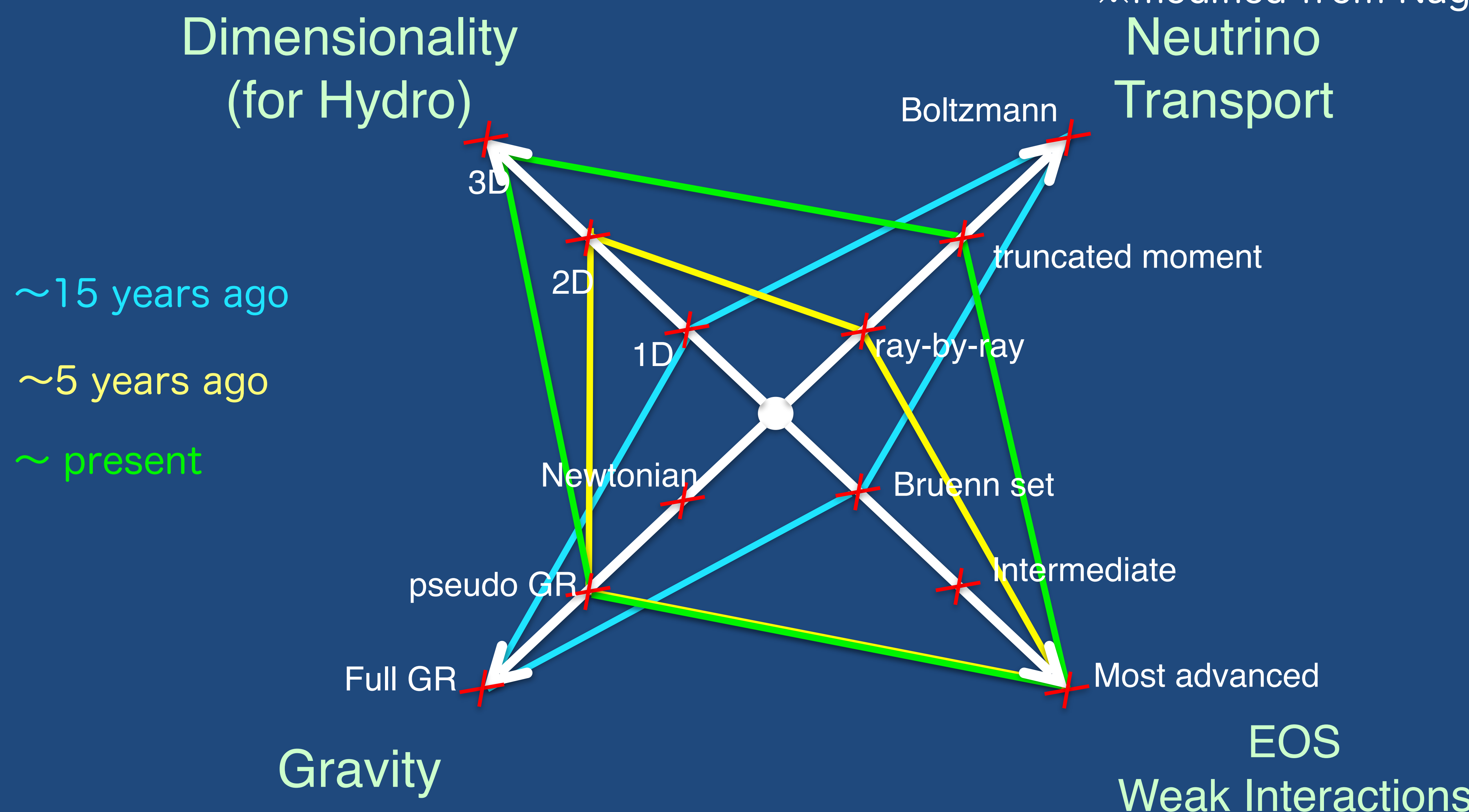
Fast Neutrino-Flavor Conversions in Core-Collapse Supernovae

C03: Applications of Boltzmann Simulations

Shoichi YAMADA
Waseda University

Progress in CCSN Modeling

※modified from Nagakura's slide

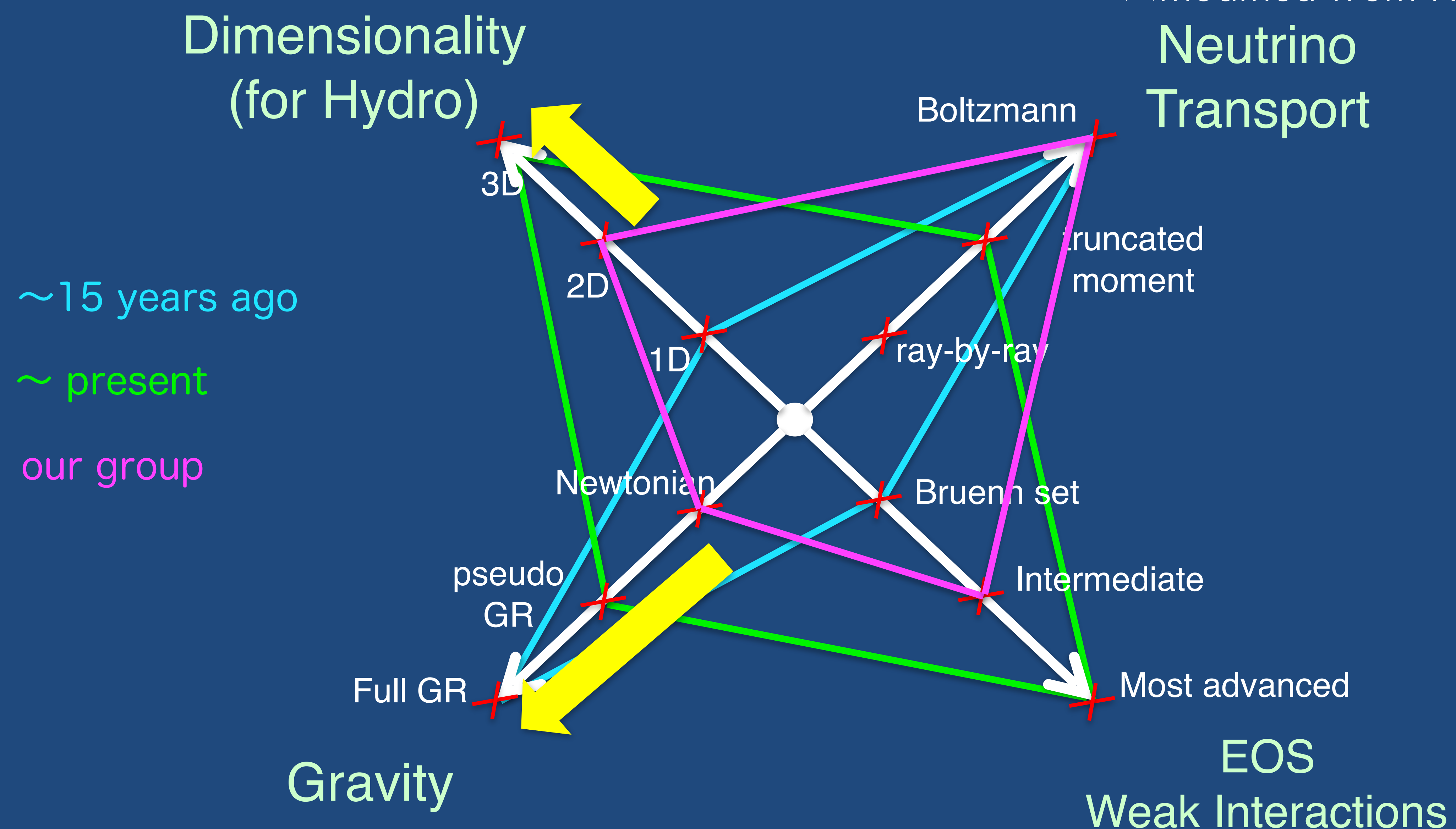


✓ In the last decade, most of the world major groups modeling core-collapse supernovae (CCSNe) numerically have proceeded to 3D in space.

- neutrino transport with the truncated moment method with some closure relation imposed by hand

Progress in CCSN Modeling

※modified from Nagakura's slide

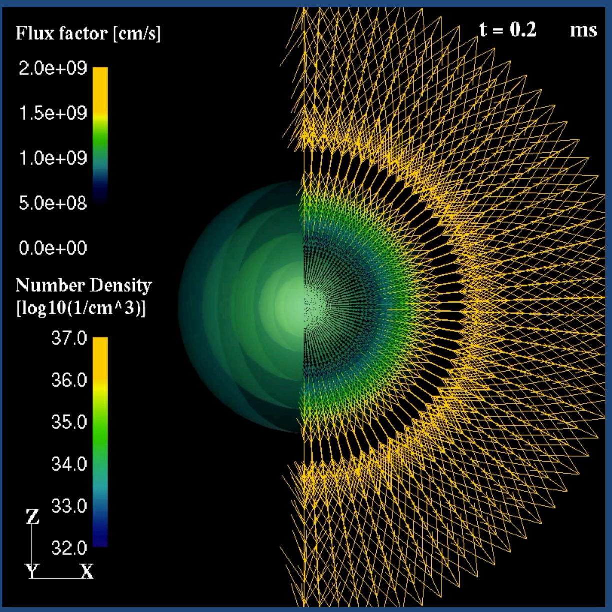


✓ We have stuck to the Boltzmann solver for ν transport.

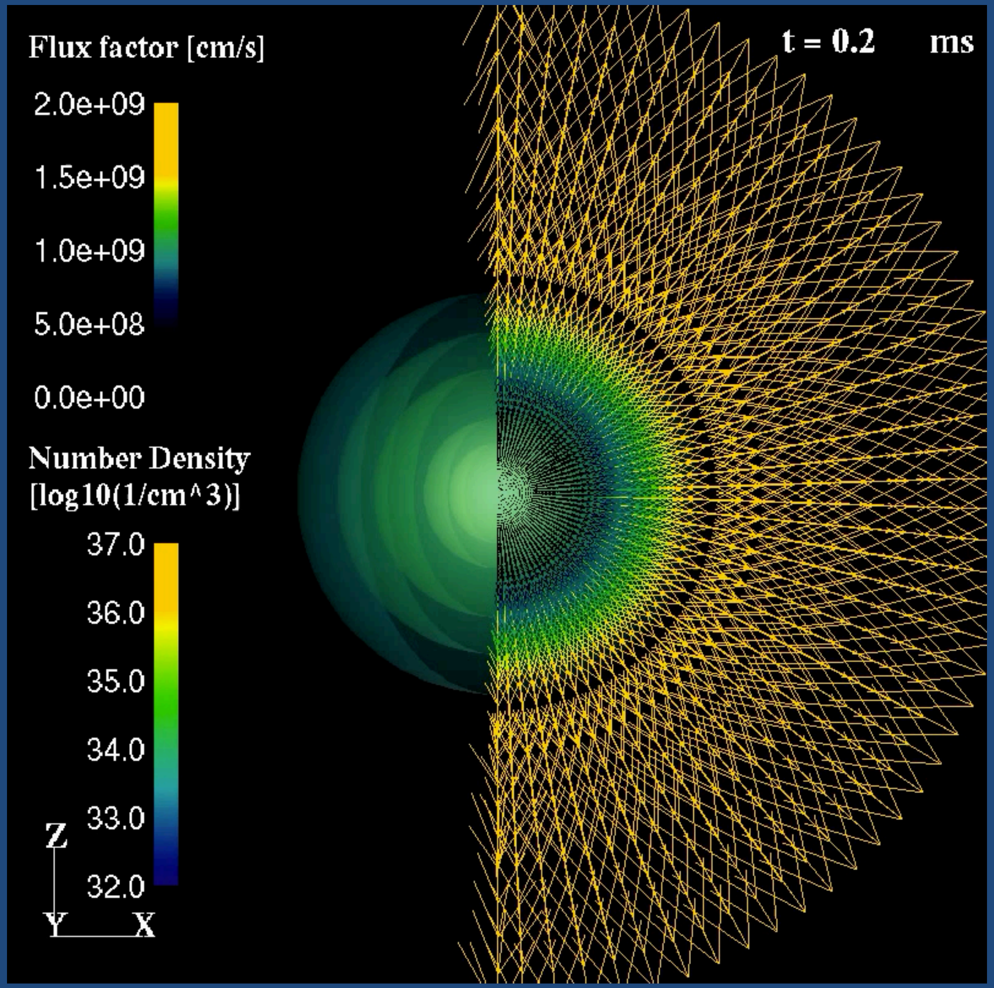
- 2 spatial dimensions under axisymmetry
- currently pushing for full GR and 3D

Neutrino number densities (left) and flux factors (right)

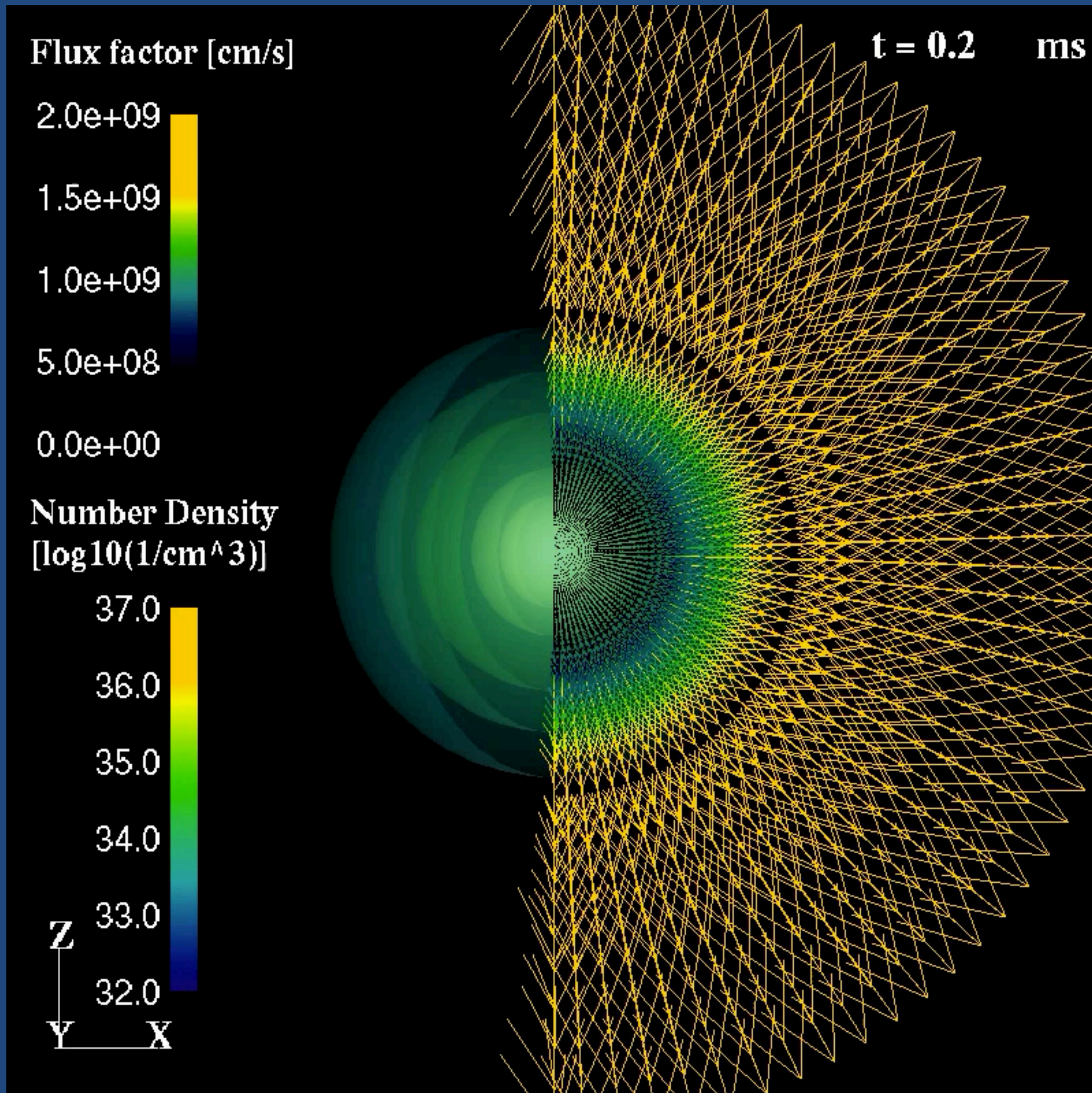
1D



2D



3D



Iwakami et al. '19

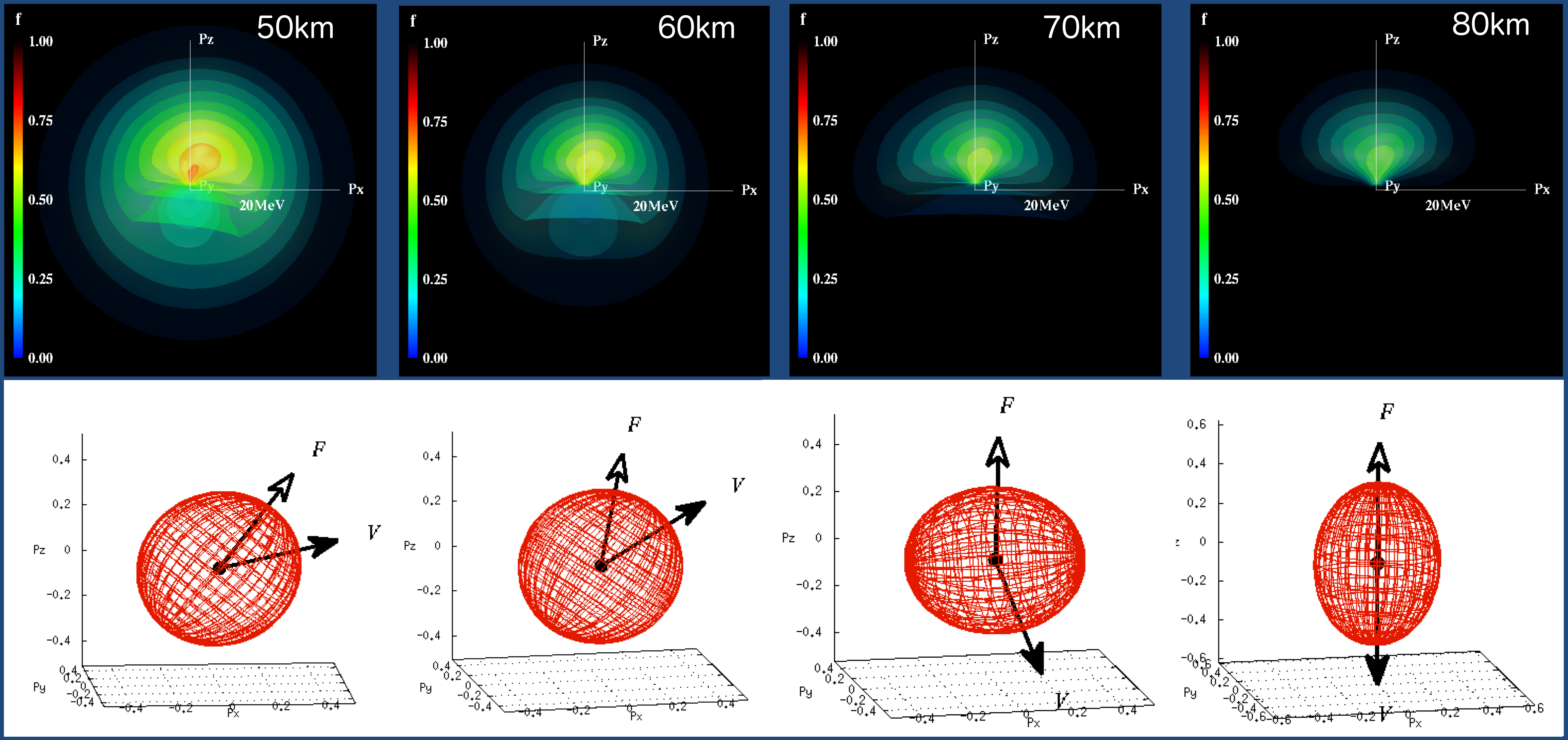
$$Nr \times N_\theta \times N_\phi \times N_{\nu e} \times N_{\nu\theta} \times N_{\nu\phi} = 256 \times 48 \times 96 \times 16 \times 6 \times 6$$

Introduction

✓ The Boltzmann solver enables us to study neutrino distributions in momentum space in detail.

$t_{\rm pb} = 10{\rm ms},\ \theta \approx 90^\circ,\ \phi \approx 340^\circ$

Iwakami et al. '19



✓ It will help us calibrate the closure relation.

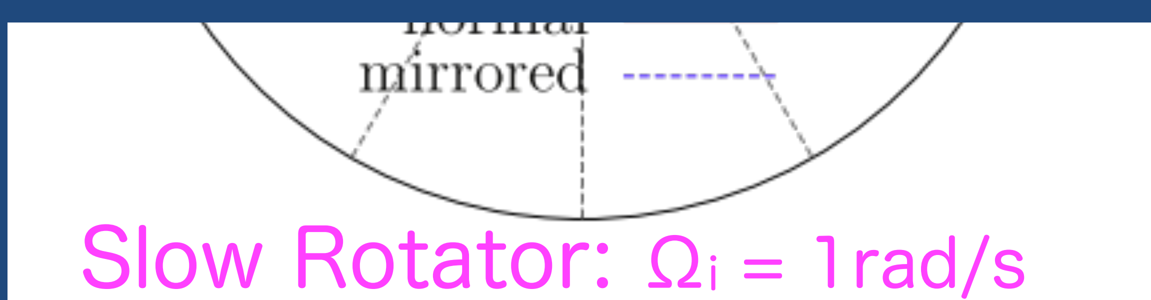
Introduction

✓ Neutrino distributions in momentum space are non-axisymmetric in general even in 2D in space.

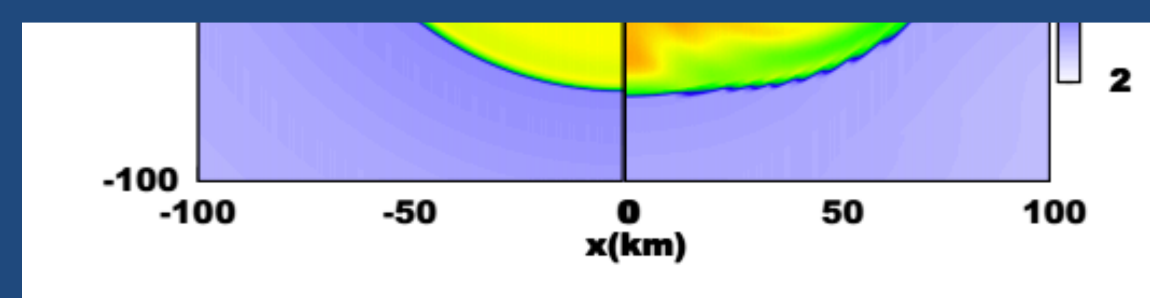
✓ The principal axes are not aligned with coordinates.

The Boltzmann solver puts us also in a unique position in the study of neutrino oscillations in CCSNe.

- The fast conversion mode feeds on the angular distribution difference between ν_e and $\bar{\nu}_e$.

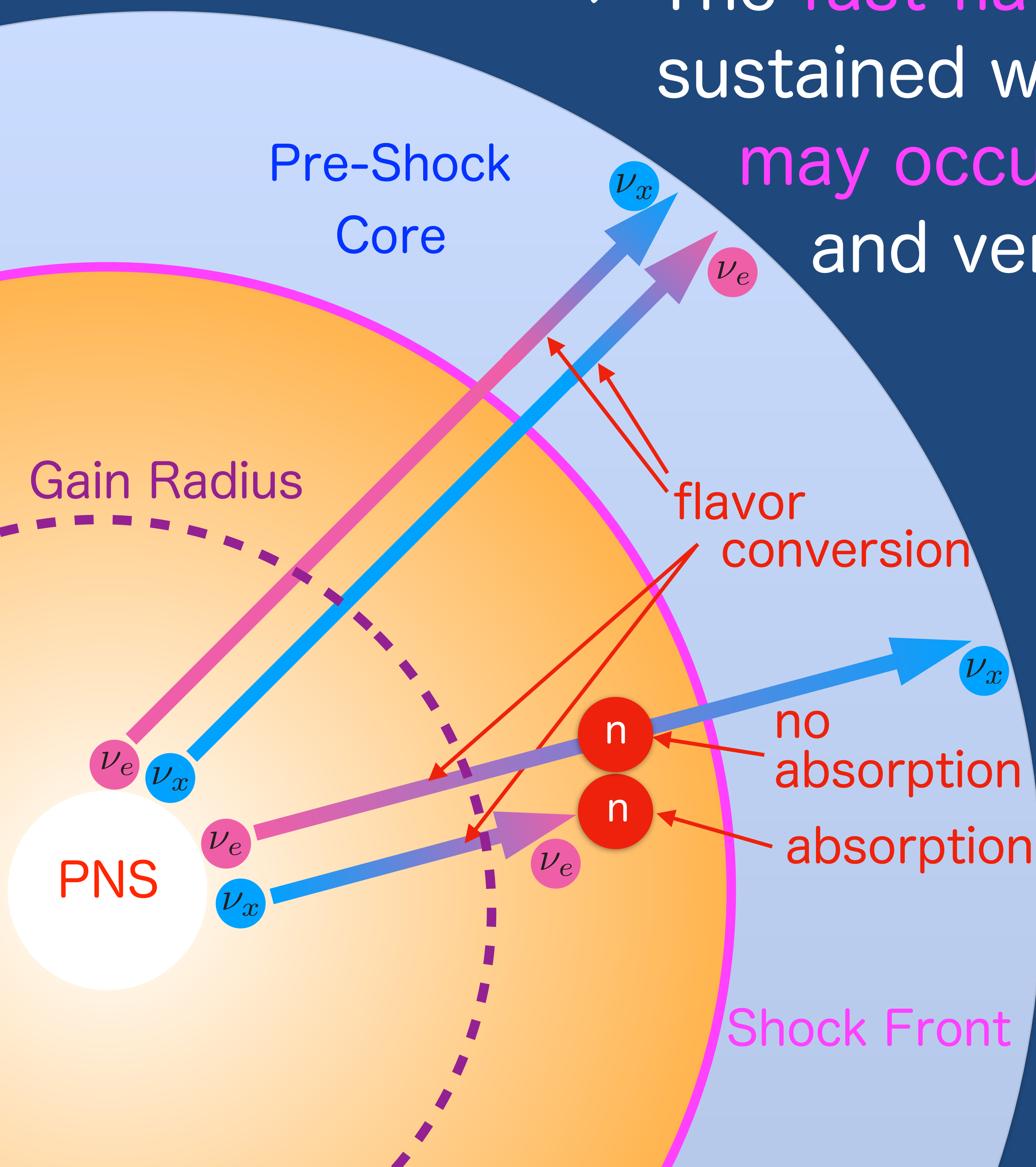


Harada et al. ’19



Nagakura et al. ’19

Fast Neutrino-Flavor Conversion



- ✓ The fast flavor conversion is self-sustained without the mass term and may occur near the neutrino sphere and very rapidly. Sawyer '05

  ν-potential: $\mu \sim \sqrt{2}\, G_{\rm F}\, n_\nu$

- ✓ If true, it may affect the supernova explosion, nucleosynthesis and the observation of ν.

- ✓ It feeds on the difference in angular distributions of ν_e and $\bar{\nu}_e$.

Crossing in ν Angular Distributions

✓ The fast flavor conversion is a nonlinear phenomenon but its onset can be studied linearly.

Density Matrix (in 2-flavor approx.)

$$\rho = \frac{f_{\nu_e} + f_{\nu_x}}{2} + \frac{f_{\nu_e} - f_{\nu_x}}{2} \begin{pmatrix} s_{\mathbf{p}} & S_{\mathbf{p}} \\ S^*_{\mathbf{p}} & -s_{\mathbf{p}} \end{pmatrix}$$

EOM for the small off-diagonal component

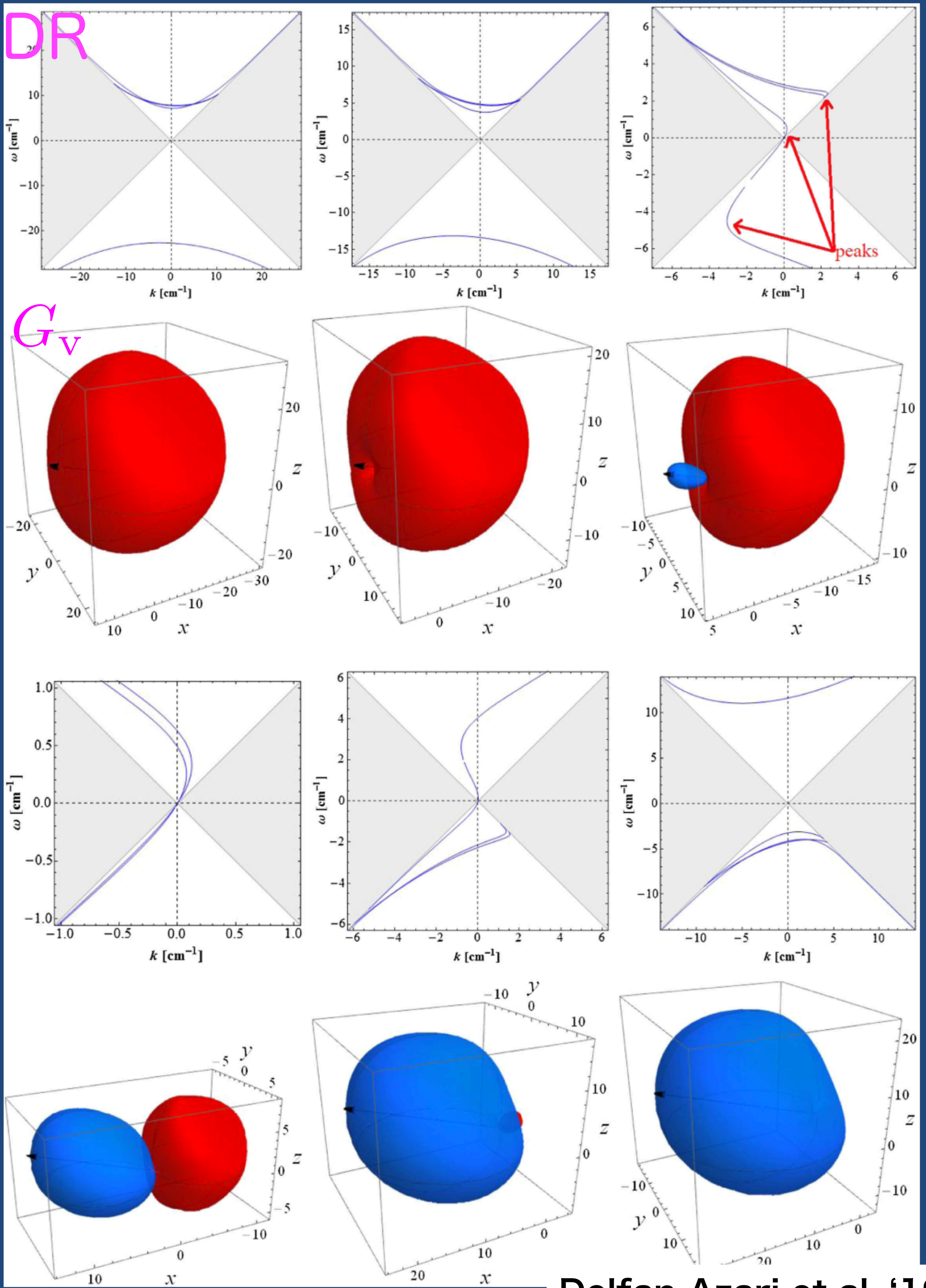
$$i(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{r}})S_{\mathbf{v}} = v^{\mu}(\Lambda_{\mu} + \Phi_{\mu})S_{\mathbf{v}} - \int d\Upsilon' v^{\mu} v'_{\mu} G_{\mathbf{v}'} S_{\mathbf{v}'}$$

→ Dispersion Relation (DR)

$$D(\omega, \boldsymbol{k}) = 0$$

✓ The angular distributions are the important ingredient for the fast flavor conversion.

$$G_{\mathbf{v}} = \sqrt{2} G_F \int_0^{\infty} \frac{dE E^2}{2\pi^2} [f_{\nu_e}(E,\mathbf{v}) - f_{\bar{\nu}_e}(E,\mathbf{v})]$$



Delfan Azari et al. '19

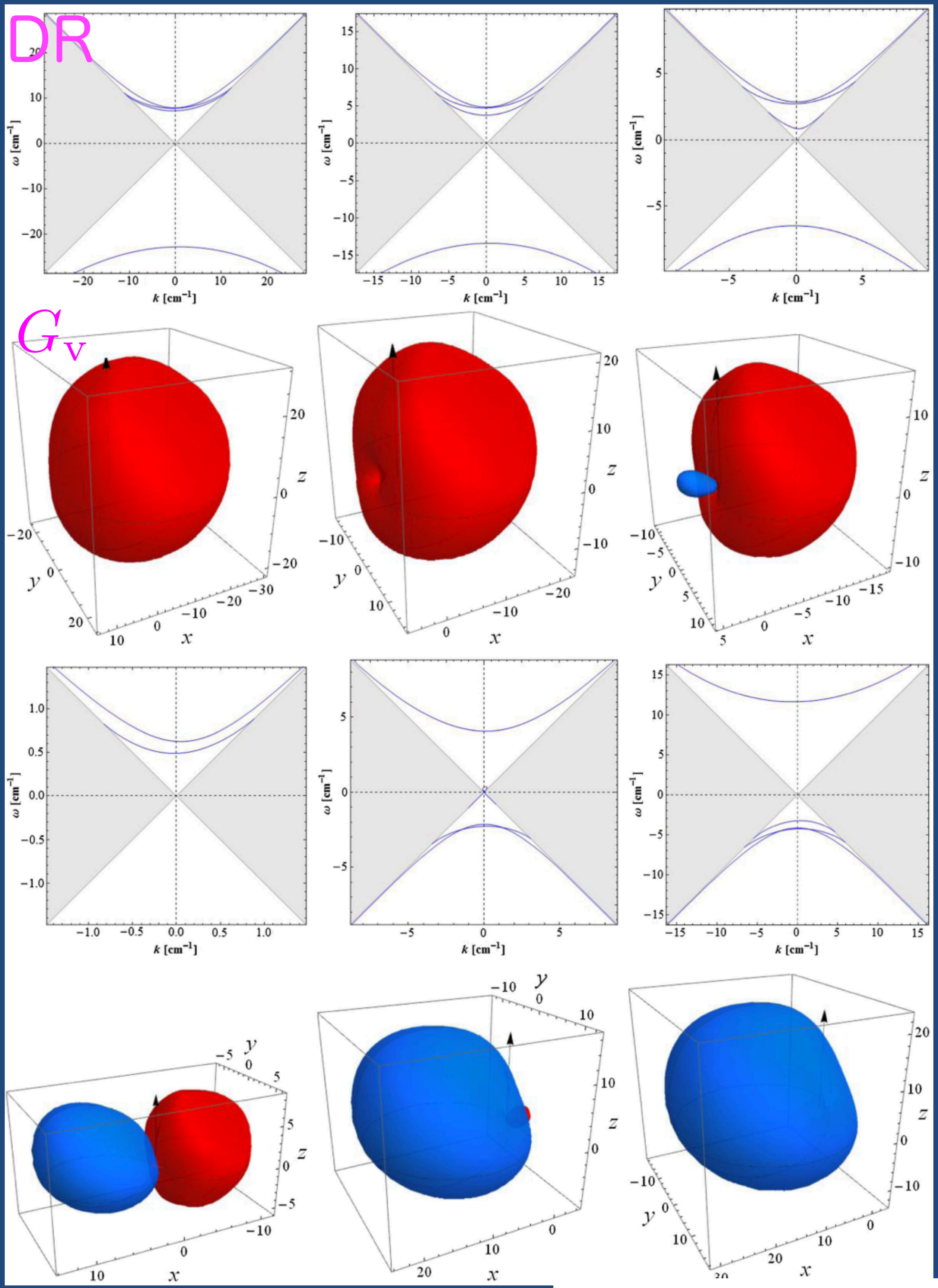
Crossing in ν Angular Distributions

DR depends on the direction of **k**.

✓ The sign change of G_v, or the crossing in angular distributions, seems indeed to be the indication of the fast flavor conversion.

$$G_{\mathbf{v}} = \sqrt{2} G_F \int_0^\infty \frac{dE E^2}{2\pi^2} [f_{\nu_e}(E, \mathbf{v}) - f_{\bar{\nu}_e}(E, \mathbf{v})]$$

We will hence look for the crossing first.

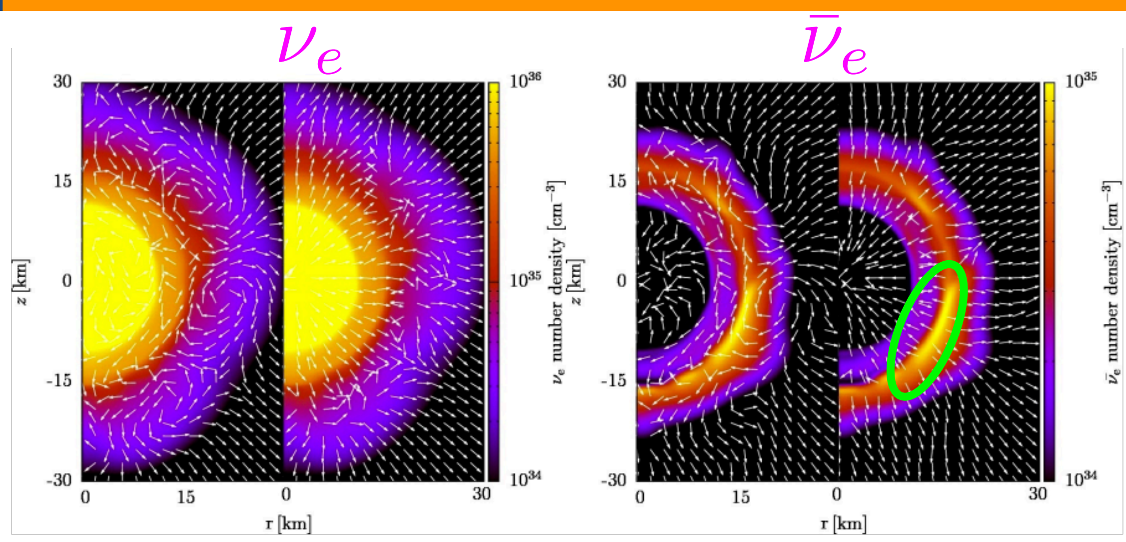


Delfan Azari et al. '19

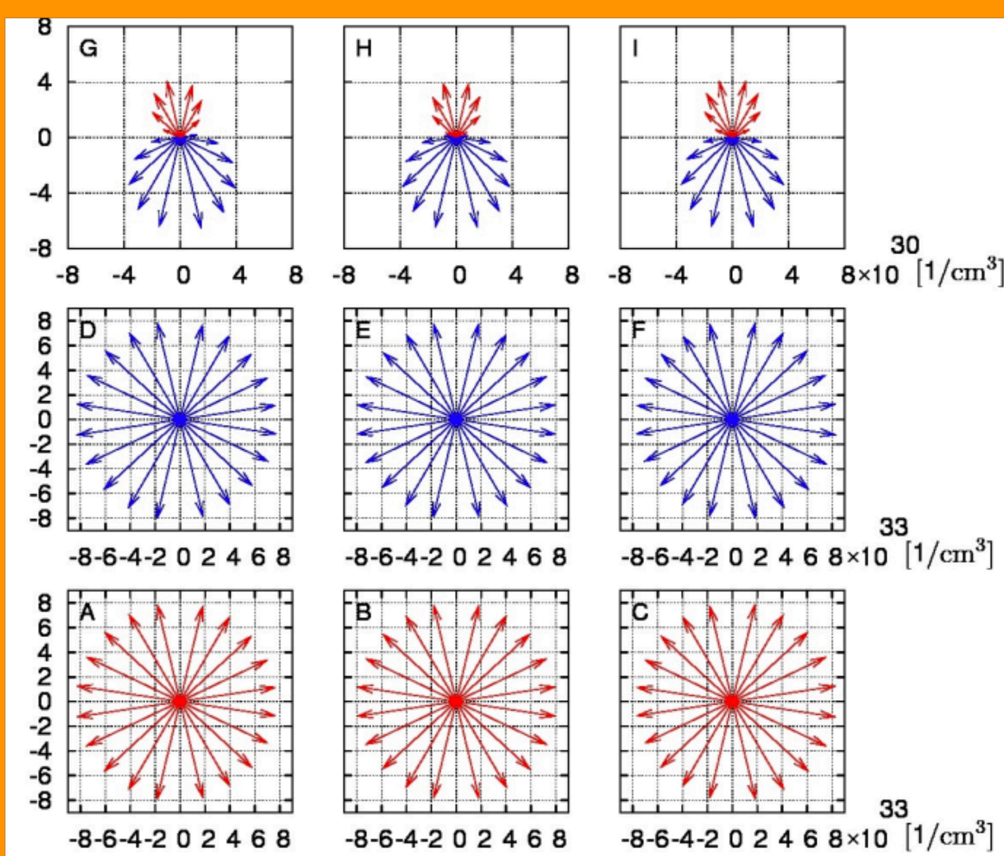
We have spotted three possible cases.

✓ Insie ν sphere

Delfan Azari et al. '19

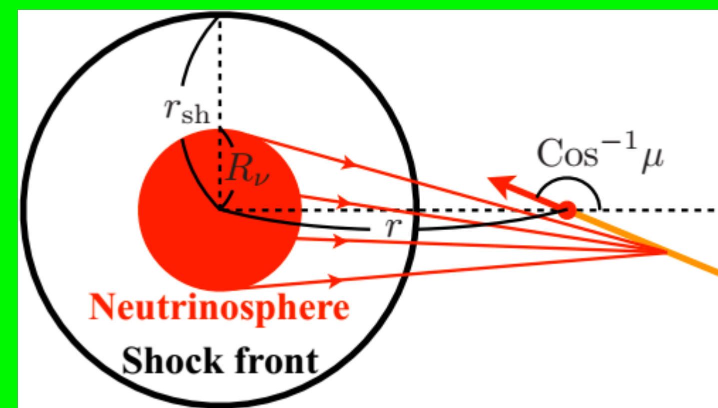


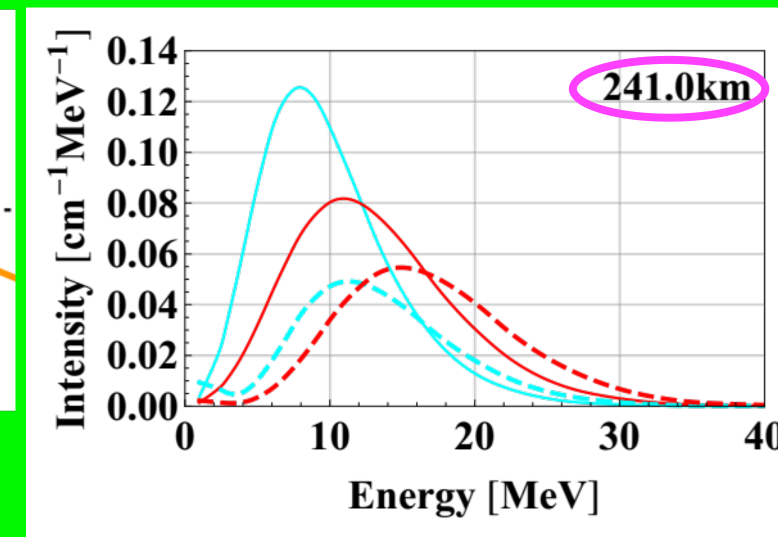
Angular Distributions of ν_eと$\bar{\nu}_e$



✓ Pre-Shock Region

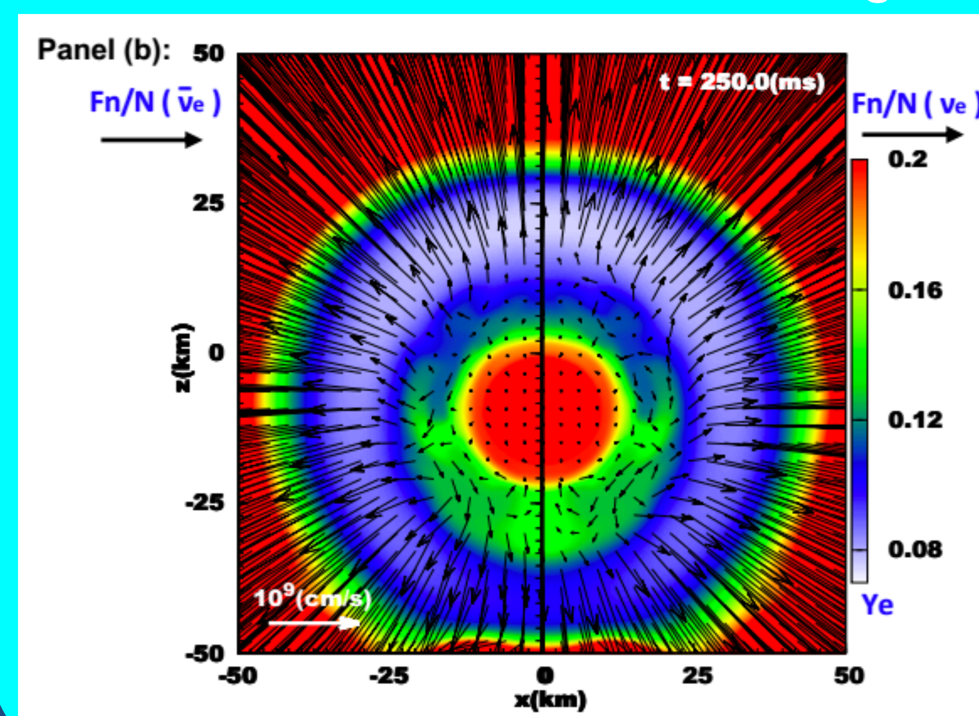
Morinaga et al. '19

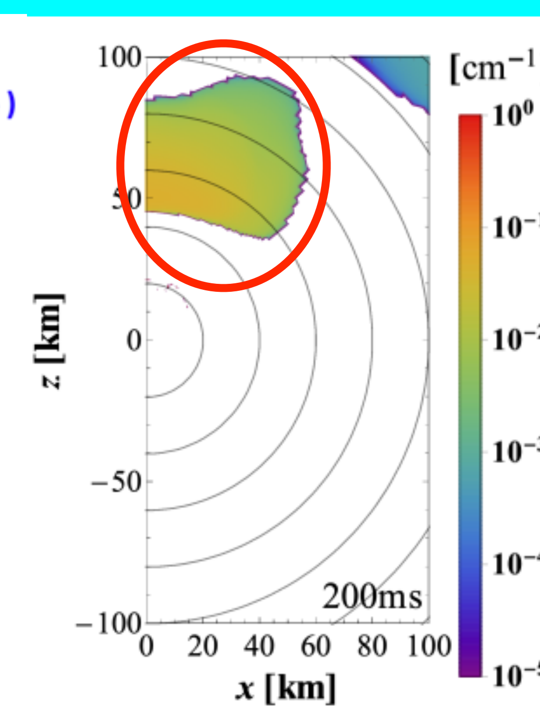




✓ Post-Shock Region

Nagakura et al. '19



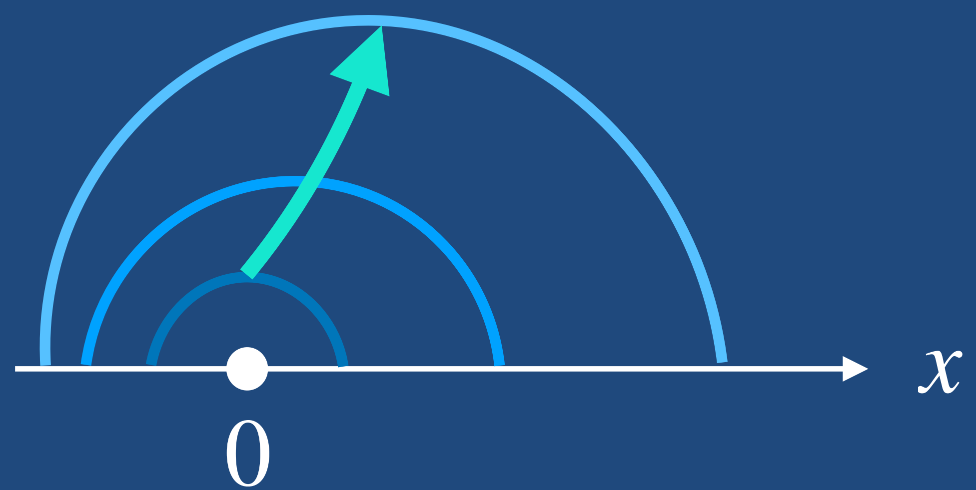


Spatio-Temporal Instability in (3+1)-Dimensional Spacetime

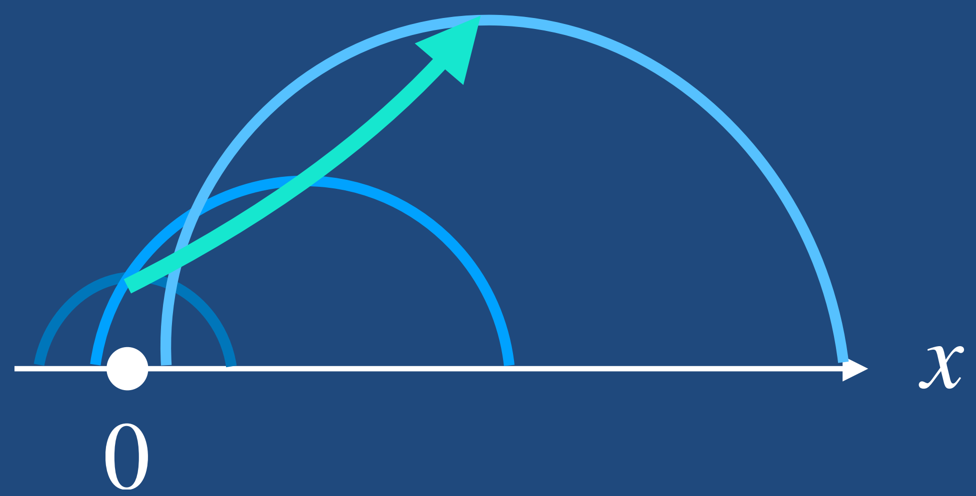
✓ The fast flavor conversion may be described as a temporal growth of a wave packet of perturbation.

✓ It is given by the asymptotic behavior of the Green function: $D(i\partial_t, -i\partial_x)S(t,x) = 0$ ⟹ $D(i\partial_t, -i\partial_x)G(t,x) = \delta(t)\delta(x)$

- There are two types of instability:



Absolute Instability

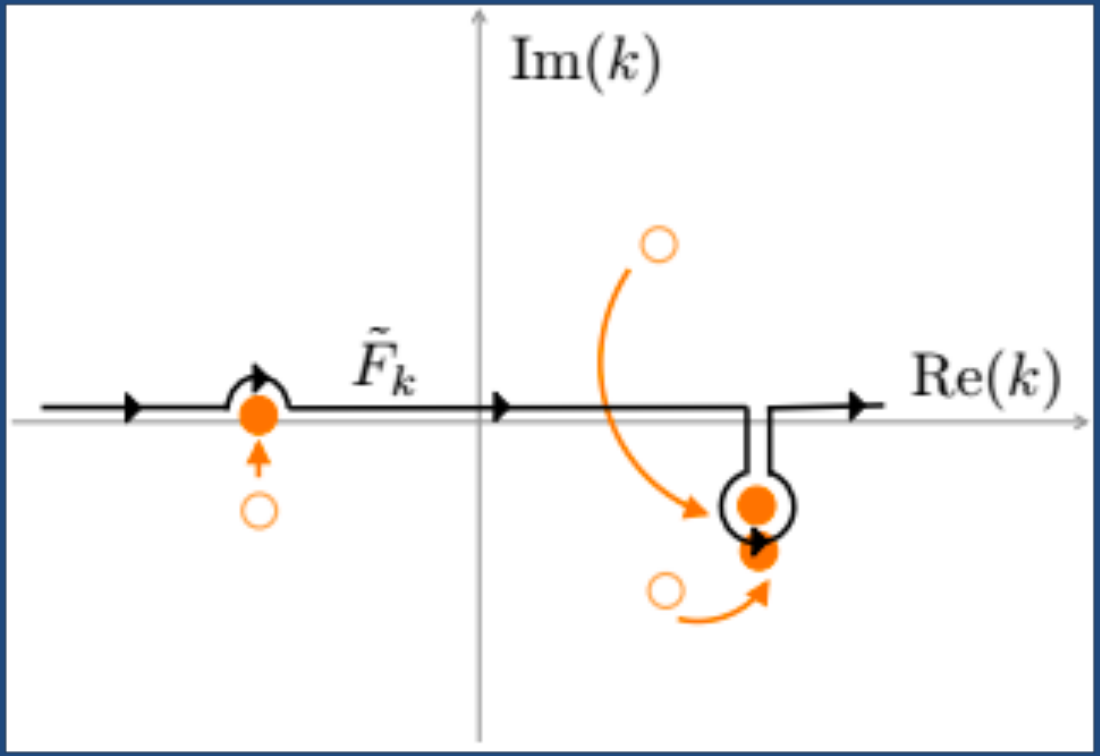


Convective Instability

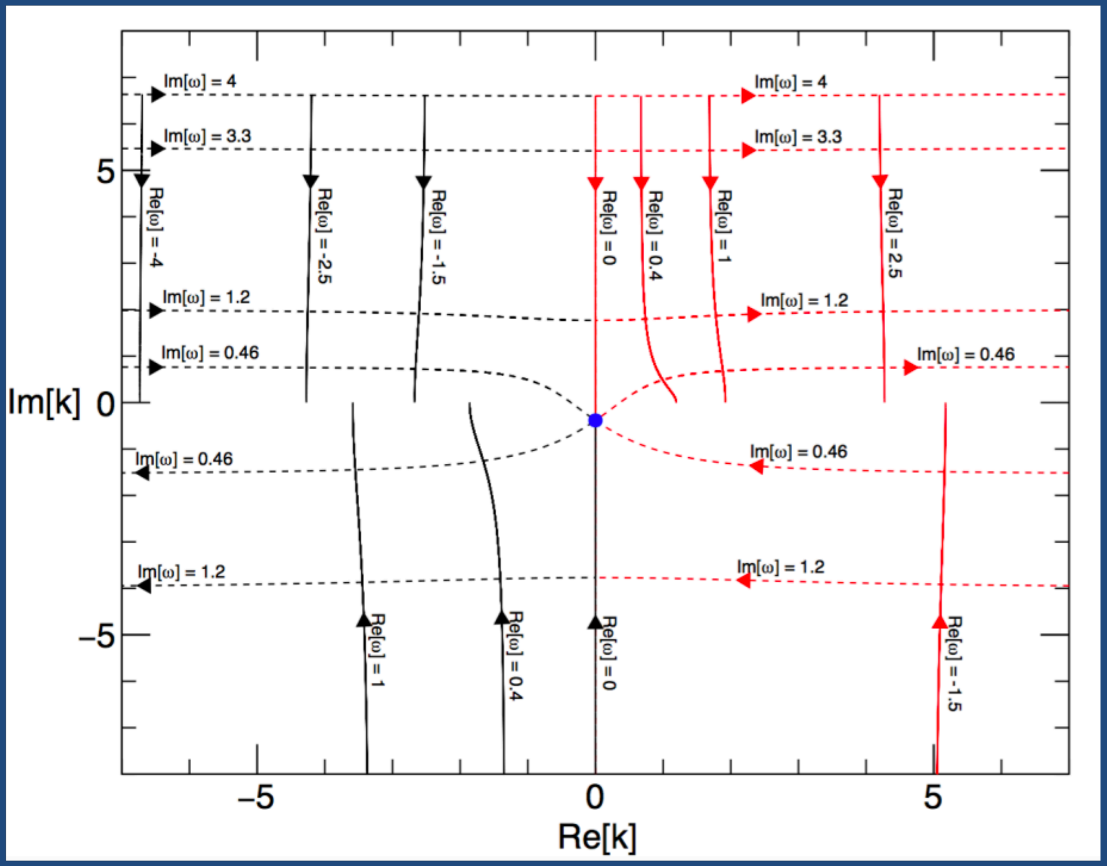
✓ Its linear stability analysis has been conducted so far only in (1+1)-dimensional spacetime.

Spatio-Temporal Instability in (3+1)-Dimensional Spacetime

- ✓ The classical theory of Briggs, which is based on the pinching criterion and is developed in (1+1)-D, is difficult to extend to (3+1)-D.
  - We need to look for pinching by solving DR in the complex space of (ω, **k**), which is all but impossible for a realistic DR.



Capozzi et al. '17



Spatio-Temporal Instability in (3+1)-Dimensional Spacetime

Morinaga & Yamada '19

✓ We propose a new method based on the Lefschetz thimble, which is a generalization of the steepest descent method to complex manifolds.

- We have only to look for critical points that intersect the original integral path via the dual thimble.
- The integral along the steepest descent path can be performed analytically.

Lefschetz Thimble Method

E. Witten (2010)

$$\int_{\mathcal{C}} d^d\boldsymbol{k}\, e^{S(\boldsymbol{k})} f(\boldsymbol{k}) = \int_{\sum_\sigma \langle \mathcal{C}, \mathcal{K}_\sigma \rangle \mathcal{J}_\sigma} d^d\boldsymbol{k}\, e^{S(\boldsymbol{k})} f(\boldsymbol{k})$$

Intersection # of $\mathcal{C}$ and $\mathcal{K}_\sigma$

$\boldsymbol{k}_\sigma$: critical point $(\partial_{\boldsymbol{k}} S(\boldsymbol{k}_\sigma) = \boldsymbol{0})$

$\mathcal{J}_\sigma$: Lefschetz thimble (the steepest descent path of $\mathrm{Re}\, S(\boldsymbol{k})$ from $\boldsymbol{k}_\sigma$

$\mathcal{K}_\sigma$: Dual thimble (the steepest ascent path of $\mathrm{Re}\, S(\boldsymbol{k})$ from $\boldsymbol{k}_\sigma$

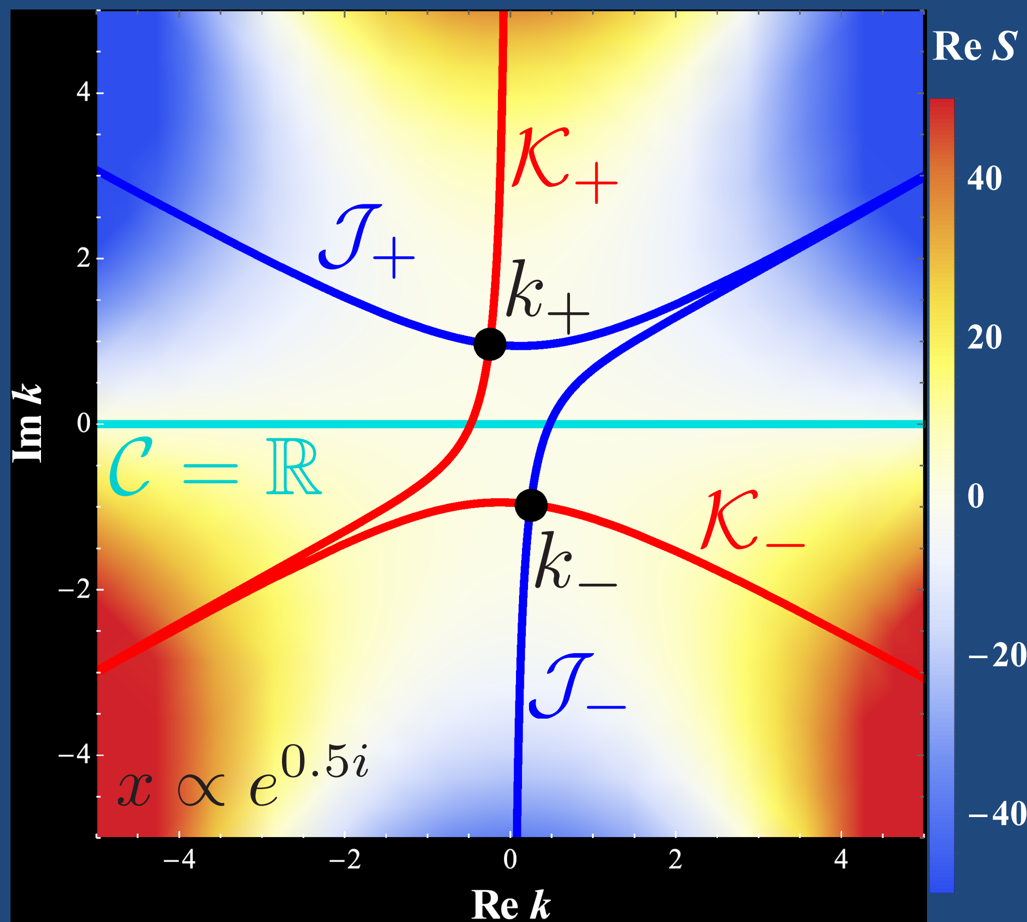
Ex. Airy function

$$\left[S(k,x) \equiv i\left(\frac{k^3}{3} + xk\right)\right]$$

$$\mathrm{Ai}(x) \equiv \int_{\mathbb{R}} \frac{dk}{2\pi} e^{S(k,x)}$$

$$= \int_{\mathcal{J}_+} \frac{dk}{2\pi} e^{S(k,x)}$$

$$\sim \frac{\exp\left(-\frac{2}{3}x^{3/2}\right)}{2\sqrt{\pi}x^{1/4}} \quad \left(\begin{array}{c} x \propto e^{0.5i} \\ |x| \to \infty \end{array}\right)$$



Formulation

Linearized Eq. $\mathbf{D}(i\partial)\boldsymbol{S}(x) = \mathbf{0}$

Green fn. $\mathbf{D}(i\partial)\mathbf{G}(x) = \delta^{(d+1)}(x)\mathbf{I}_N$

$$\mathbf{G}(t, \boldsymbol{x} + \boldsymbol{u}t) = \int_{\mathcal{M}} \frac{d^{d+1}k}{(2\pi)^{d+1}} e^{-ik\cdot ut} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \mathbf{D}(k)^{-1}$$

Residue form theory

$\mathcal{M}$: Laplace-Fourier contour

$$\mathbf{G}(t, \boldsymbol{x} + \boldsymbol{u}t) = \frac{\theta(t)}{(2\pi)^d i} \int_{\mathcal{C}} d^d\boldsymbol{k} \frac{e^{-ik\cdot ut} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}}{\partial_0 \Delta(k)} \operatorname{adj} \mathbf{D}(k)$$

Lefschetz thimble method

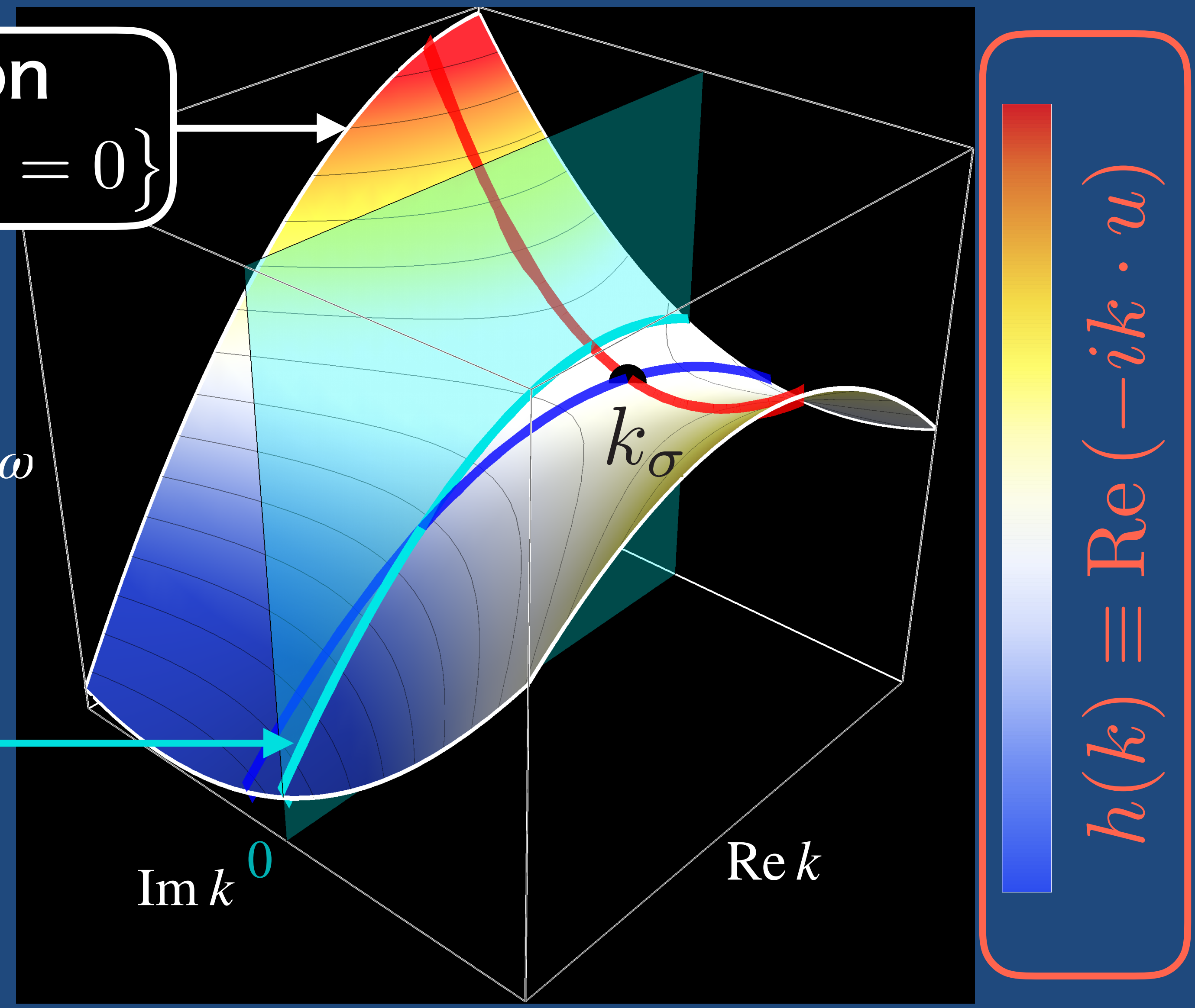
$\Delta(k) \equiv \det \mathbf{D}(k)$

$$\mathbf{G}(t, \boldsymbol{x} + \boldsymbol{u}t) \sim \frac{1}{(2\pi i t)^{d/2} i} \sum_{\sigma} \langle \mathcal{C}, \mathcal{K}_\sigma \rangle \frac{e^{-ik_\sigma\cdot ut} e^{i\boldsymbol{k}_\sigma\cdot\boldsymbol{x}}}{\sqrt{H_\sigma} \partial_0 \Delta(k_\sigma)} \operatorname{adj} \mathbf{D}(k_\sigma)$$

$$H_\sigma \equiv \det \left[\frac{(\partial_i - u_i\partial_0)(\partial_j - u_j\partial_0)\Delta}{\partial_0 \Delta} \right]_{k=k_\sigma}$$

Dispersion Relation

$\mathcal{D} \equiv \{k \in \mathbb{C}^{d+1} | \Delta(k) = 0\}$



$\mathcal{C} \equiv \mathcal{D} \cap (\mathbb{C} \times \mathbb{R}^d)$

We solve Eq. for Dual thimble $\mathcal{K}_\sigma$.

$$\frac{dK^\alpha(s)}{ds} = iu_\beta \left[\delta^{\beta\alpha} - \delta^{\beta\gamma} \frac{\partial_\gamma \Delta \overline{\partial_\delta \Delta}}{\|\partial \Delta\|^2} \delta^{\delta\alpha}\right]_{k=K(s)}$$

Boundary Condition: $\lim_{s \to -\infty} K(s) = k_\sigma$

Maximum Growth Rate

$$\mathbf{G}(t, \boldsymbol{x} + \boldsymbol{u}t) \sim \frac{1}{(2\pi i t)^{d/2} i} \sum_{\sigma} \langle \mathcal{C}, \mathcal{K}_\sigma \rangle \frac{e^{-ik_\sigma \cdot ut} e^{i\boldsymbol{k}_\sigma \cdot \boldsymbol{x}}}{\sqrt{H_\sigma} \partial_0 \Delta(k_\sigma)} \operatorname{adj} \mathbf{D}(k_\sigma)$$

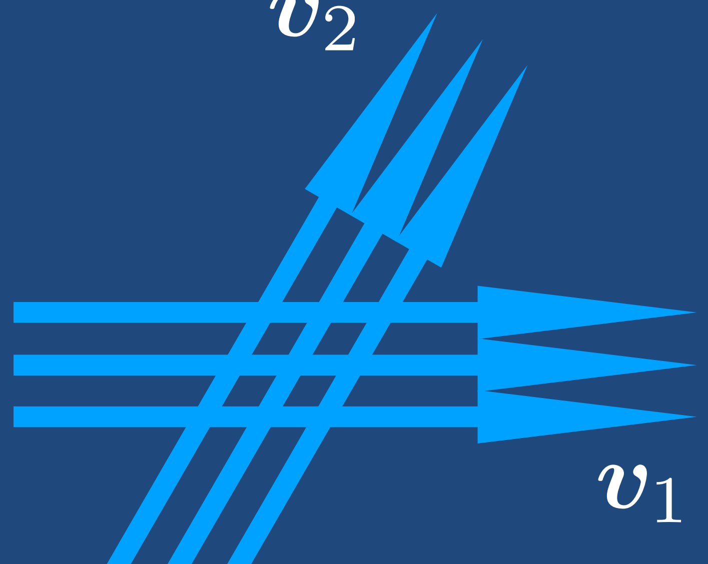
- ✓ Growth Rates of Instability: $\mathrm{Im}(k_\sigma \cdot u)$
  - valid for all u
  - absolutely unstable if $\mathrm{Im}(k_\sigma \cdot u) > 0 \quad \text{for } \boldsymbol{v} = \mathbf{0}$
  - convectively stable if $\mathrm{Im}(k_\sigma \cdot u) > 0 \quad \text{for } \boldsymbol{v} \neq \mathbf{0}$

- ✓ The maximum growth rate is obtained for

$$v_{\max} = \left(-\frac{\partial_i \Delta}{\partial_0 \Delta} \right) = \frac{\partial \omega_\sigma}{\partial \boldsymbol{k}_\sigma} : \text{group velocity}$$

as in (1+1)-D case but for (3+1)-D.

Application to 2-beam model



$$\mathscr{G}_{\boldsymbol{v}} = 4\pi \left[\mathscr{G}_1 \delta(\boldsymbol{v} - \boldsymbol{v}_1) + \mathscr{G}_2 \delta(\boldsymbol{v} - \boldsymbol{v}_2) \right]$$

$$\begin{cases} \mathscr{G}_i > 0 & \text{for } \nu_e \\ \mathscr{G}_i < 0 & \text{for } \bar{\nu}_e \end{cases}$$

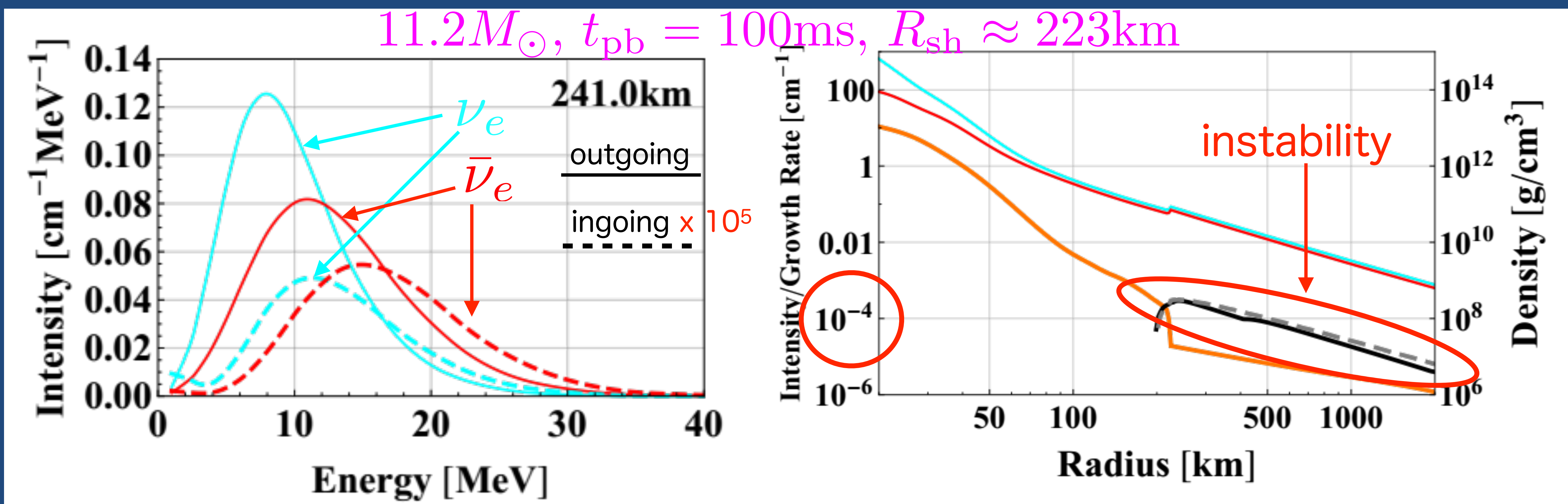
The result is qualitatively changed!

For (3+1)-dimensions: Capozzi et al. '19
For (1+1)-dimensions: Morinaga & Yamada '19

stable	absolutely unstable	convectively unstable
$\mathscr{G}_1\mathscr{G}_2 < 0$	$\mathscr{G}_1\mathscr{G}_2 > 0,\ v_{1x}v_{2x} < 0$	$\mathscr{G}_1\mathscr{G}_2 > 0$ / $\mathscr{G}_1\mathscr{G}_2 > 0,\ v_{1x}v_{2x} > 0$

Fast Flavor Conversion Ahead of Shock Wave

✓ We found the sign change in ELN for the ingoing ν's outside the stalled shock at $t_{pb} \gtrsim$ 100ms in our 1D simulations.



✓ We found it also in the 1D models of the MPA group that Tamborra et al. '17 employed in their survey.

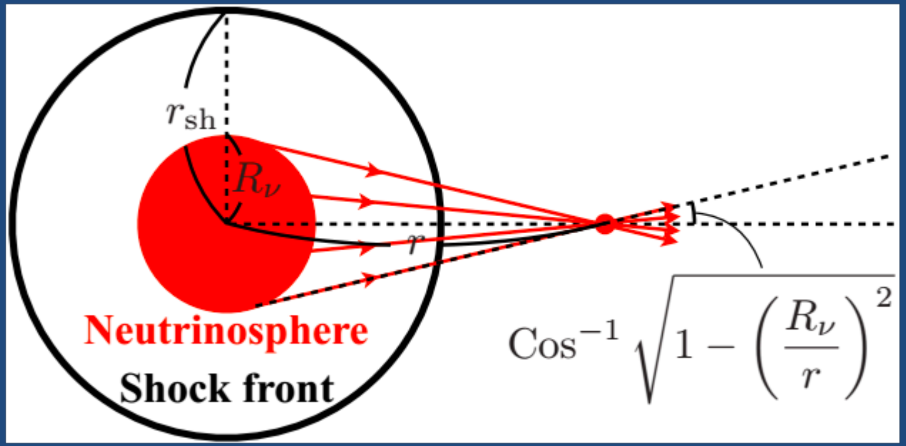
Morinaga et al. '19

✓ It turns out that the crossing is induced by coherent back-scatterings of neutrinos on heavy nuclei.

Bulb Model

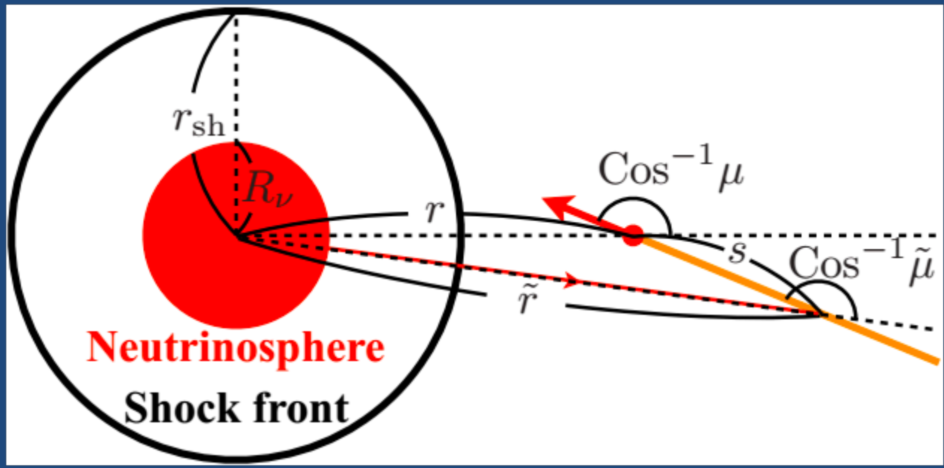
outgoing component

$$\mathscr{G}_\nu^{\rm bulb}(\mu) = 2\,{\rm cm}^{-1} \left(\frac{50\,{\rm km}}{R_\nu}\right)^2 \left(\frac{L_\nu}{10^{52}\,{\rm erg/s}}\right) \left(\frac{10\,{\rm MeV}}{\bar{E}_\nu}\right) \times \Theta\left(\mu - \sqrt{1-(R_\nu/r)^2}\right)$$



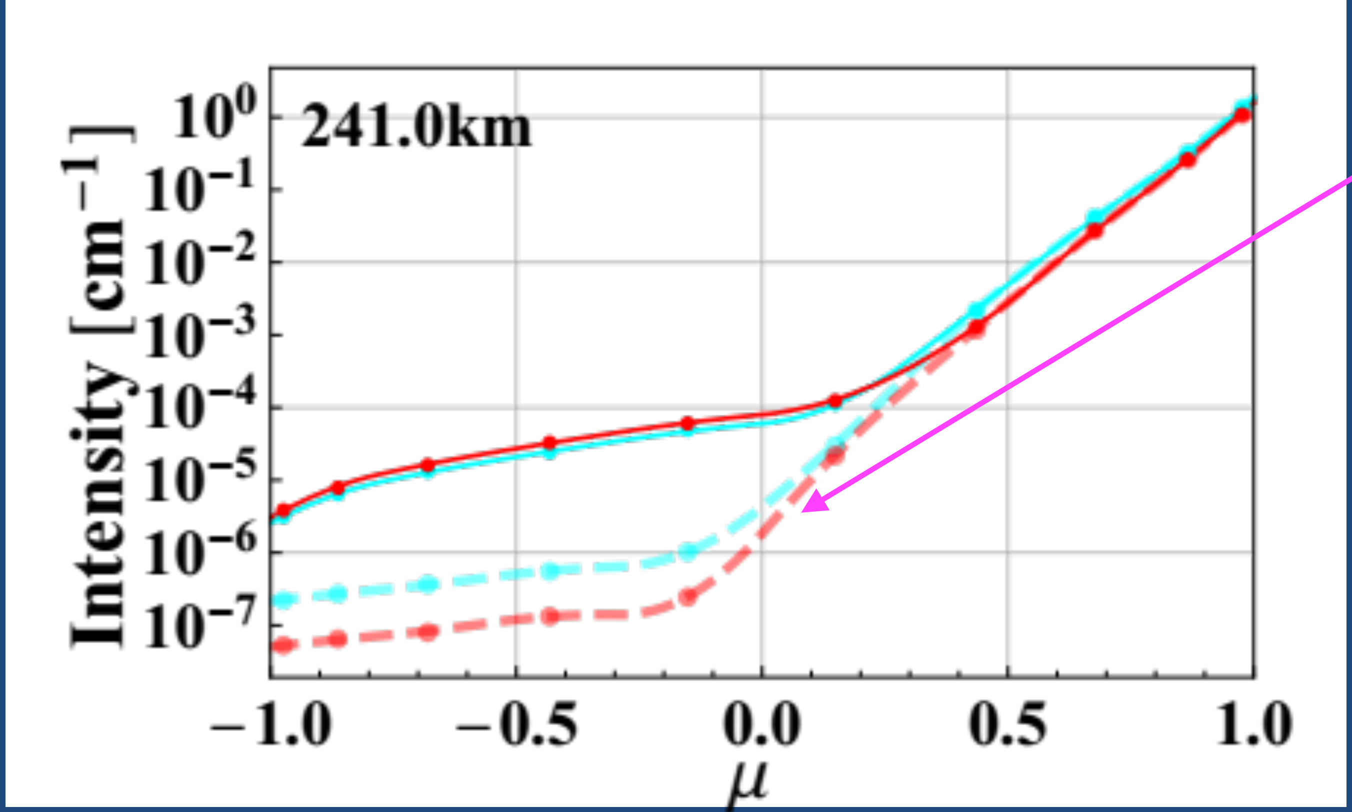
ingoing component

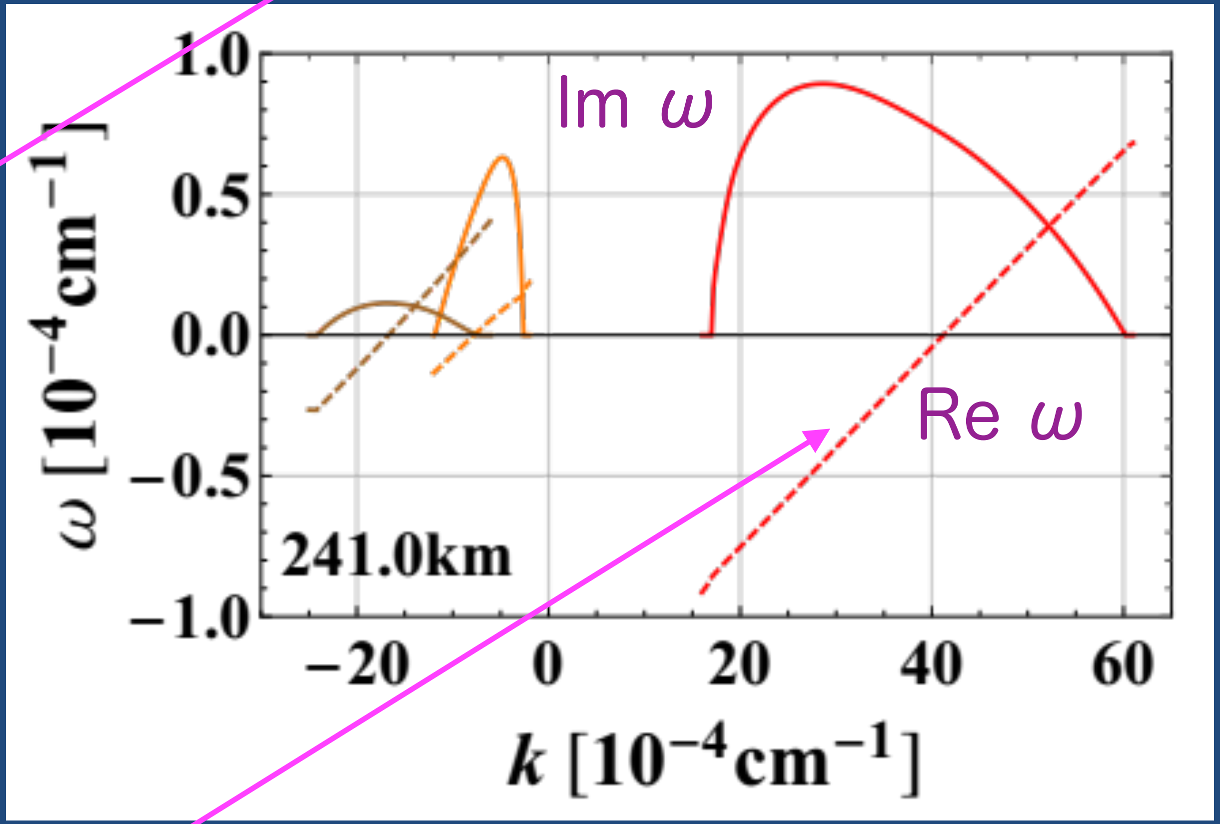
$$\mathscr{G}_\nu^{\rm scat}(\mu) \simeq 2\times 10^{-4}\,{\rm cm}^{-1} \frac{4+\alpha_\nu}{(3+\alpha_\nu)(3+\beta)} \left(\frac{A}{56}\right) \times \left(\frac{\rho_{\rm sh}}{10^7\,{\rm g/cm^3}}\right) \left(\frac{R_{\rm sh}}{200\,{\rm km}}\right)^\beta \left(\frac{200\,{\rm km}}{r}\right)^{1+\beta} \times \left(\frac{L_\nu}{10^{52}\,{\rm erg/s}}\right) \left(\frac{\bar{E}_\nu}{10\,{\rm MeV}}\right) \left[(\mu+1) + \frac{1}{4}\left(\frac{R_\nu}{r}\right)^2\right]$$



➡ If $L_{\nu_e}\bar{E}_{\nu_e} < L_{\bar{\nu}_e}\bar{E}_{\bar{\nu}_e}$, then ELN becomes negative.

✓ When the coherent scattering is turned off in the simulation, then the ELN crossing disappears.

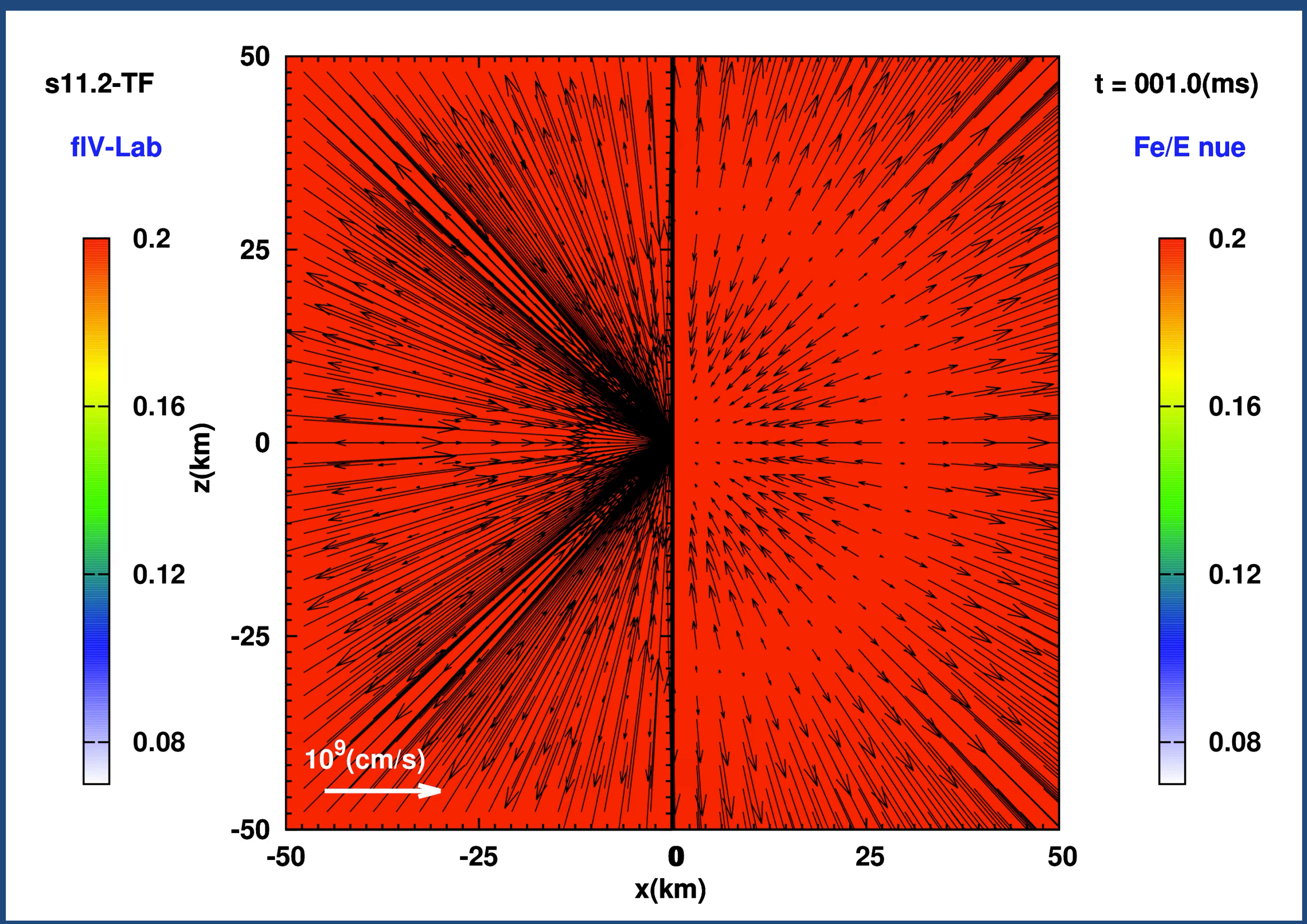


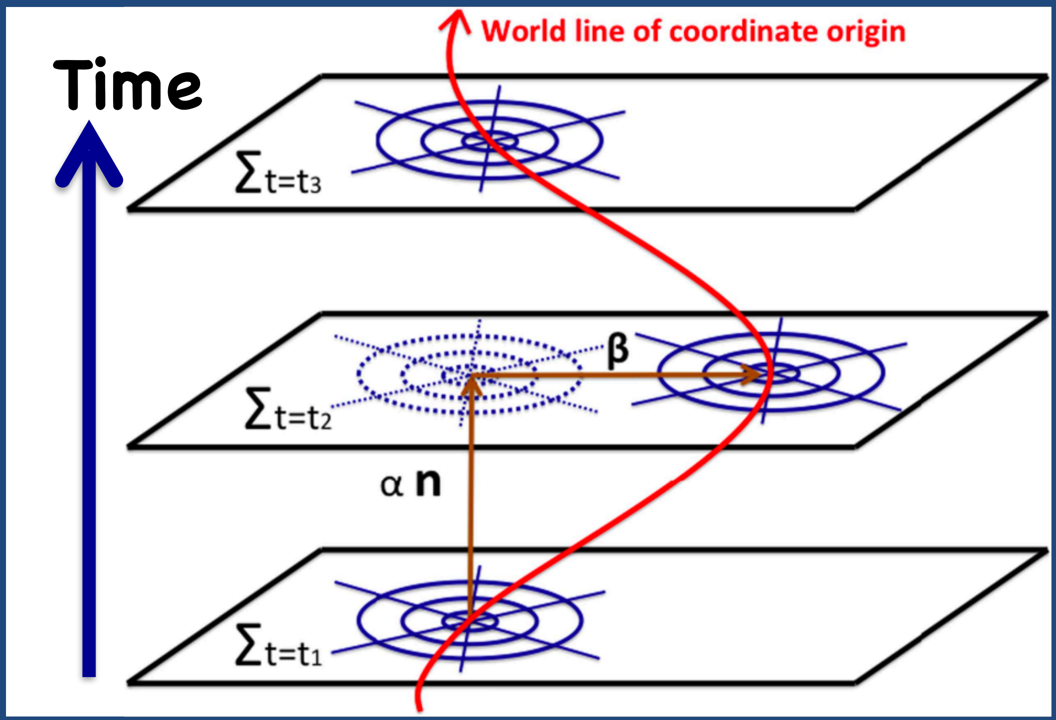


✓ Interestingly, the group velocity is always positive irrespective of the phase velocity and hence the flavor conversion may have an observational impact.

Fast Flavor Conversion in the Post-Shock Region

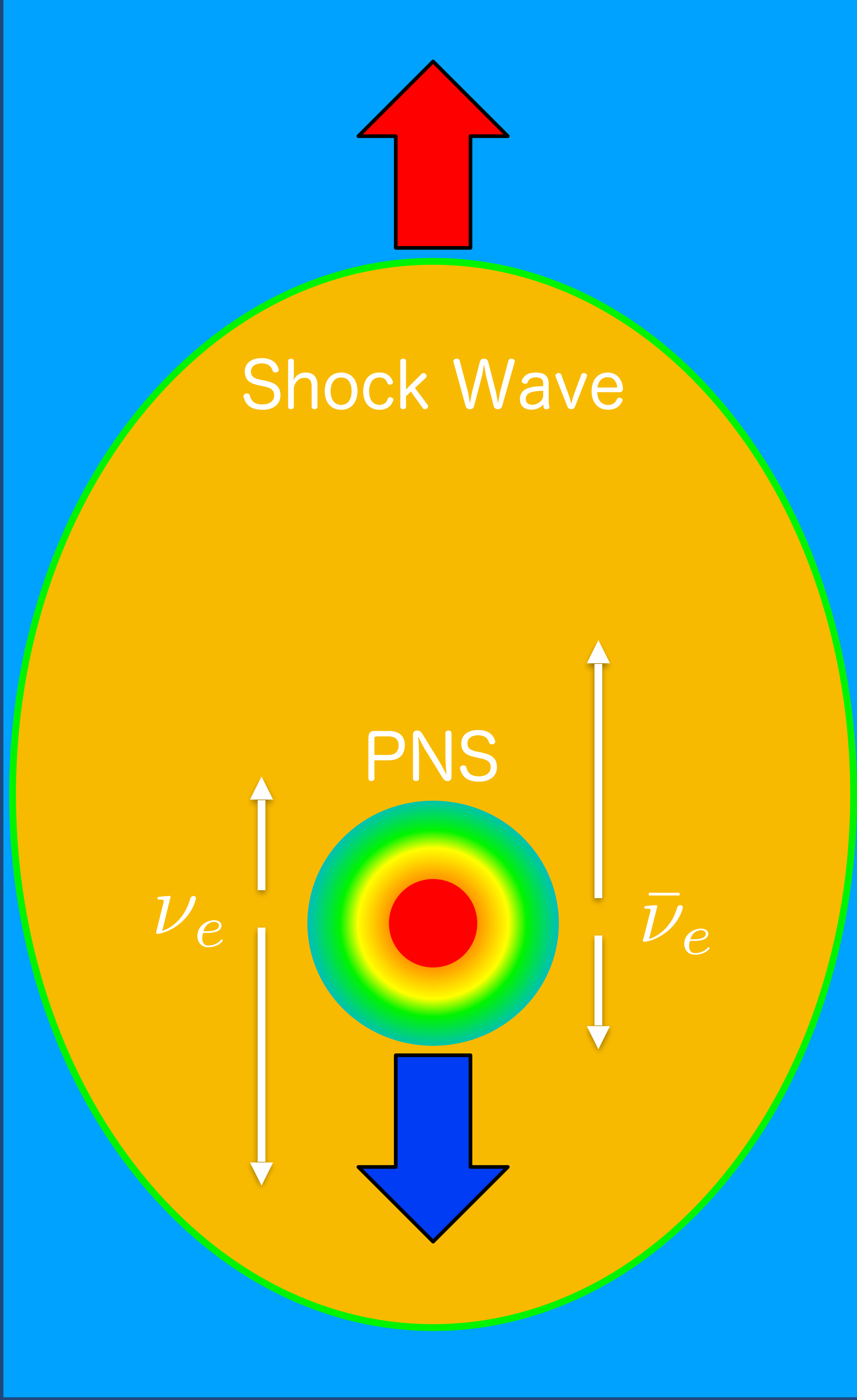
- ✓ We found the ELN crossing inside the shock wave at $t_{pb} \gtrsim 200$ms in one of our latest 2D simulations for 11.2M model.

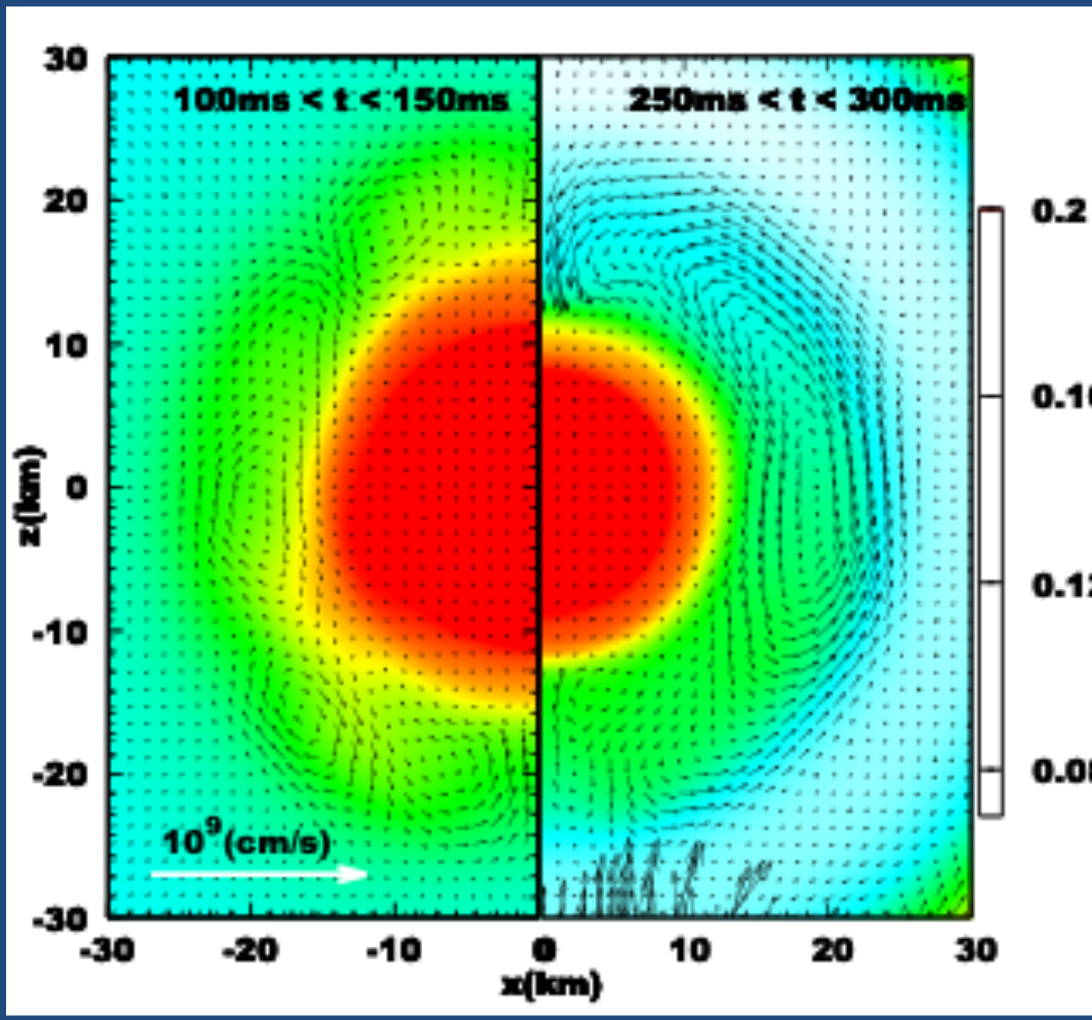




We shift the coordinates using a GR-feature of our code. Nagakura et al. '16

- ✓ In this model, PNS starts to move southward at around the same time.

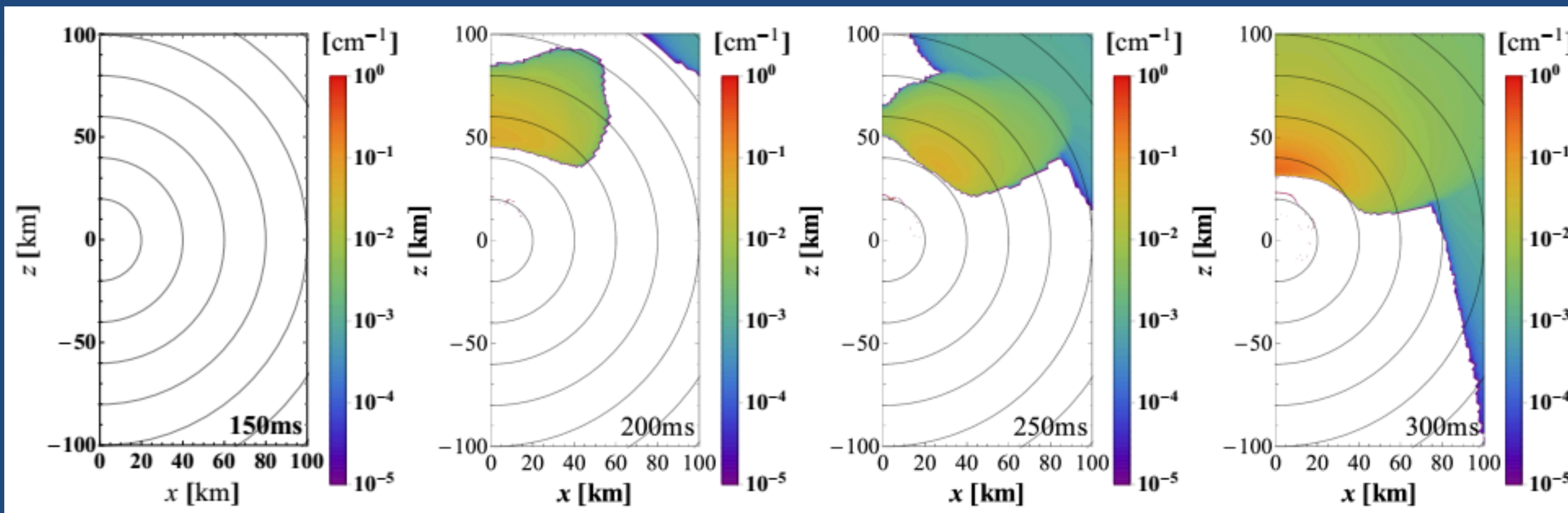




✓ Interplays between PNS motions and asymmetric neutrino emissions in operation

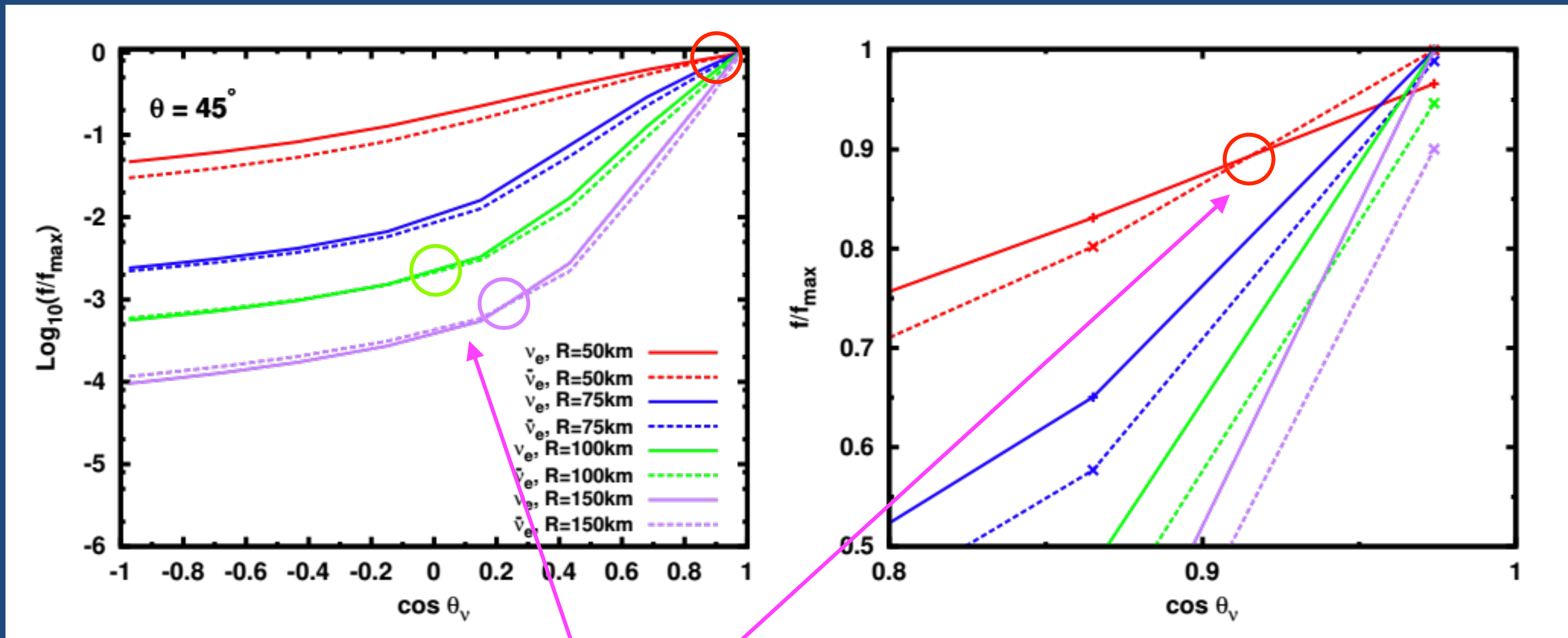
1. Breaking of up-down symmetry in matter distributions by PNS motions
2. Appearing of sustained lateral circular matter motion in the envelope of PNS
3. Sustained asymmetries in the Ye distribution and neutrino emissions

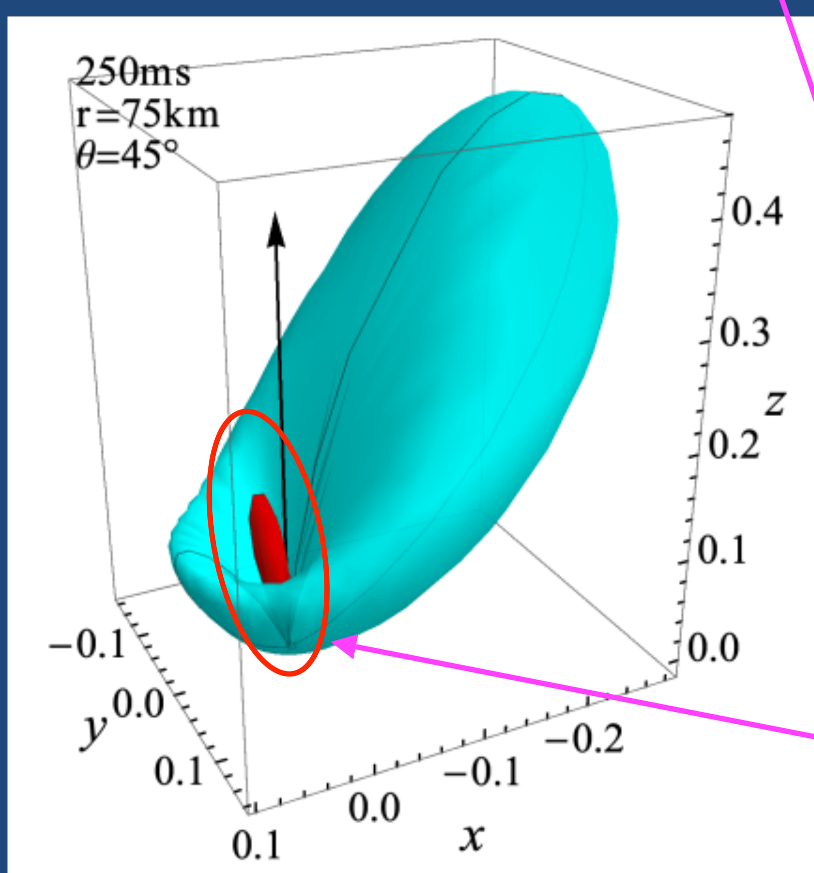
Locations of Fast Flavor Conversion in the Post-Shock Region



The domains of possible fast flavor conversion are expanding with time in the direction of the stronger shock expansion.

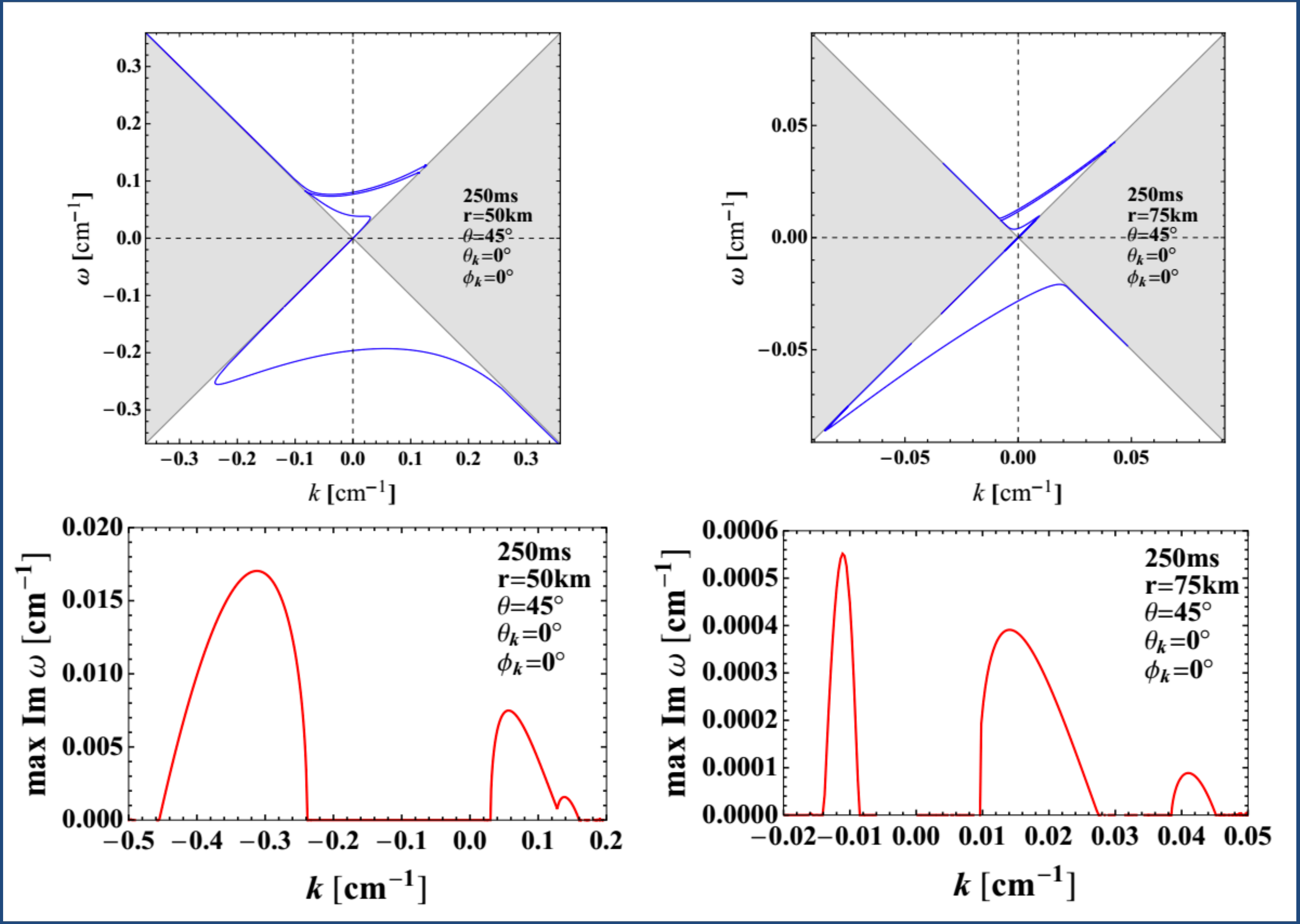
ELN Crossings in the Post-Shock Region





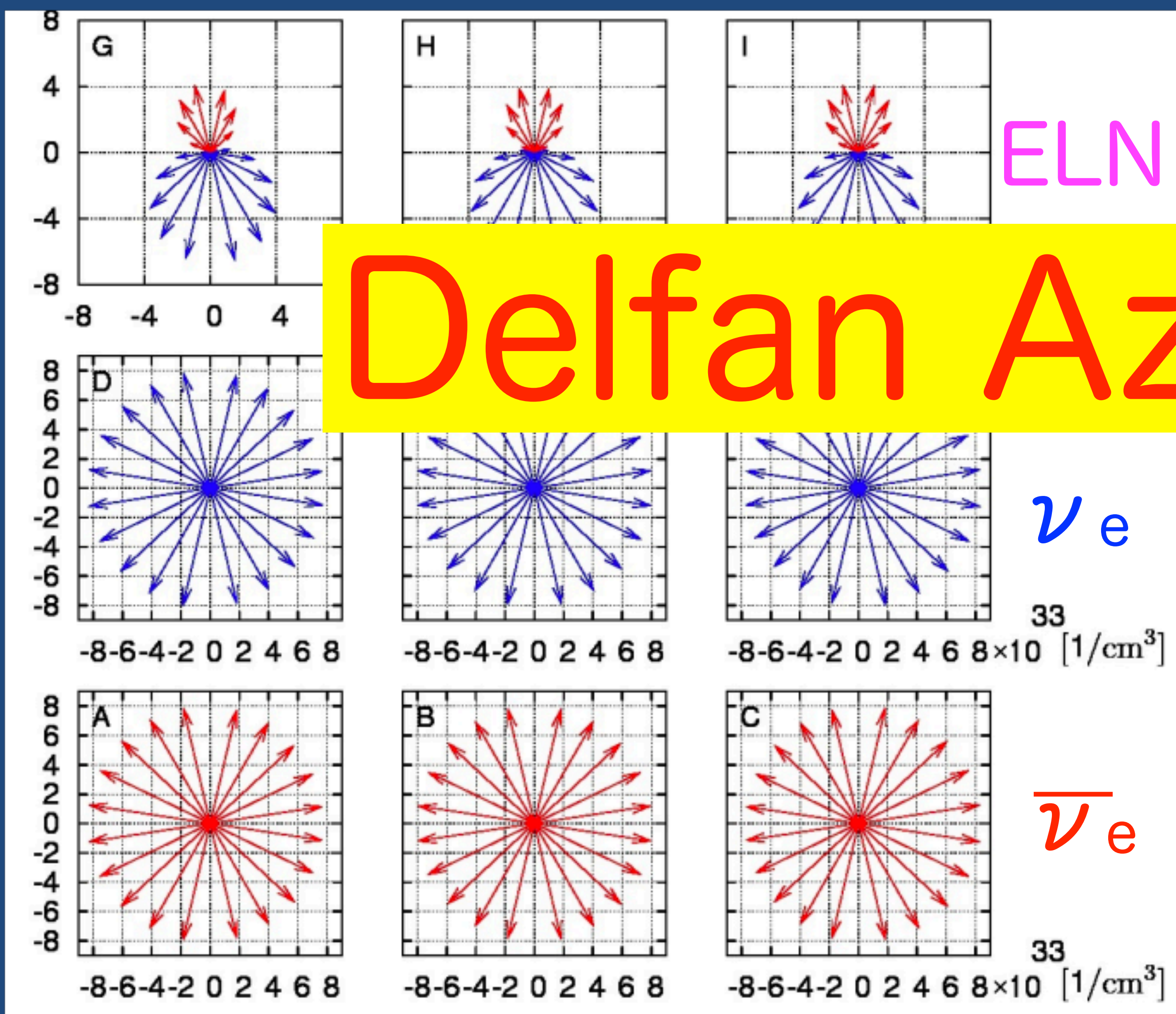
- ✓ $\bar{\nu}_e$ is dominant in the outward direction at r ~ 50km, since it is more forward-peaked.
- ✓ It is dominant in the inward direction at r ≳ 100km, since it is emitted or scattered more frequently.
- ✓ It is dominant in the non-radial direction at r ~ 75km.

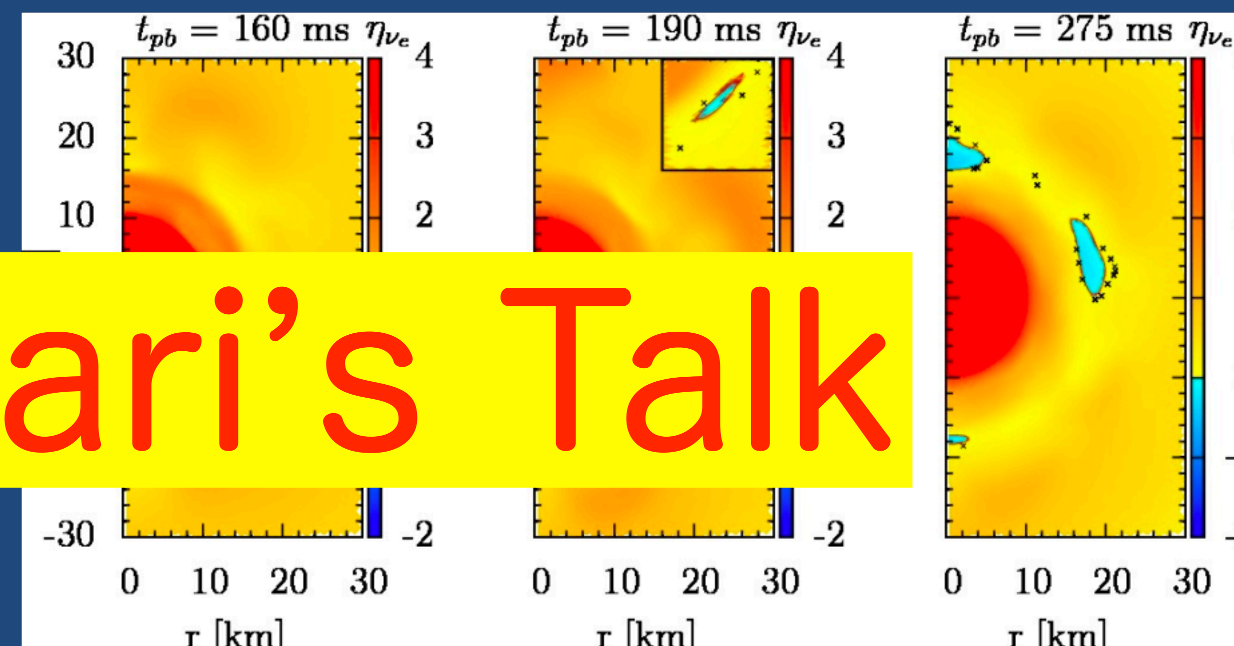
Linear Growth rates of Fast Flavor Conversion in the Post-Shock Region

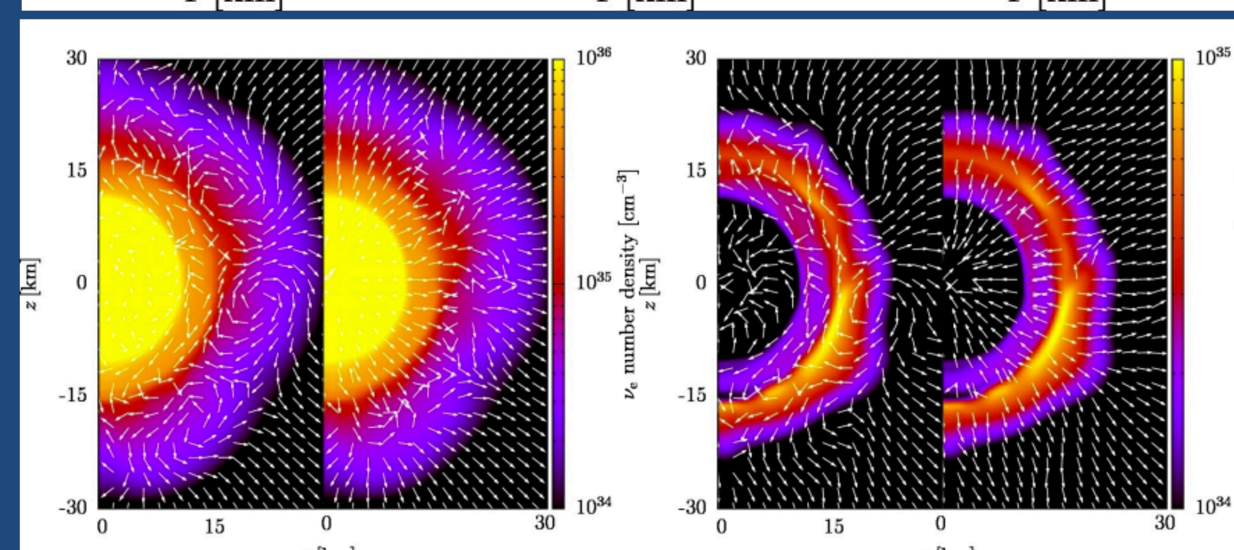


Fast Flavor Conversion inside the Neutrino Sphere

✓ We found the ELN crossing inside the neutrino sphere (r ~15-20km) at $t_{pb} \gtrsim 200$ms in one of our 2D simulations for 11.2M model.







Delfan Azari's Talk

Delfan Azari et al. '19

Summary

✓ We have been developing the radiation-hydrodynamics code that solves the Boltzmann equations as they are in multi-spatial dimensions.

- we currently pushing for full GR, 3D and better microphysics

The existence of the fast conversion is one thing and its implication for CCSNe is quite another! And the latter will be the next focus.

in core-collapse supernovae.

- ELN crossing in angular distributions is crucial.

✓ We have found 3 possibilities, which are worth further investigations.

- pre- and post-shock regions, inside ν sphere