collapse Supernovae

(M. Oertel et al., Rev. Mod. Phys. 89 (2017) 015007)

Model	Nuclear Interaction	Degrees of Freedom	$M_{\max}$ ($M_\odot$)	$R_{1.4M_\odot}$ (km)	Ξ	publ. avail.	References
H&W	SKa	$n, p, \alpha, \{(A_i, Z_i)\}$	2.21[a]	13.9 [a]		n	El Eid and Hillebrandt (1980); Hillebrandt *et al.* (1984)
LS180	LS180	$n, p, \alpha, (A, Z)$	1.84	12.2	0.27	y	Lattimer and Swesty (1991)
LS220	LS220	$n, p, \alpha, (A, Z)$	2.06	12.7	0.28	y	Lattimer and Swesty (1991)
LS375	LS375	$n, p, \alpha, (A, Z)$	2.72	14.5	0.32	y	Lattimer and Swesty (1991)
STOS	TM1	$n, p, \alpha, (A, Z)$	2.23	14.5	0.26	y	Shen *et al.* (1998); Shen *et al.* (1998, 2011)
FYSS	TM1	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.22	14.4	0.26	n	Furusawa *et al.* (2013b)
HS(TM1)	TM1*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.21	14.5	0.26	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(TMA)	TMA*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.02	13.9	0.25	y	Hempel and Schaffner-Bielich (2010)
HS(FSU)	FSUgold*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.74	12.6	0.23	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(NL3)	NL3*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.79	14.8	0.31	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(DD2)	DD2	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.42	13.2	0.30	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(IUFSU)	IUFSU*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.95	12.7	0.25	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
SFHo	SFHo	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.06	11.9	0.30	y	Steiner *et al.* (2013a)
SFHx	SFHx	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.13	12.0	0.29	y	Steiner *et al.* (2013a)
SHT(NL3)	NL3	$n, p, \alpha, \{(A_i, Z_i)\}$	2.78	14.9	0.31	y	Shen *et al.* (2011b)
SHO(FSU)	FSUgold	$n, p, \alpha, \{(A_i, Z_i)\}$	1.75	12.8	0.23	y	Shen *et al.* (2011a)
SHO(FSU2.1)	FSUgold2.1	$n, p, \alpha, \{(A_i, Z_i)\}$	2.12	13.6	0.26	y	Shen *et al.* (2011a)

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Effective interactions (Skyrme or RMF model)

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Microscopic EOS with bare nuclear potentials

Uniform EOS: cluster variational method with AV18 + UIX potentials

Non-uniform EOS: Thomas-Fermi method (Single nucleus approximation)

(HT, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki, M. Takano, NPA961 (2017) 78)

- *Extended to* ***Nuclear statistical equilibrium (NSE) model***

(S. Furusawa, HT, H. Nagakura, K. Sumiyoshi, S. Yamada, H. Suzuki, M. Takano, J. Phys. G 44 (2017) 094001)

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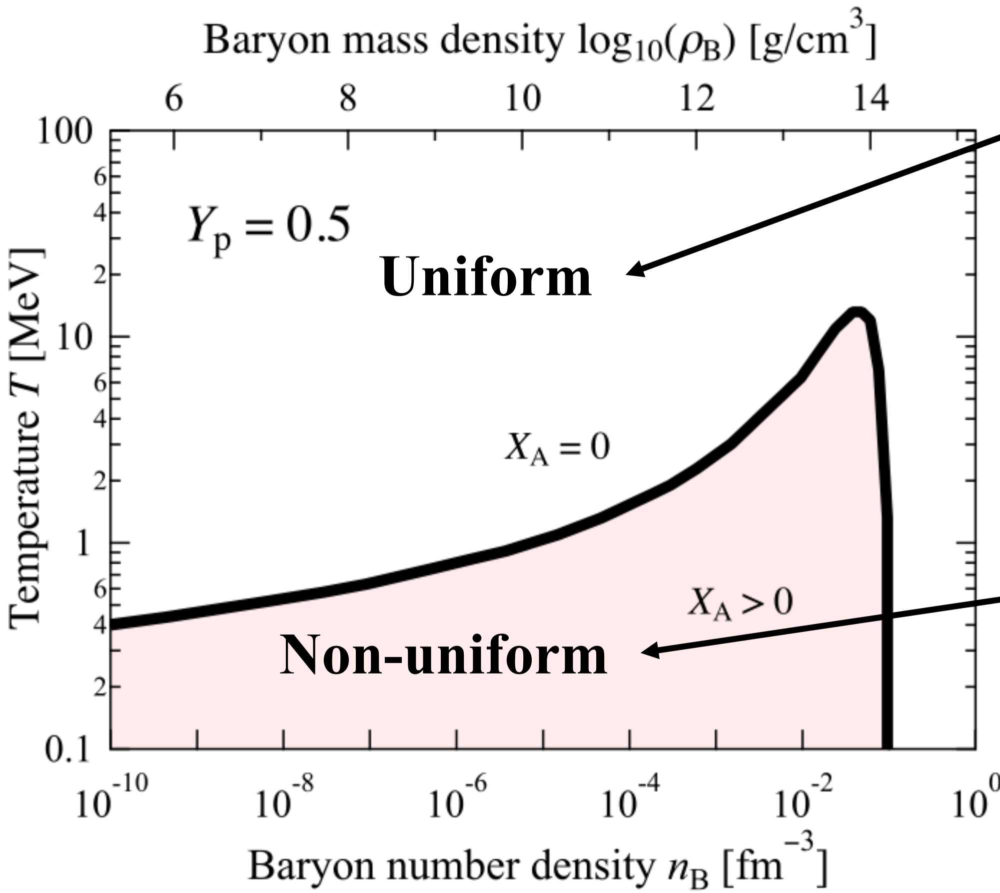
These EOSs are not suitable for the systematic study of supernovae in terms of the nuclear saturation properties.

Our plan for a systematic study of supernova EOSs

We extend the systematic study for cold β-stable matter (Neutron Star Crust).
(K. Oyamatsu & K. Iida PRC 75 (2007) 015801)



Phase diagram of nuclear matter [based on HT *et al*., NPA 961 (2017) 78]

1: **Parameterized EOSs**

Parameters are fine-tuned
with the Thomas-Fermi method
for stable nuclei

★We are here.★

2: **Thomas-Fermi calculation for non-uniform matter**
(Single nucleus approximation)

3: **Numerical simulations for core-collapse supernovae with obtained EOSs**

2. Supernova EOS with Macroscopic Nuclear Model

Macroscopic EOS for uniform matter by Oyamatsu & Iida

(PRC 75 (2007) 015801)

Energy per nucleon at zero temperature

$$E(n_\mathrm{B}, Y_\mathrm{p}) = E_\mathrm{F} + [1-(1-2Y_\mathrm{p})^2]v_\mathrm{s}(n_\mathrm{B}) + (1-2Y_\mathrm{p})^2 v_\mathrm{n}(n_\mathrm{B})$$

– E_F: One-body kinetic energy per particle for the Fermi-gas

- Potential energy per particle for symmetric and neutron matter

$$v_\mathrm{s}(n_\mathrm{B}) = a_1 n_\mathrm{B} + \frac{a_2 n_\mathrm{B}^2}{1 + a_3 n_\mathrm{B}} \qquad v_\mathrm{n}(n_\mathrm{B}) = b_1 n_\mathrm{B} + \frac{b_2 n_\mathrm{B}^2}{1 + b_3 n_\mathrm{B}}$$

Parameters (a_1, a_2, a_3, b_1, b_2, b_3) are determined

Thomas-Fermi calculation of **isolated atomic nuclei** with E reproduces the gross feature of **experimental data (masses and radii)**.

2. Supernova EOS with Macroscopic Nuclear Model

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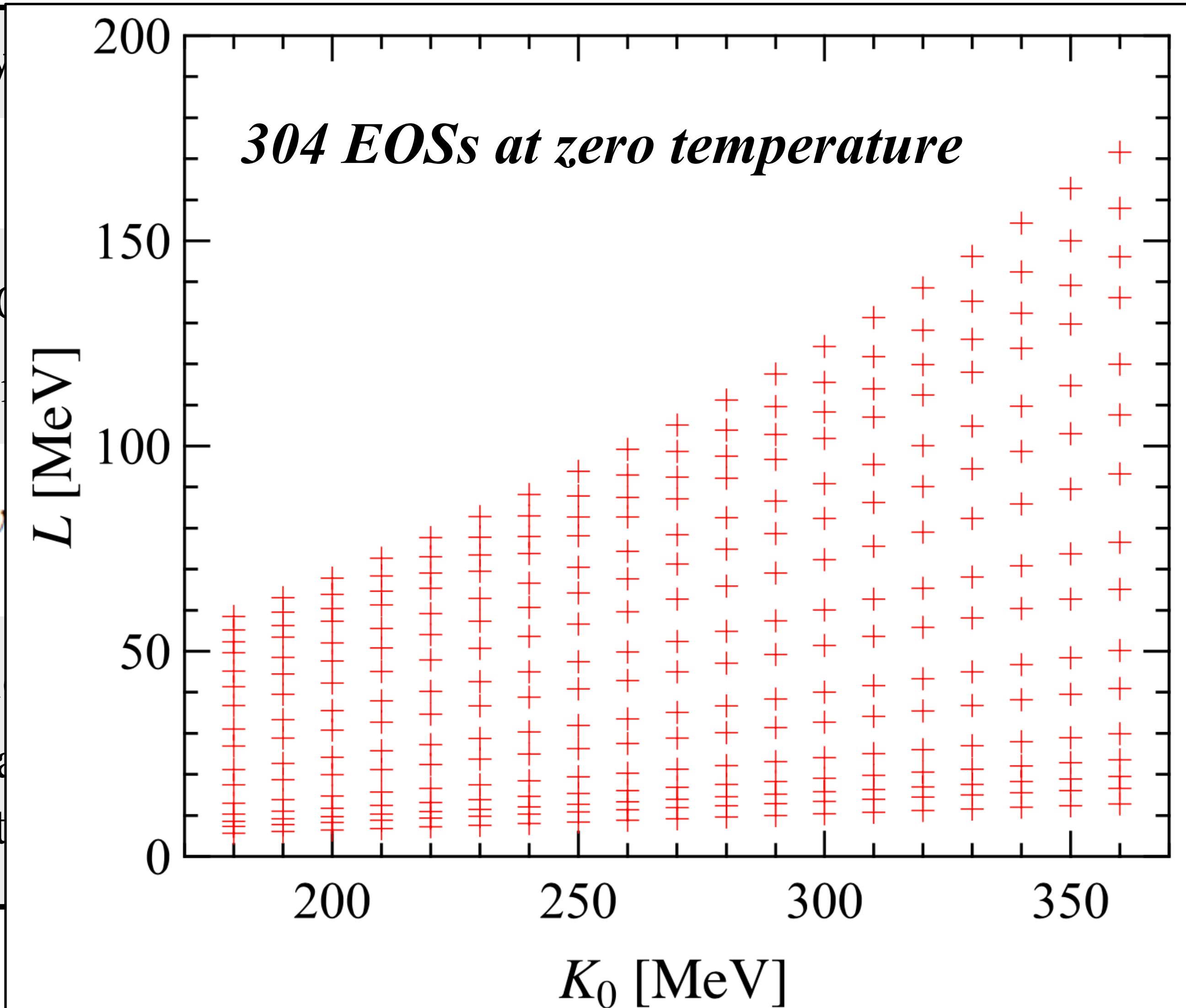
Energy

$E(n_{\mathrm{B}}$ … $)^2 v_{\mathrm{n}}(n_{\mathrm{B}})$

– E_{F}: (

- Pote

v … n_{B}^2 / … $_3 n_{\mathrm{B}}$



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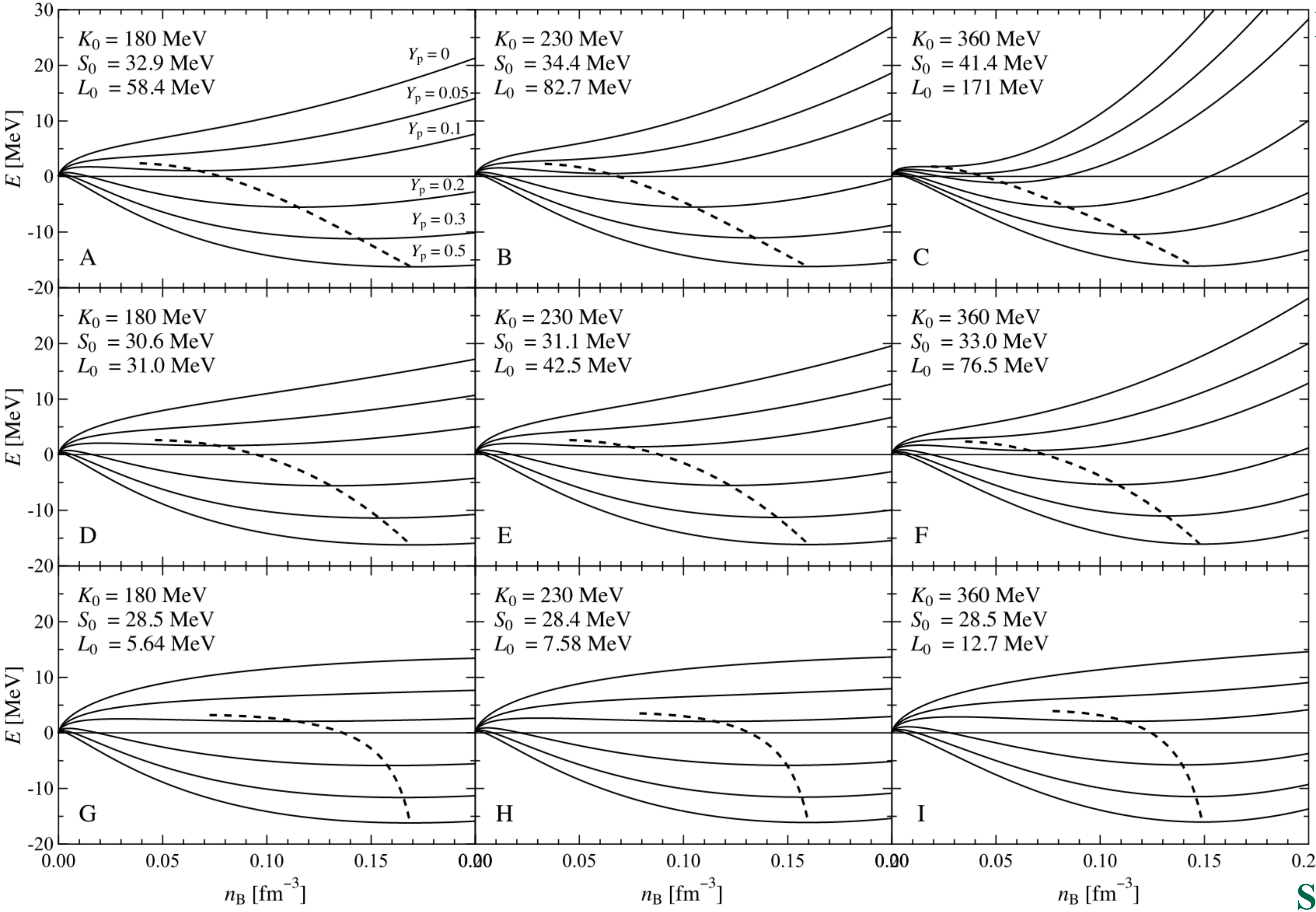
Energies per particles for macroscopic EOSs

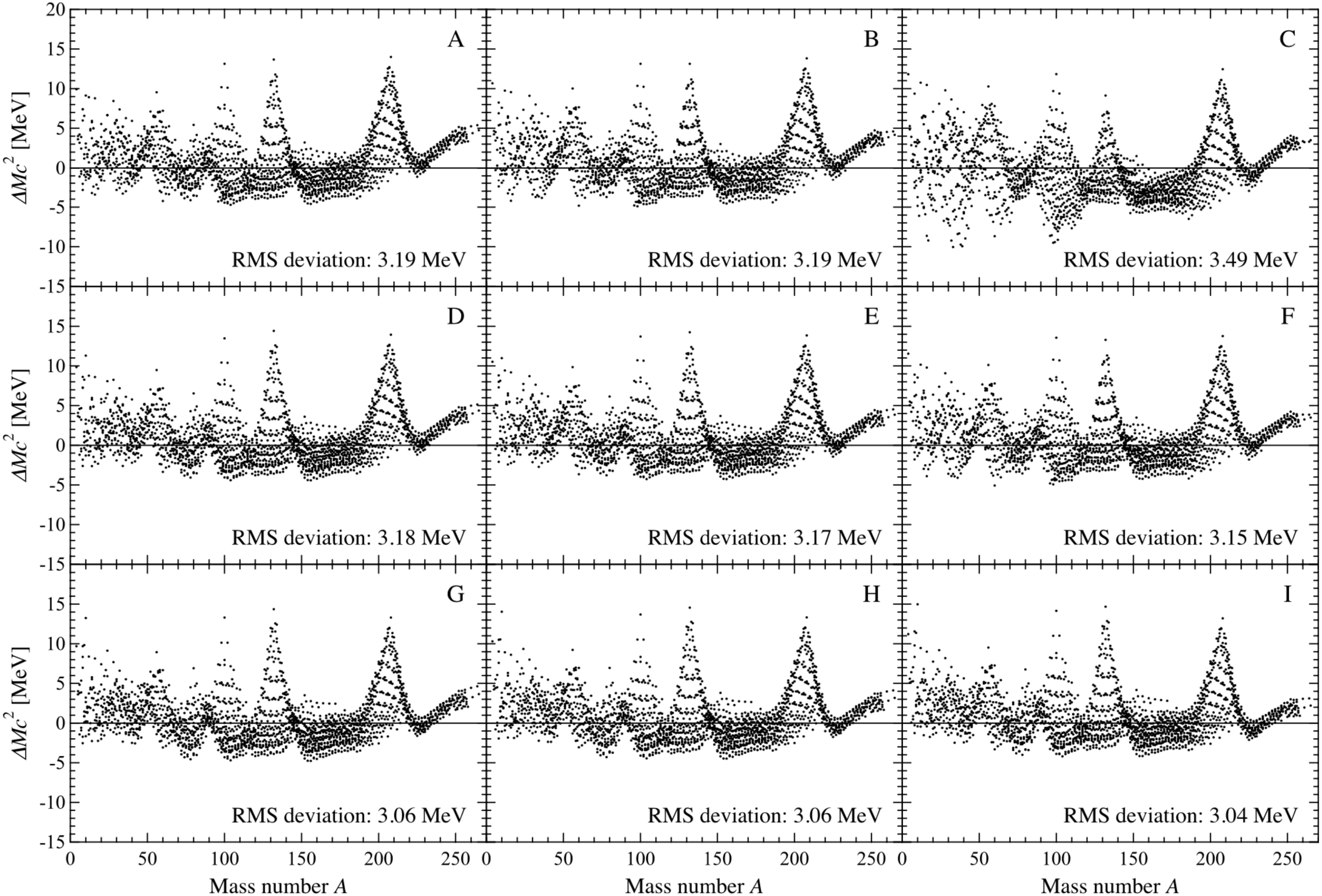


Larger L_0

Smaller L_0

Smaller K_0 ⟷ Larger K_0

Thomas-Fermi calculation for isolated atomic nuclei



2. Supernova EOS with Macroscopic Nuclear Model

Macroscopic EOS for uniform matter by Oyamatsu & Iida

(PRC 75 (2007) 015801)

Energy per nucleon at zero temperature

$$E(n_{\mathrm{B}}, Y_{\mathrm{p}}) = E_{\mathrm{F}} + [1 - (1-2Y_{\mathrm{p}})^2] v_{\mathrm{s}}(n_{\mathrm{B}}) + (1-2Y_{\mathrm{p}})^2 v_{\mathrm{n}}(n_{\mathrm{B}})$$

– E_{F}: One-body kinetic energy per particle for the Fermi-gas

- Potential energy per particle for symmetric and neutron matter

$$v_{\mathrm{s}}(n_{\mathrm{B}}) = a_1 n_{\mathrm{B}} + \frac{a_2 n_{\mathrm{B}}^2}{1 + a_3 n_{\mathrm{B}}} \qquad v_{\mathrm{n}}(n_{\mathrm{B}}) = b_1 n_{\mathrm{B}} + \frac{b_2 n_{\mathrm{B}}^2}{1 + b_3 n_{\mathrm{B}}}$$

Parameters (a_1, a_2, a_3, b_1, b_2, b_3) are determined

Thomas-Fermi calculation of **isolated atomic nuclei** with E reproduces the gross feature of **experimental data (masses and radii)**.

2. Supernova EOS with Macroscopic Nuclear Model

Macroscopic EOS for uniform matter by Oyamatsu & Iida

(PRC 75 (2007) 015801)

Free energy per nucleon at finite temperature

$$F(n_{\mathrm{B}}, Y_{\mathrm{p}}, T) = F_{\mathrm{F}} + [1-(1-2Y_{\mathrm{p}})^2]v_{\mathrm{s}}(n_{\mathrm{B}}) + (1-2Y_{\mathrm{p}})^2 v_{\mathrm{n}}(n_{\mathrm{B}})$$

– F_{F}: Free energy per particle for the Fermi-gas

- Potential energy per particle for symmetric and neutron matter

$$v_{\mathrm{s}}(n_{\mathrm{B}}) = a_1 n_{\mathrm{B}} + \frac{a_2 n_{\mathrm{B}}^2}{1 + a_3 n_{\mathrm{B}}} \qquad v_{\mathrm{n}}(n_{\mathrm{B}}) = b_1 n_{\mathrm{B}} + \frac{b_2 n_{\mathrm{B}}^2}{1 + b_3 n_{\mathrm{B}}}$$

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Thomas-Fermi calculation of **isolated atomic nuclei** with E reproduces the gross feature of **experimental data (masses and radii)**.

Nuclear EOS for non-uniform matter

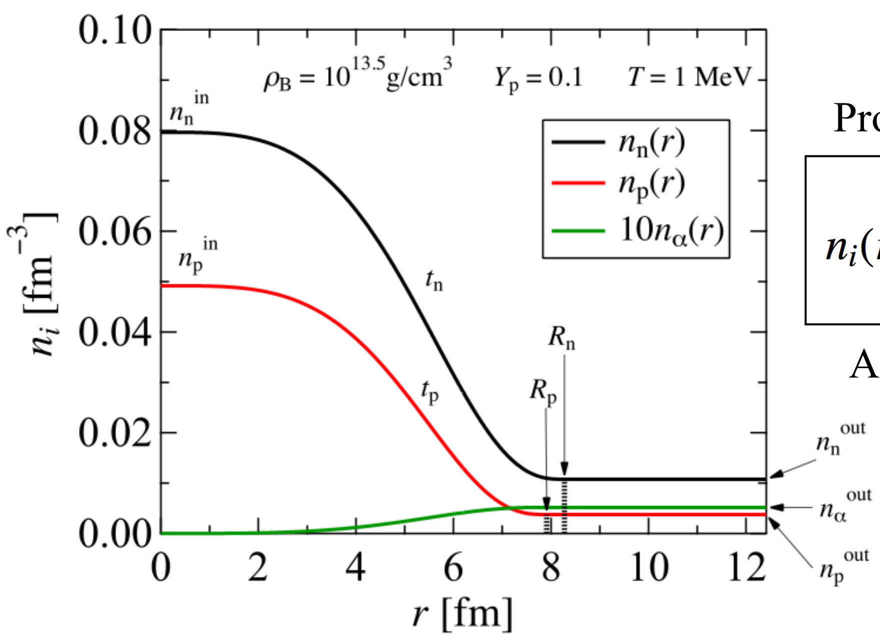
*We use the **Thomas-Fermi method**.* (PTP 100 (1998) 1013, APJS 197(2011) 20)

Free energy of a Wigner-Seitz cell

$$F = \int d\boldsymbol{r} \underbrace{f(n_{\mathrm{p}}(r), n_{\mathrm{n}}(r), n_\alpha(r))}_{\text{Bulk energy}} + \underbrace{F_0 \int d\boldsymbol{r} |\nabla(n_{\mathrm{p}}(r) + n_{\mathrm{n}}(r))|^2}_{\text{Gradient energy}}$$

$$+ \underbrace{\frac{e^2}{2} \int d\boldsymbol{r} \int d\boldsymbol{r}' \frac{[n_{\mathrm{p}}(r) + 2n_\alpha(r) - n_{\mathrm{e}}][n_{\mathrm{p}}(r') + 2n_\alpha(r') - n_{\mathrm{e}}]}{|\boldsymbol{r} - \boldsymbol{r}'|} + c_{\mathrm{bcc}} \frac{(Ze)^2}{a}}_{\text{Coulomb energy}}$$

Free energy density of uniform matter: $f = f_{\mathrm{N}} + f_\alpha$



Particle number density distributions

Protons and neutrons (i = p, n)

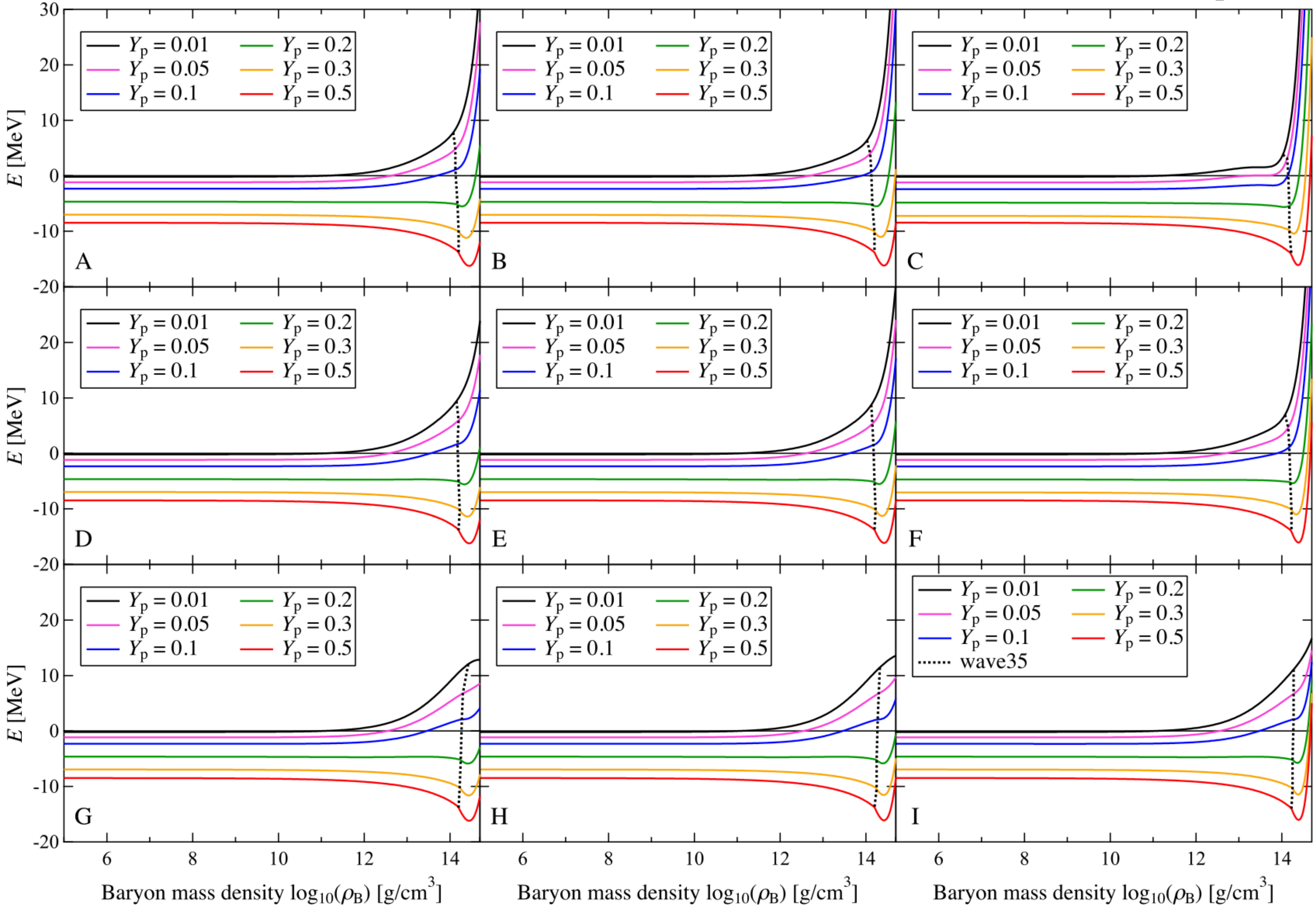
$$n_i(r) = \begin{cases} (n_i^{\mathrm{in}} - n_i^{\mathrm{out}})[1 - (r/R_i)^{t_i}]^3 + n_i^{\mathrm{out}} & (0 \le r \le R_i) \\ n_i^{\mathrm{out}} & (R_i \le r \le R_{\mathrm{cell}}) \end{cases}$$

Alpha-particles

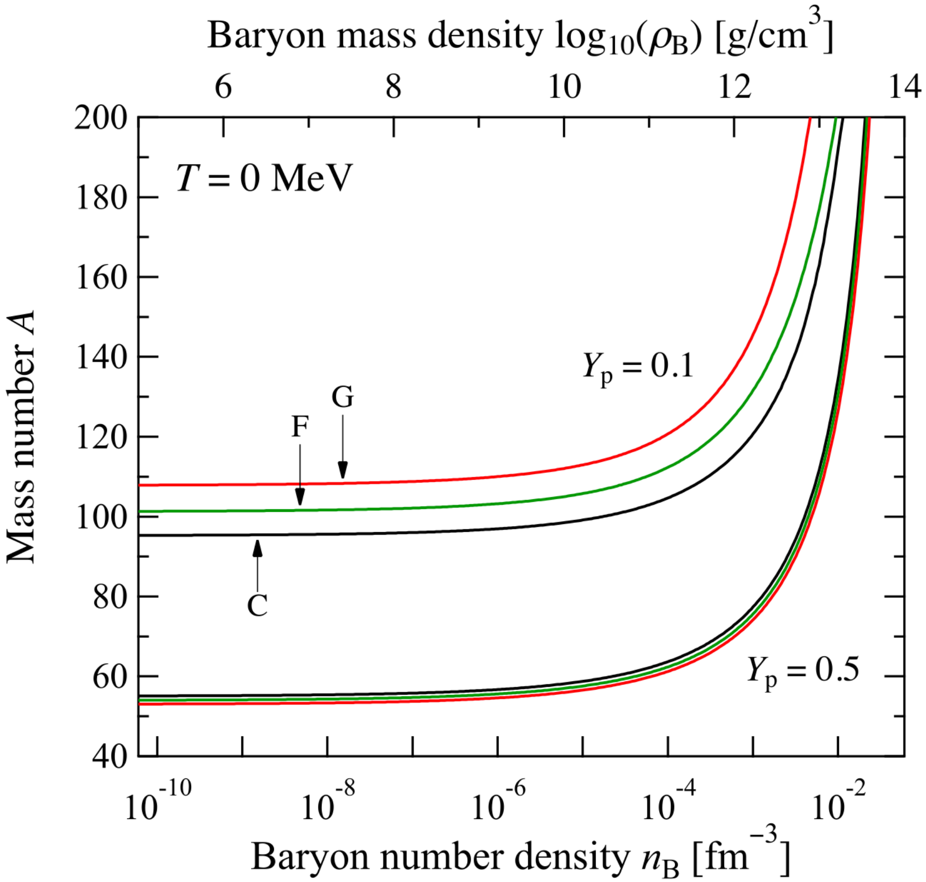
$$n_\alpha(r) = \begin{cases} -n_\alpha^{\mathrm{out}}[1 - (r/R_{\mathrm{p}})^{t_{\mathrm{p}}}]^3 + n_\alpha^{\mathrm{out}} & (0 \le r \le R_{\mathrm{p}}) \\ n_\alpha^{\mathrm{out}} & (R_{\mathrm{p}} \le r \le R_{\mathrm{cell}}) \end{cases}$$

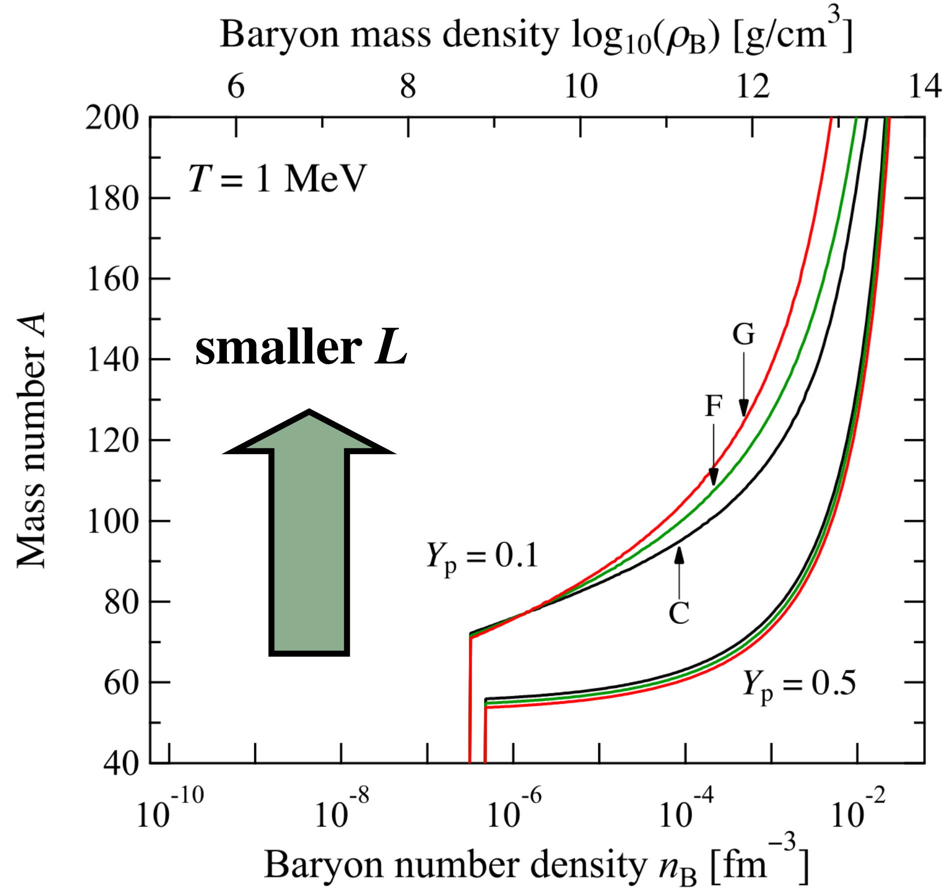
3. Numerical Results

Energy per particle at 0 MeV for various proton fractions Y_p



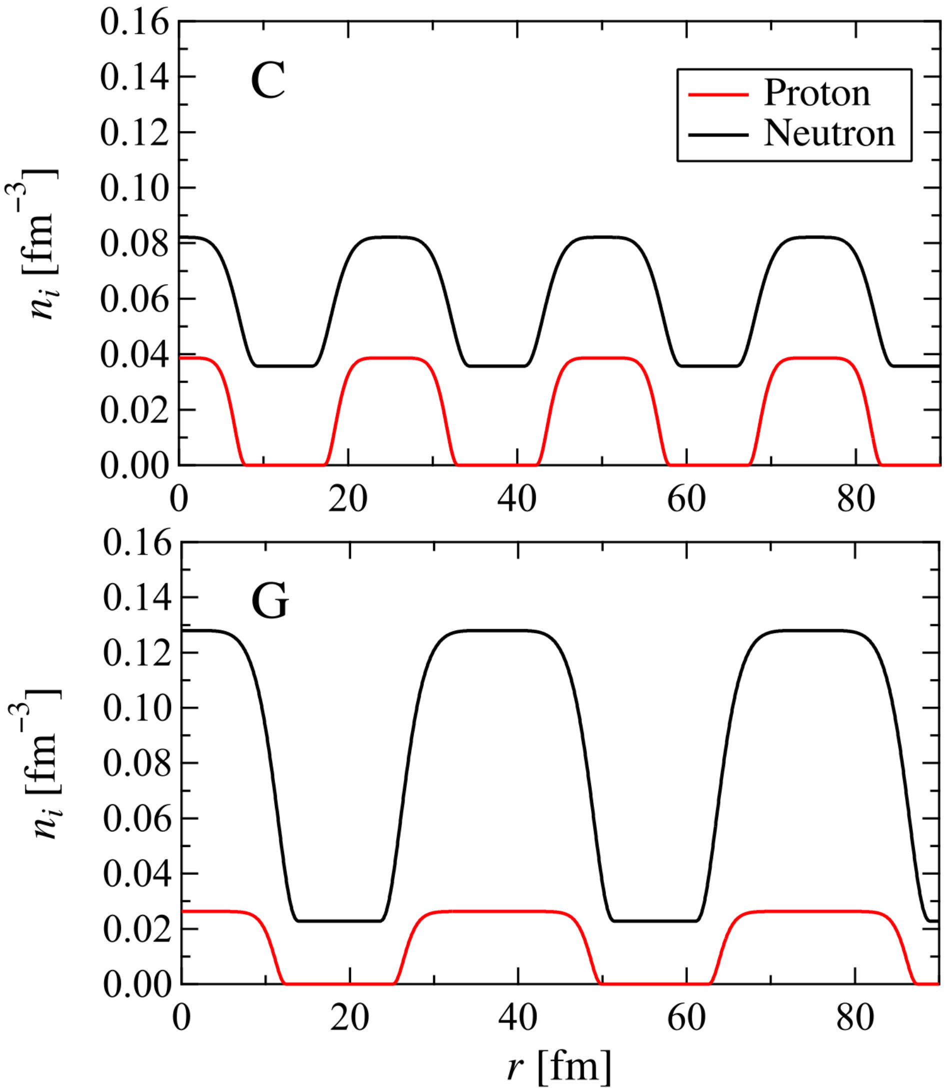
Mass number of heavy nuclei in non-uniform phase



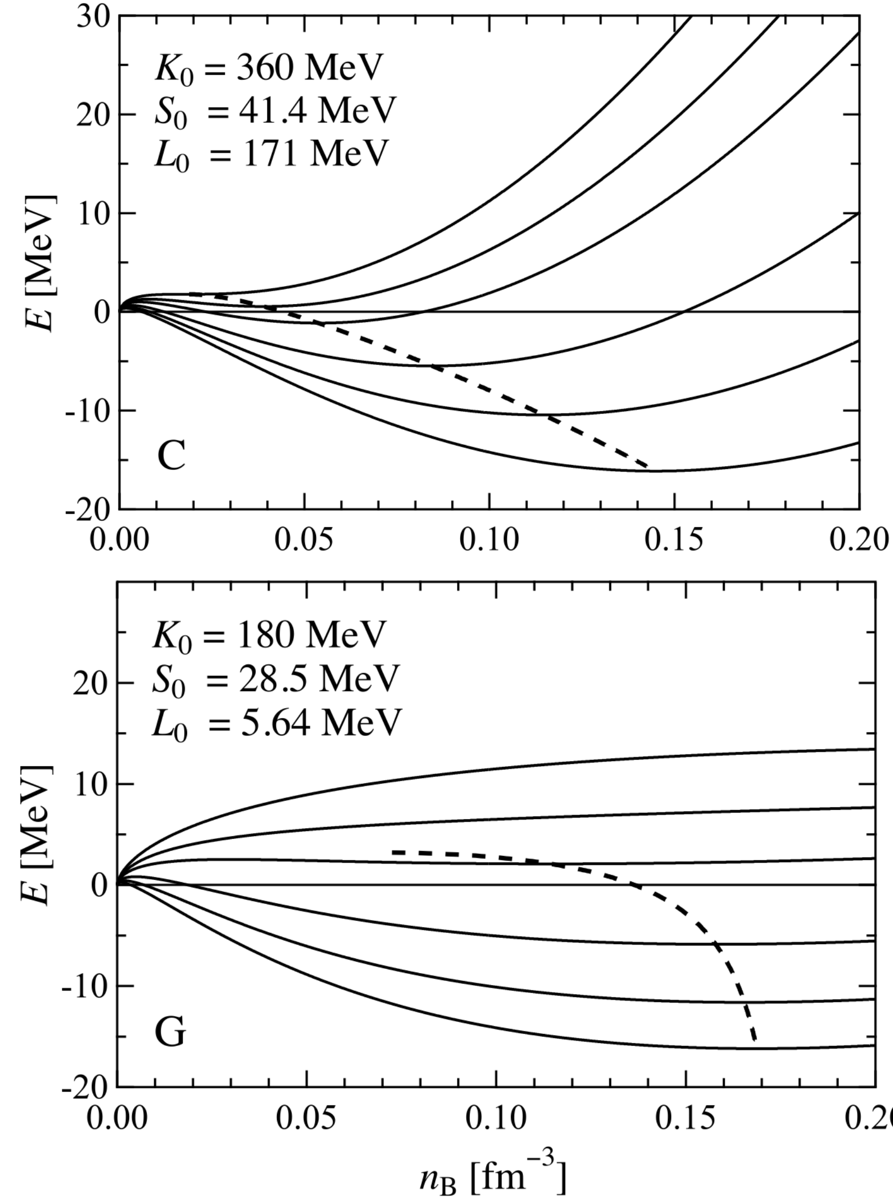


The smaller value of L → Larger mass number in neutron-rich matter

Density distributions in a Wigner-Seitz cell

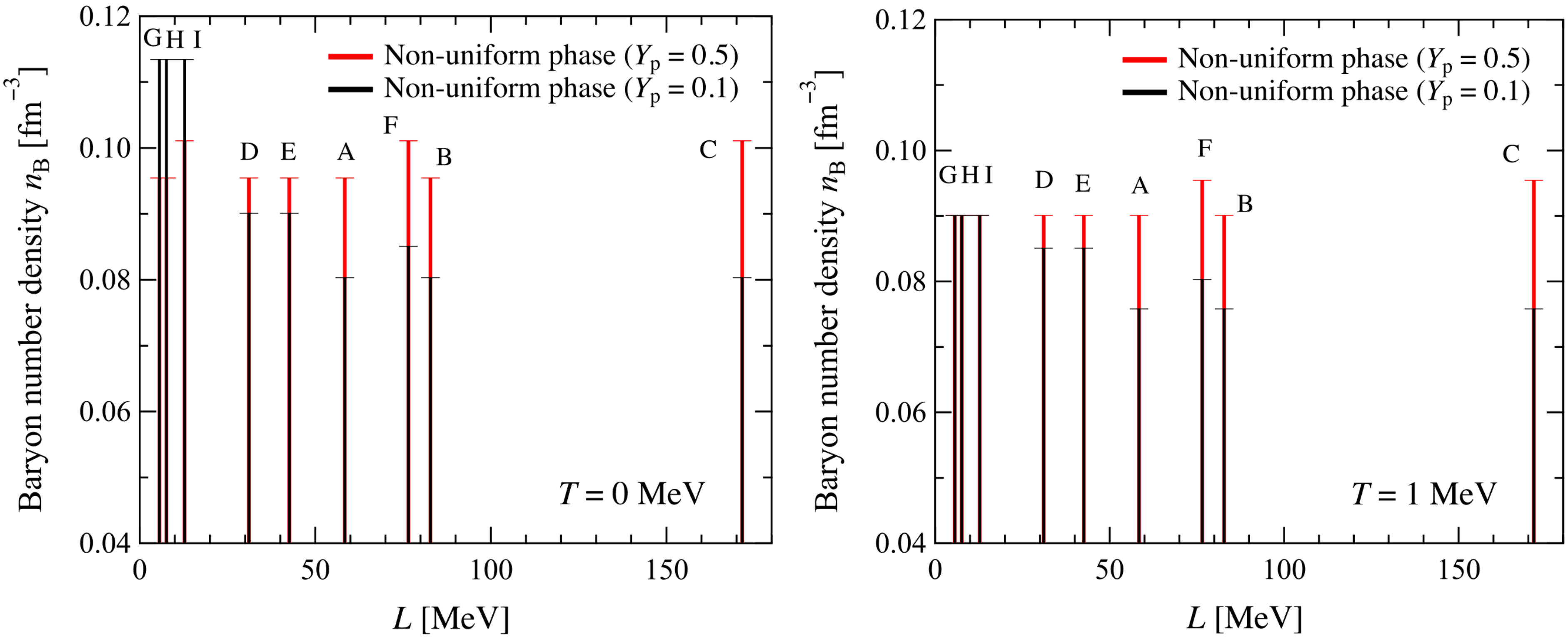


Density distributions in a Wigner-Seitz cell
(Y_p =0.1 $\rho = 10^{13.9}$ g/cm^3)



Energy per particle for uniform matter

Critical density for phase transition



The smaller value of *L* → Higher critical density in neutron-rich matter

Summary

I am constructing a new family of supernova EOSs to investigate the effects of nuclear saturation properties on the mechanism of core-collapse supernovae.

- **EOS for uniform matter: parameterized macroscopic model**
- **EOS for non-uniform matter: Thomas-Fermi method**

As the value of *L* becomes smaller,

- The critical density from non-uniform matter to uniform matter is higher.
- Masses of heavy nuclides are slightly larger in neutron-rich nuclear matter.

Future Plans

- Completion of the EOS tables for core-collapse supernova
- Application of the EOSs to supernova simulations
- Taking into account the pasta phase in the low-temperature region