Chiral transport and turbulence in core-collapse supernovae

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“Multi-dimensional Modeling and Multi-Messenger observation from Core-Collapse Supernovae,” October 24, 2019

Main topics

- Chiral transport phenomena
- Chiral plasma instability
- Chiral turbulence in supernovae
- Chiral radiation transport for neutrinos

Units: $\hbar = c = k_{\rm B} = e = 1$

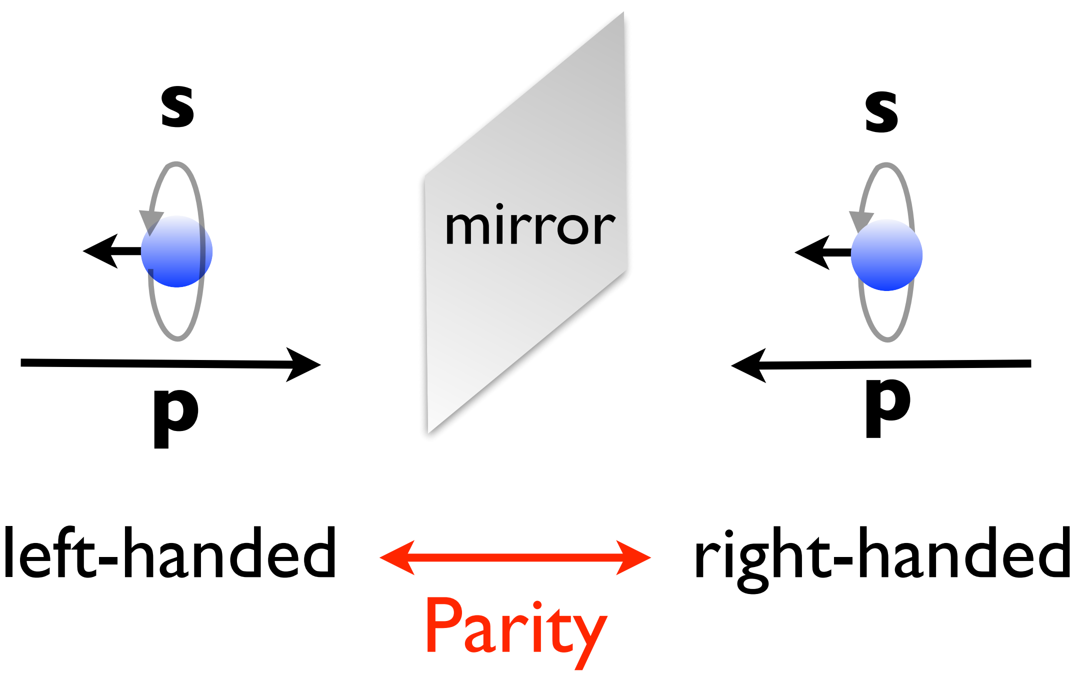
Core-collapse supernova explosions

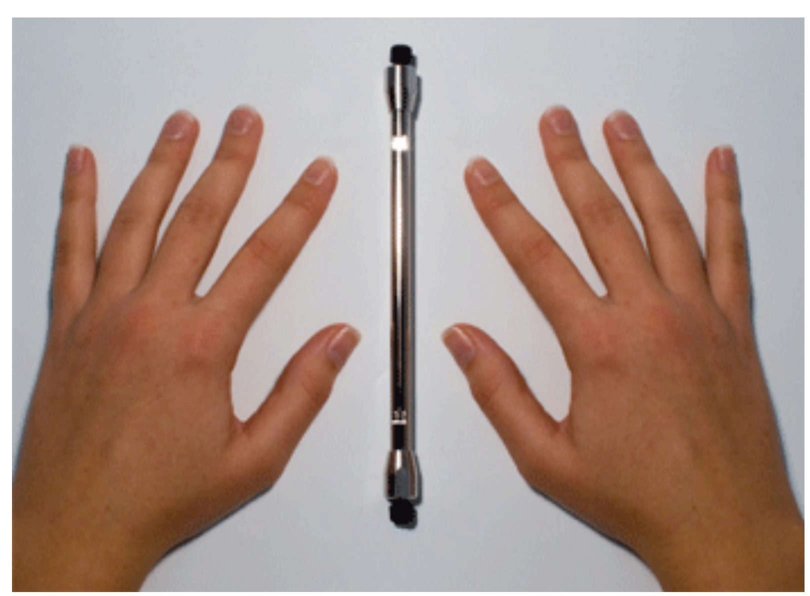
- One of the most energetic phenomena in the Universe
- But explosion is difficult in conventional 3D hydrodynamic theory

One of the puzzles in astrophysics

http://www.riken.jp/pr/press/2009/20091211/

Chirality of fermions





Why is “God” left-handed?

The laws of physics are left-right symmetric except for the weak interaction that acts only on left-handed particles.

W. Pauli

“God is just a weak left-hander.”

From micro to macro

Microscopic parity violation is reflected in macroscopic behavior:

Micro — Chirality of fermions (e, ν) in Standard Model

↓

Chiral kinetic theory

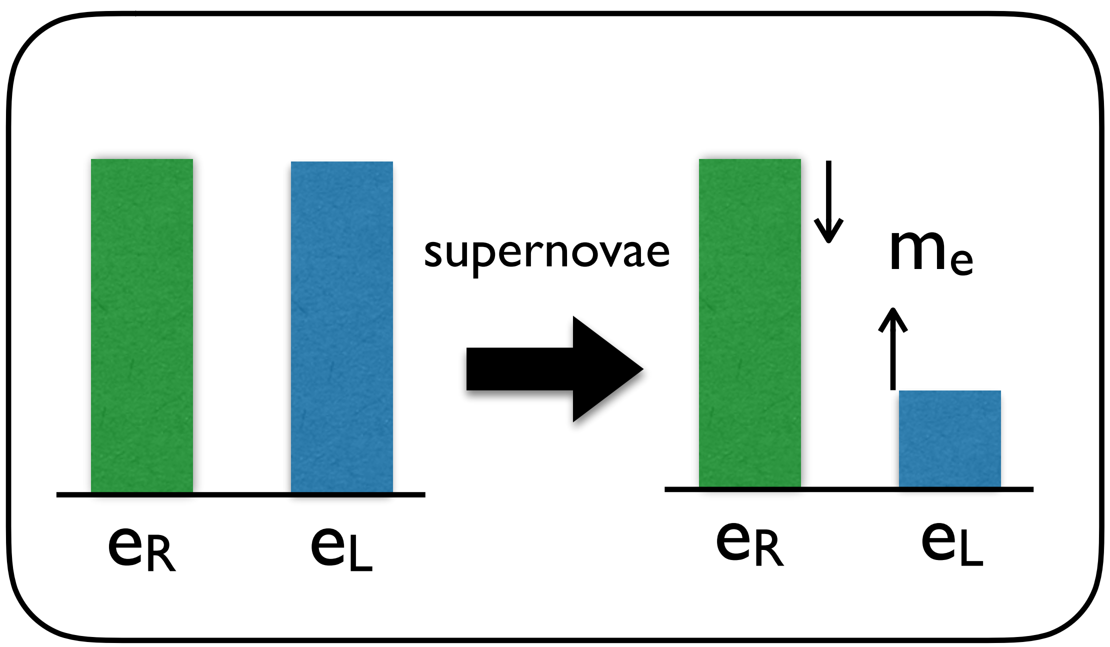
↓ Son, Yamamoto (2012); Stephanov, Yin (2012), …

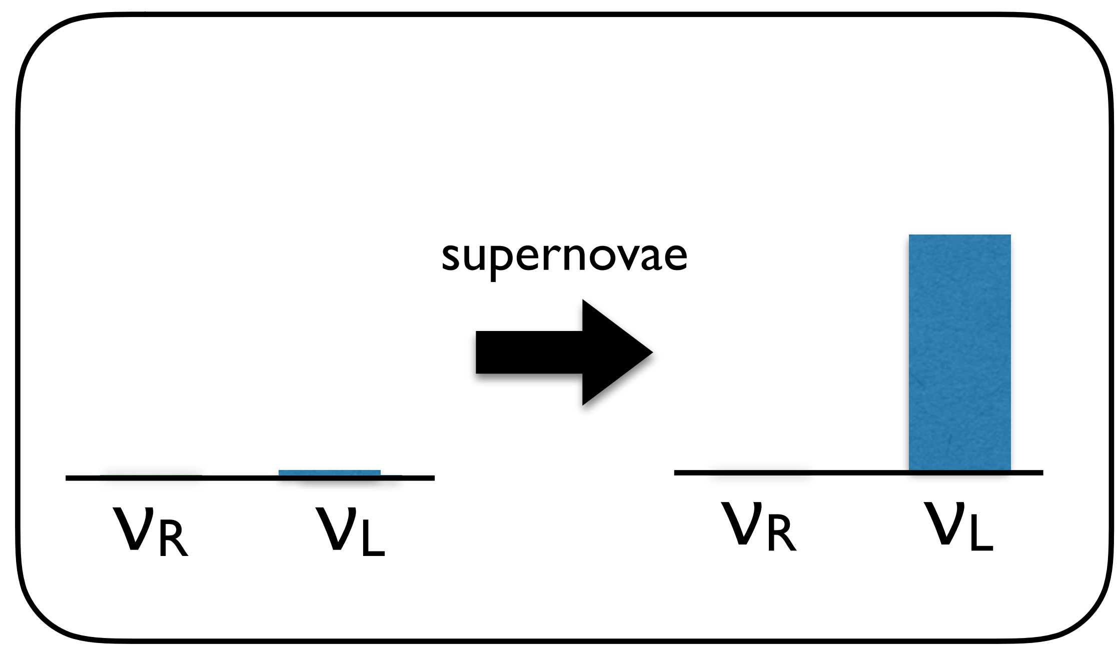
Macro — Evolution of core-collapse supernovae (giant P violation)

Yamamoto (2016)

Supernova = Giant Parity Breaker

$$p + e^{\mathrm{L}} \rightarrow n + \nu_e^{\mathrm{L}}$$





Ohnishi, Yamamoto (2014); Grabowska, Kaplan, Reddy (2015); Sigl, Leite (2016), …

Chiral transport phenomena

Transport phenomena

- Classical and familiar examples:
    - Ohm's law: $\boldsymbol{j}_e = \sigma \boldsymbol{E}$
    - Fourier's law: $\boldsymbol{j}_Q = \kappa(-\boldsymbol{\nabla} T)$

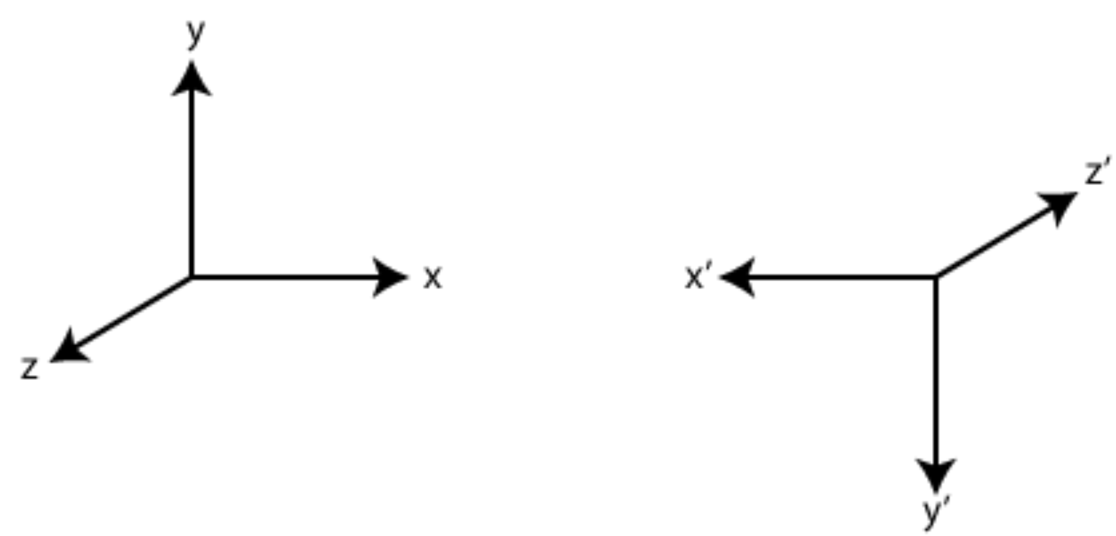
Quiz

- Is it possible to have the following current in metals?

$$\boldsymbol{j}_e \sim \boldsymbol{B}$$

Parity

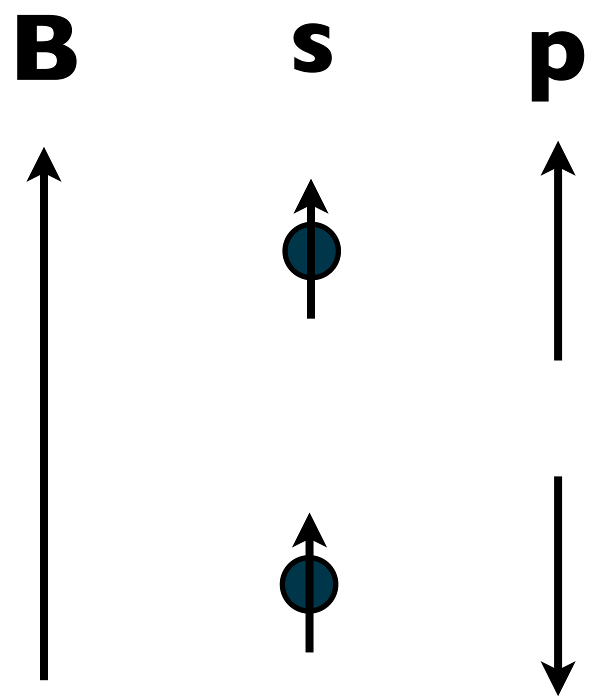
- Assume the relation: $\boldsymbol{j}_e = \kappa \boldsymbol{B}$
- Under the parity, $-\boldsymbol{j}_e = \kappa \boldsymbol{B}$ ($\because$ ***B*** is axial-vector)
- It does not occur in our daily life.



Relativistic systems are different

- Possible in chiral matter: $\boldsymbol{j}_e \sim (\mu_{\mathrm{R}} - \mu_{\mathrm{L}})\boldsymbol{B}$
- This is called the chiral magnetic effect (CME).

Chiral magnetic effect



right-handed $j_{\mathrm{R}} = \frac{\mu_{\mathrm{R}}}{4\pi^2}\boldsymbol{B}$

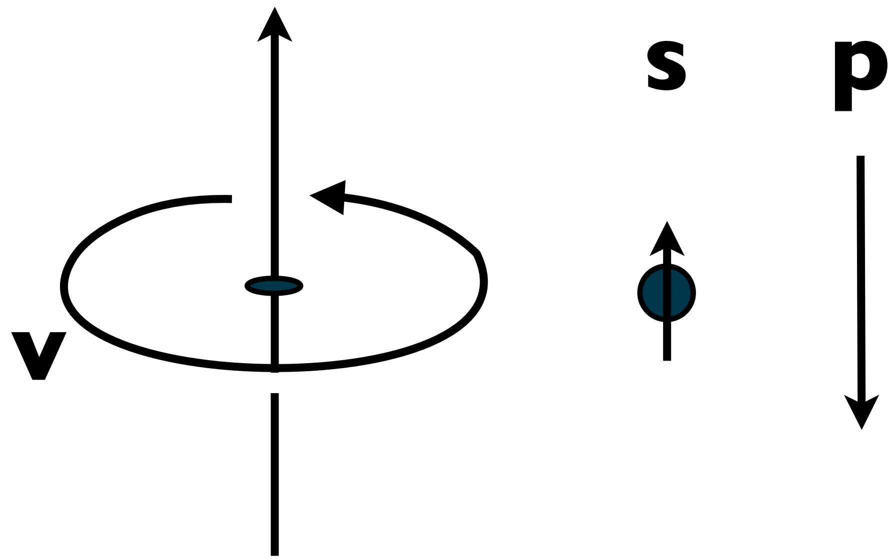
left-handed $j_{\mathrm{L}} = -\frac{\mu_{\mathrm{L}}}{4\pi^2}\boldsymbol{B}$

$$\boldsymbol{j} = \frac{\mu_{\mathrm{R}} - \mu_{\mathrm{L}}}{4\pi^2}\boldsymbol{B} \equiv \frac{\mu_5}{2\pi^2}\boldsymbol{B}$$

Vilenkin (1980); Nielsen, Ninomiya (1983); Fukushima, Kharzeev, Warringa (2008), …

Chiral Vortical Effect

$$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$$



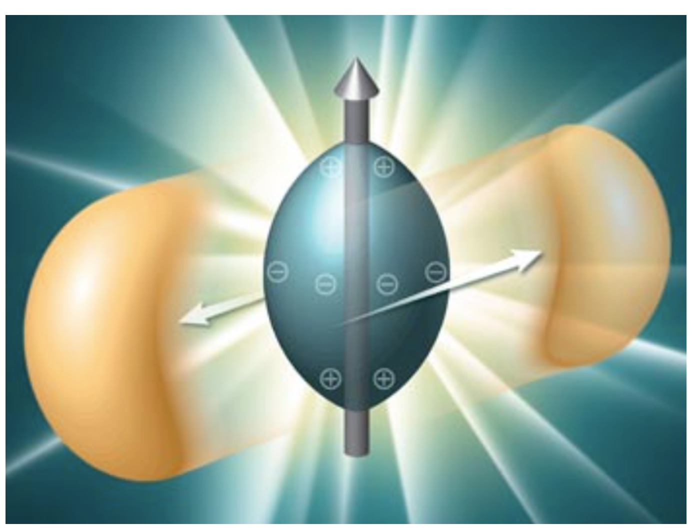
Left-handed neutrino

$$\boldsymbol{j} = -\left(\frac{\mu^2}{8\pi^2} + \frac{T^2}{24}\right)\boldsymbol{\omega}$$

Vilenkin (1979); Erdmenger et al. (2009); Banerjee et al. (2011);
Son, Surowka (2009); Landsteiner et al. (2011)

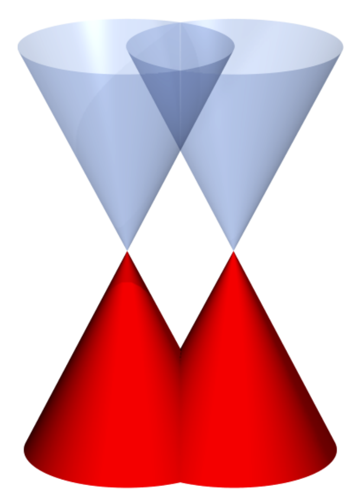
Chiral Matter

- Electroweak plasma in early Universe — Joyce, Shaposhnikov (1997), …
- Quark-gluon plasma in RHIC/LHC — Kharzeev, Mclerran, Warringa (2008), …
- Weyl semi-metals (“3D graphene”) — Nielsen, Ninomiya (1983), …
- Lepton matter in supernovae — Yamamoto (2016), …



QGP

http://www0.bnl.gov/rhic/news2/



Weyl semimetals

Supernovae

Chiral plasma instability

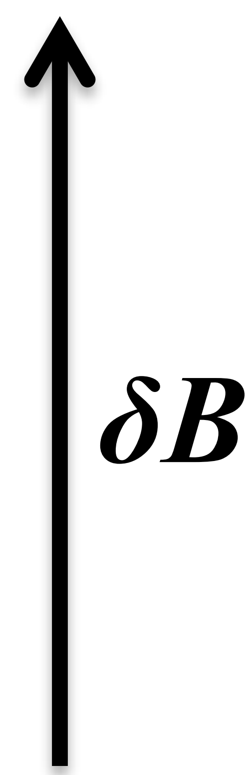
Electroweak theory Redlich, Wijewardhana (1985); Rubakov (1986); Laine (2005)

Early Universe Joyce, Shaposhnikov (1997); Boyarsky et al. (2012); Tashiro et al. (2012)

Quark-gluon plasma Akamatsu, Yamamoto (2013); Manuel, Torres-Rincon (2015), …

Neutron stars Ohnishi, Yamamoto (2014), …

Chiral plasma instability



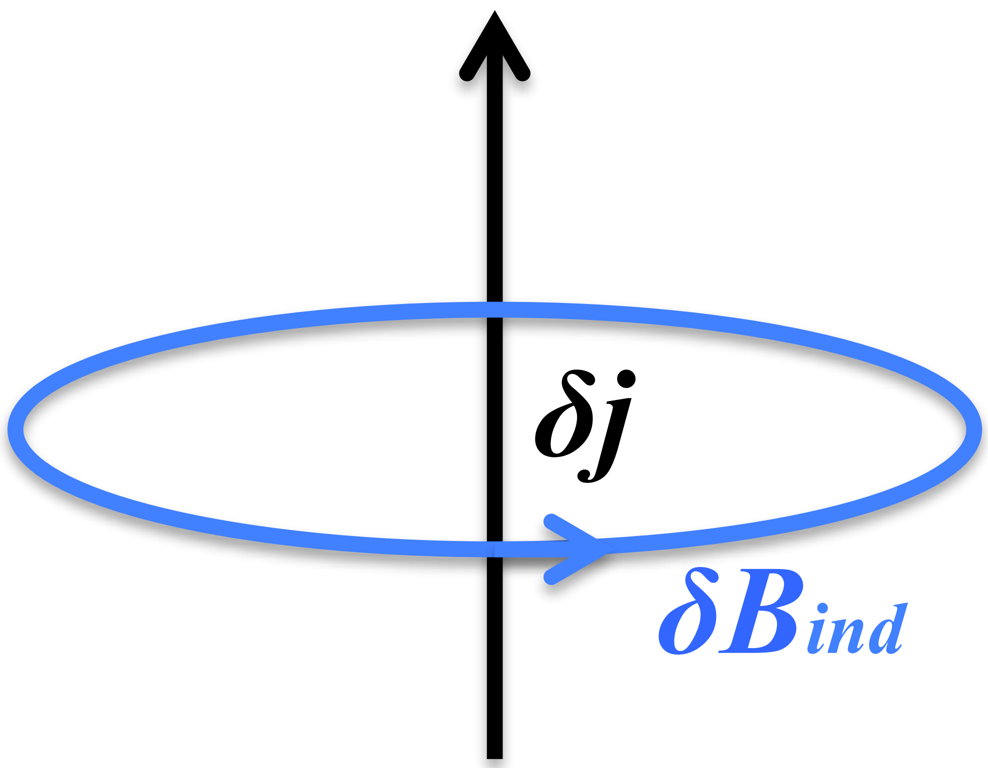
Assume homogeneous $\mu_5 \equiv \mu_\mathrm{R} - \mu_\mathrm{L}$ initially

Chiral plasma instability

Chiral magnetic effect

$$\delta \boldsymbol{j} \sim \mu_5 \delta \boldsymbol{B}$$

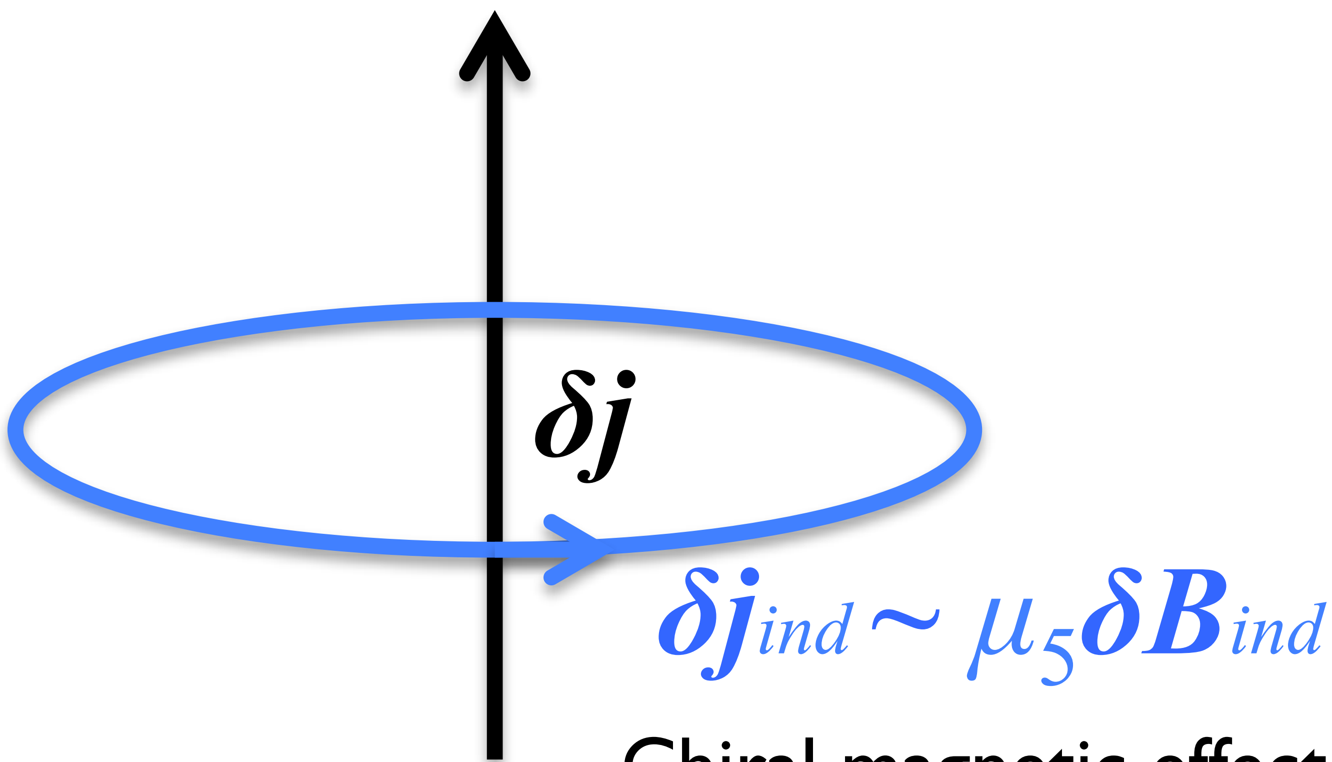
Chiral plasma instability



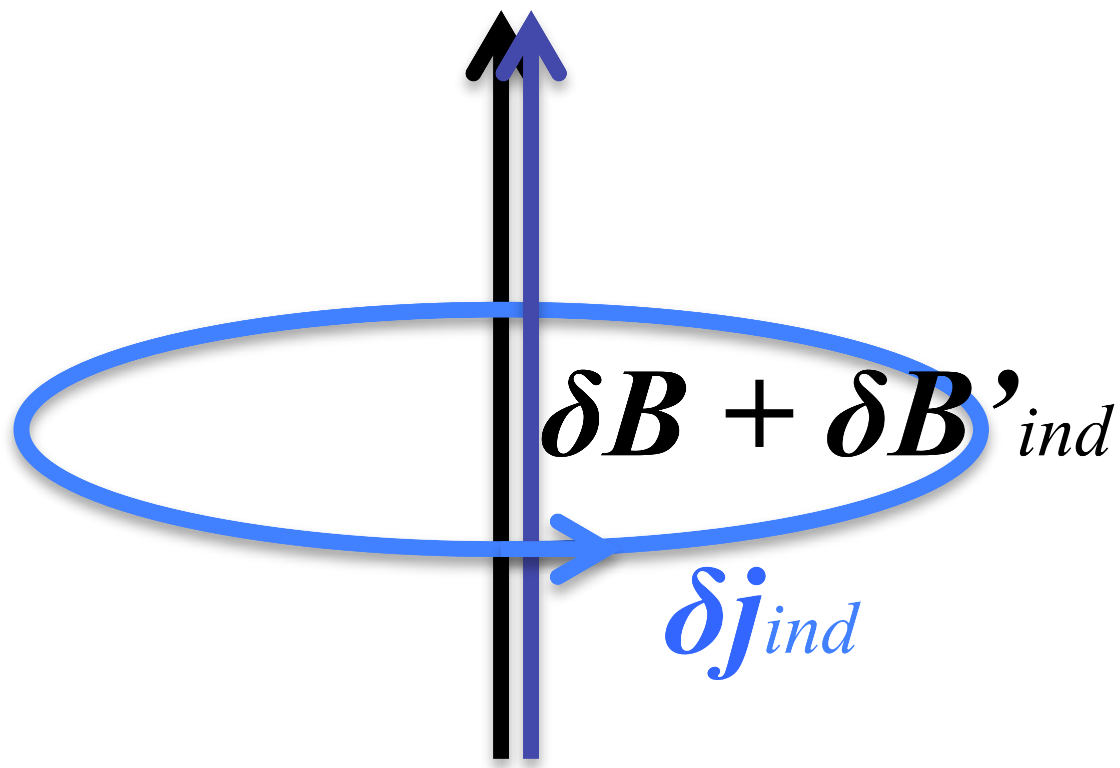
Ampere's law

$$\nabla \times B = j$$

Chiral plasma instability



Chiral plasma instability



Ampere's law

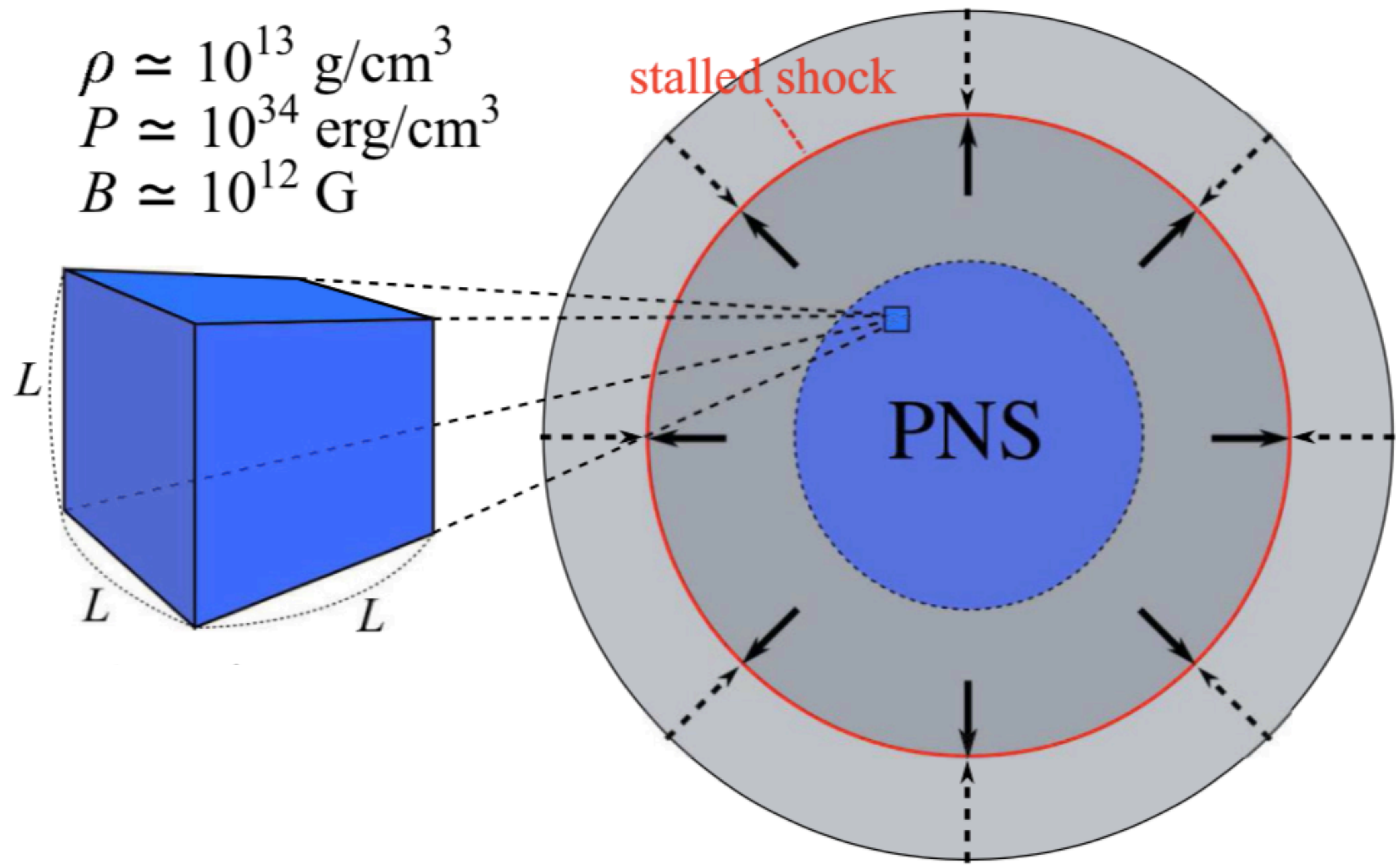
$$\nabla \times B = j$$

Positive feedback: instability

Chiral MHD turbulence in supernovae

Chiral MHD for supernovae

Masada, Kotake, Takiwaki, Yamamoto, arXiv:1805.10419



Proto-neutron star (PNS)

Chiral MHD for supernovae

Masada, Kotake, Takiwaki, Yamamoto, arXiv:1805.10419

- Chiral MHD w/o vorticity at the core (proton, e_R, e_L):

$$\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0$$

$$\partial_t (\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \boldsymbol{v}) = -\boldsymbol{\nabla} P + \boldsymbol{J} \times \boldsymbol{B} + \text{(dissipation)}$$

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta \boldsymbol{\nabla} \times \underbrace{(\xi_B \boldsymbol{B})}_{\text{CME}}$$

$$\partial_t n_5 = \underbrace{\frac{\eta}{2\pi^2} (\boldsymbol{\nabla} \times \boldsymbol{B} - \xi_B \boldsymbol{B}) \cdot \boldsymbol{B}}_{\text{chiral anomaly}}$$

- Setup for proto-neutron stars (100 MeV = 1):

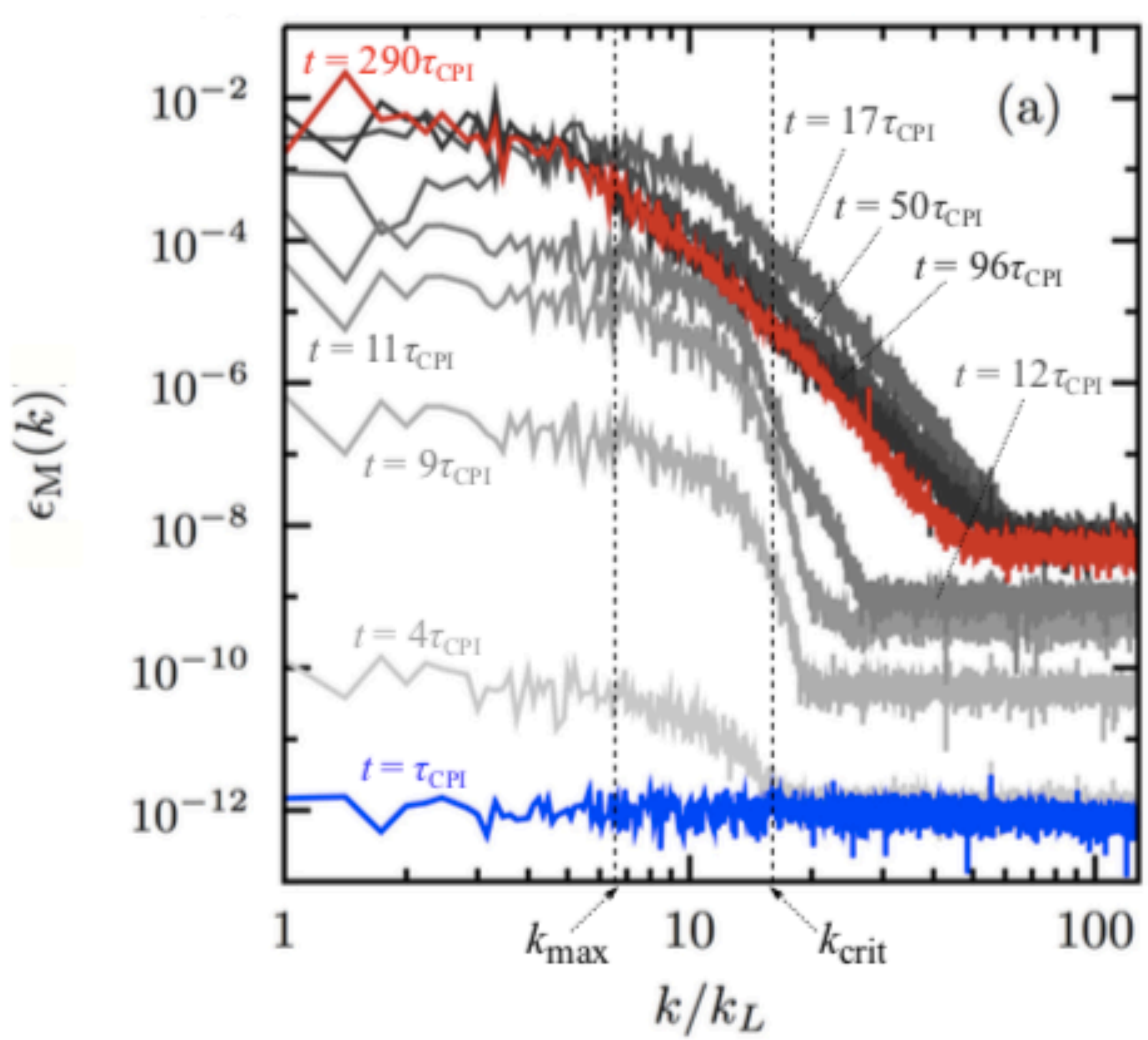
$$\rho_0 = 5.0, \ P_0 = 1.0, \ \xi_{B0} = 4.2 \times 10^{-3}, \ \eta = 100.0$$

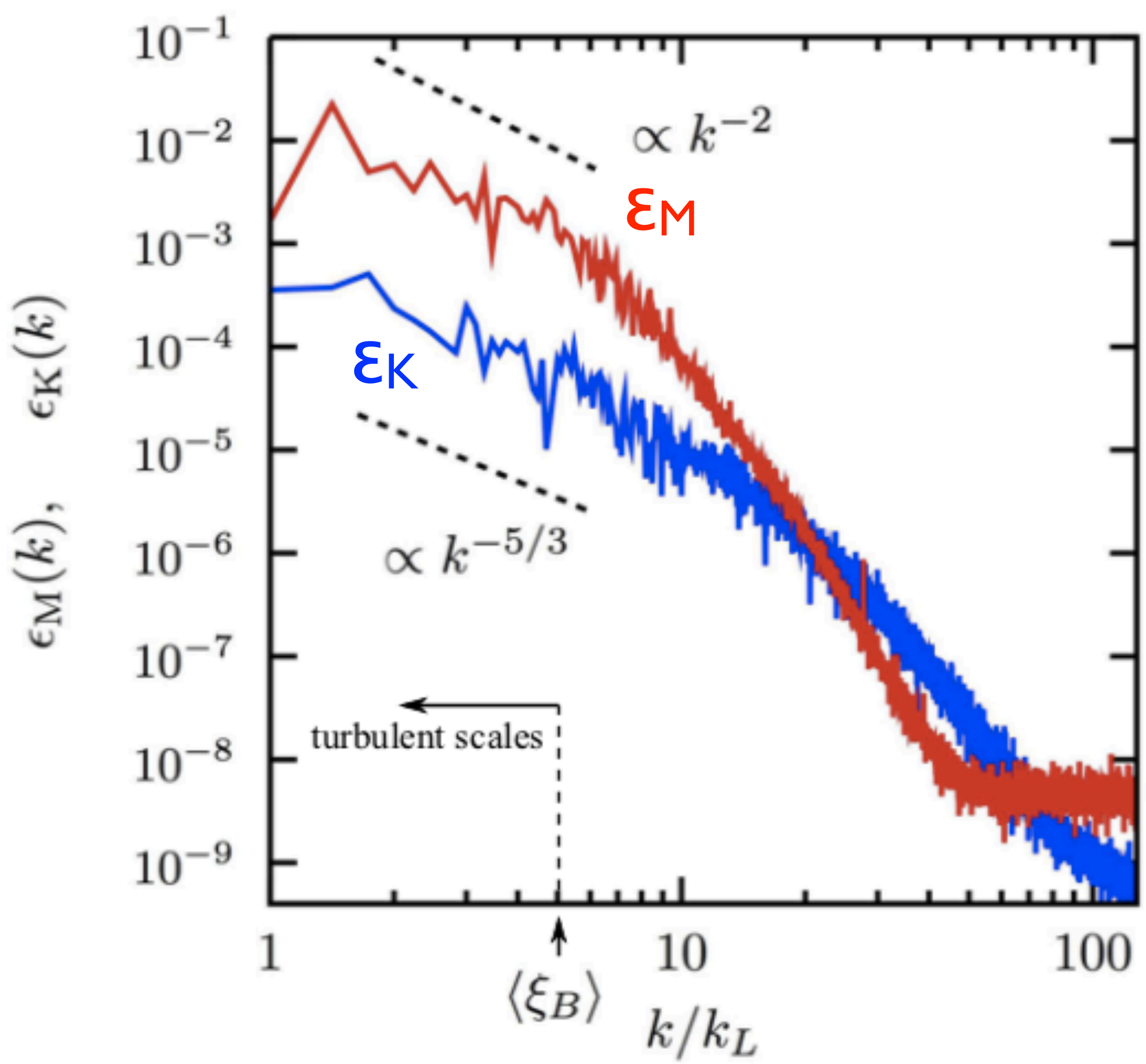
Movies of 3D simulations are available at:

http://www.kusastro.kyoto-u.ac.jp/~masada/movie.mp4

Masada, Kotake, Takiwaki, Yamamoto, arXiv:1805.10419

Energy spectra





- As time passes, energy in small-*k* and large-*k* regions grows
- Eventually, $\varepsilon_M \sim k^{-2}$, $\varepsilon_K \sim k^{-5/3}$

Masada et al., arXiv:1805.10419; see also Brandenburg et al., arXiv:1707.03385

Chiral radiation transport for neutrinos

The Nobel Prize in Physics 2016

© Trinity Hall, Cambridge University. Photo: Kiloran Howard

David J. Thouless

Prize share: 1/2

Photo: Princeton University, Comms. Office, D. Applewhite

F. Duncan M. Haldane

Prize share: 1/4

Ill: N. Elmehed. © Nobel Media 2016

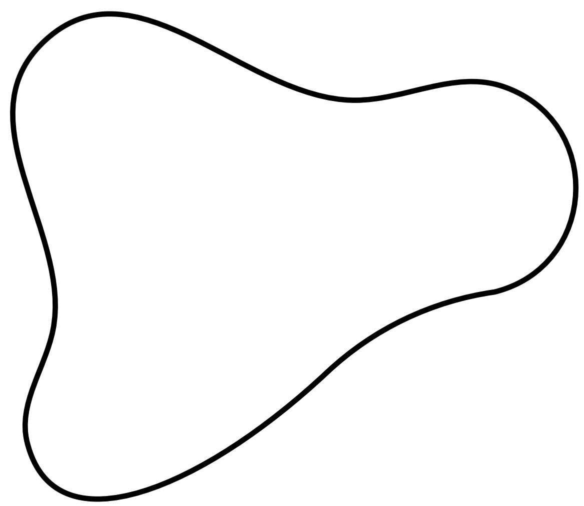
J. Michael Kosterlitz

Prize share: 1/4

The Nobel Prize in Physics 2016 was divided, one half awarded to David J. Thouless, the other half jointly to F. Duncan M. Haldane and J. Michael Kosterlitz *"for theoretical discoveries of topological phase transitions and topological phases of matter"*. (mostly in 2D)

Topology

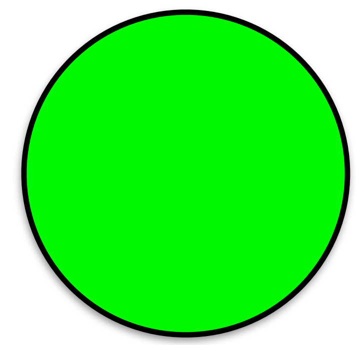
M



rubber band

$\approx$

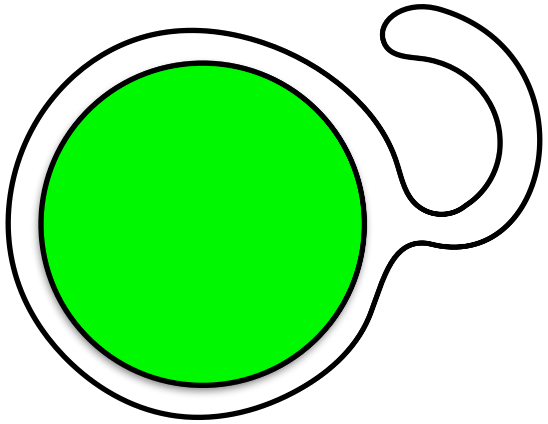
N



surface of a plastic bottle
(from the top)

M and *N* are topologically the same: S^1

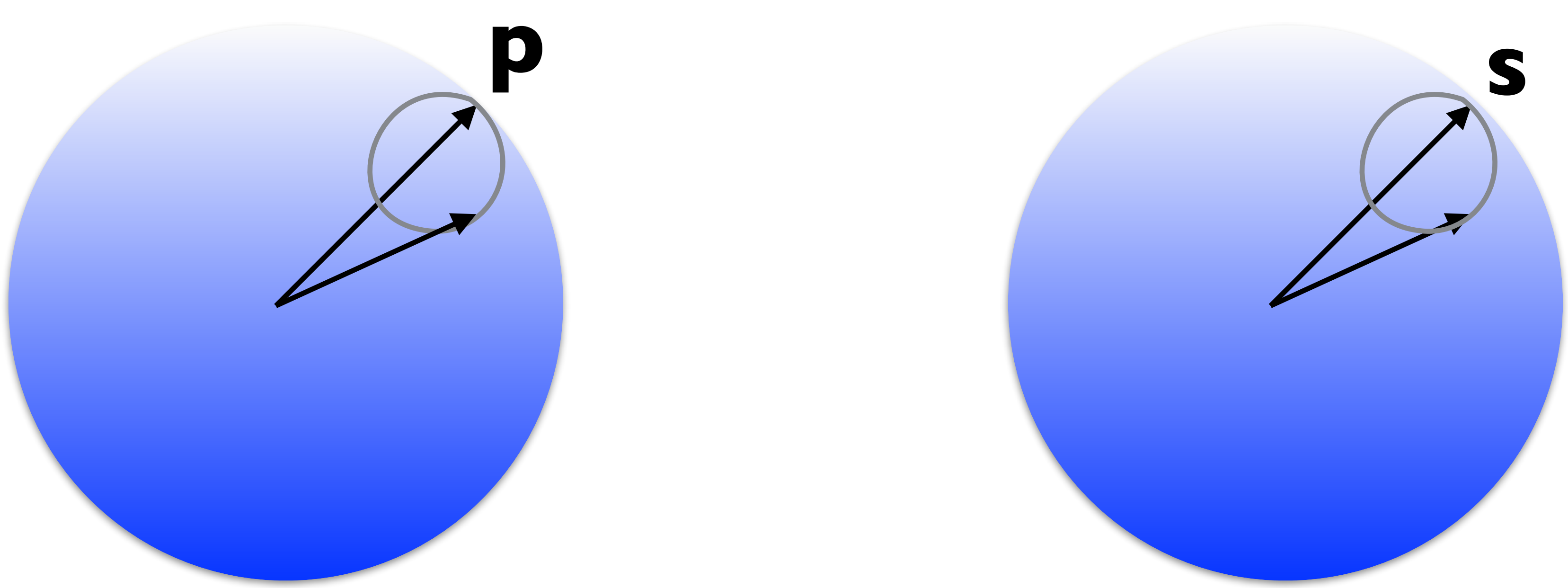
Topological invariant



Mapping from S^1 (rubber) to S^1 (bottle): winding number n

Chirality and topology

Right-handed fermions



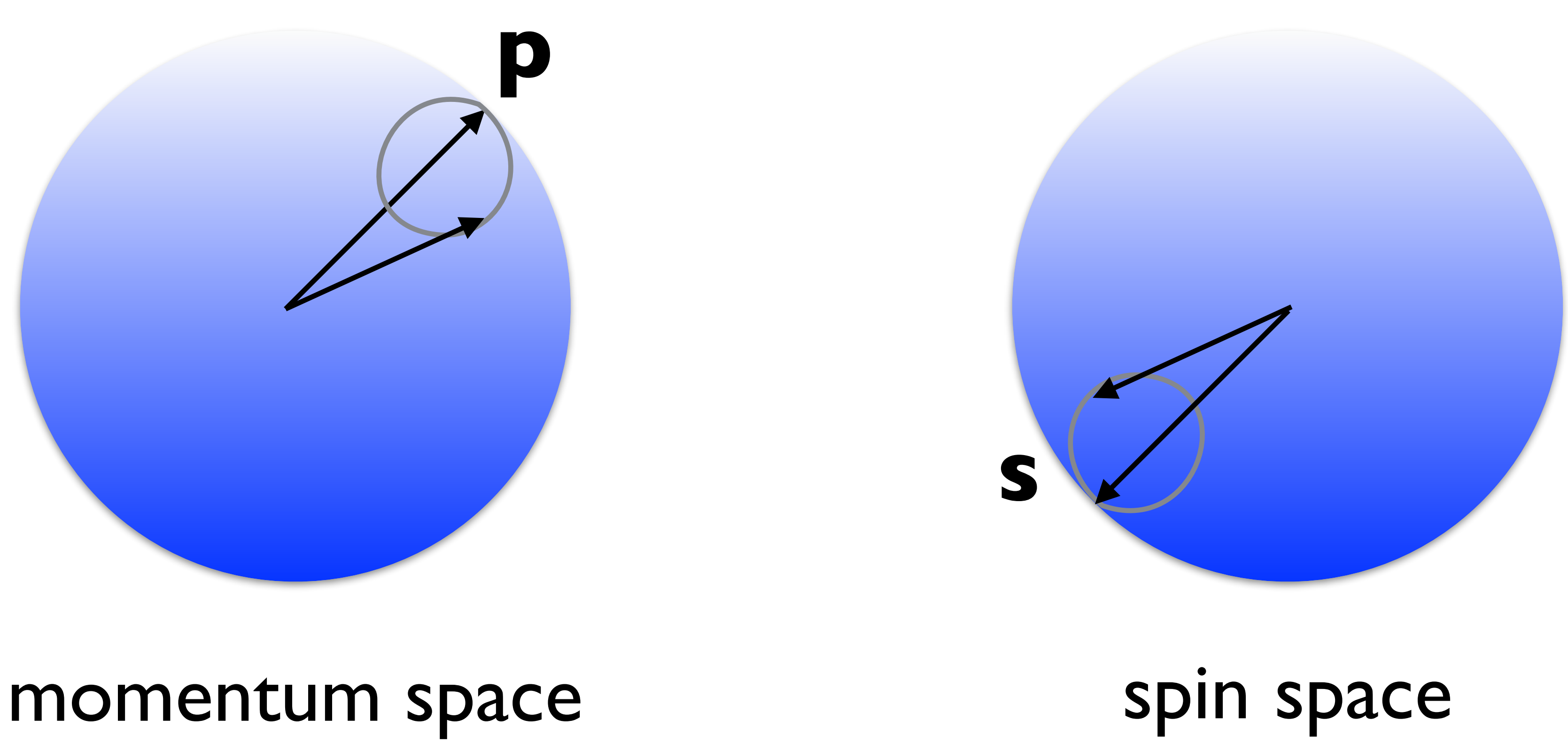
momentum space

spin space

Mapping: S^2 (**p**-space) → S^2 (spin space) winding number +1

Chirality and topology

Left-handed fermions



Mapping: S^2 ($\boldsymbol{p}$-space) → S^2 (spin space) winding number -1

Neutrino matter = 3D topological matter Yamamoto (2016)

Conventional ν radiation transfer

e.g., Mihalas, Mihalas (1984)

- In comoving frame for spherically symmetric metric,

$$E\left[\frac{\partial_t}{c} + \mu\partial_r + E\left(\frac{\mu^2}{c}\partial_t \ln\rho - (1-3\mu^2)\frac{v}{cr} - \frac{\mu\partial_t v}{c^2}\right)\partial_E \right.$$
$$\left. + (1-\mu^2)\left(\frac{1}{r}\left(1+\frac{3\mu v}{c}\right) - \frac{\partial_t v}{c^2} + \frac{\mu}{c}\partial_t \ln\rho\right)\partial_\mu\right] f = \mathcal{C}[f]$$

for the distribution function $f(t, r, E, \mu)$ where $\mu \equiv \cos\bar{\theta}$

Chiral ν radiation transfer

Yamamoto, Yang, to appear

- In comoving frame for the distribution function $f(t,r,\theta,\phi,E,\mu,\bar{\phi})$

$$
\begin{aligned}
E\Bigg[&\frac{\partial_t}{c}+\mu\partial_r+E\left(\frac{\mu^2}{c}\partial_t\ln\rho-(1-3\mu^2)\frac{v}{cr}-\frac{\mu\partial_t v}{c^2}\right)\partial_E \\
&+(1-\mu^2)\left(\frac{1}{r}\left(1+\frac{3\mu v}{c}\right)-\frac{\partial_t v}{c^2}+\frac{\mu}{c}\partial_t\ln\rho\right)\partial_\mu \\
&+\hbar\frac{\sqrt{1-\mu^2}}{2Er^2}\left(r\left(\mu\partial_t\ln\rho-\frac{\partial_t v}{c}\right)+3\mu v\right)\left(\sin\bar{\phi}\partial_\theta-\frac{\cos\bar{\phi}}{\sin\theta}\partial_\phi\right) \\
&-\frac{\hbar}{Er^2}\left(\left(r\partial_t\ln\rho+2v\right)+4\pi r^2\rho c\mu\left(1-8\pi r^2\rho+r(1-4\pi r^2\rho)\partial_r\ln\rho\right)\right)\partial_{\bar{\phi}}+\cdots\Bigg]f=\mathcal{C}[f]
\end{aligned}
$$

Chiral effects necessarily break spherical symmetry

Chiral ν radiation transfer

Yamamoto, Yang, to appear

- In inertial frame with matter in local thermal equilibrium,

$$C[f] = -R_{\rm abs} f + R_{\rm emis}(1-f)$$

$$R_{\rm abs} = R_0 + \hbar \boldsymbol{q} \cdot [R_1 \boldsymbol{\omega} + \boldsymbol{v} \times \boldsymbol{\nabla} R_2 + R_3 \boldsymbol{B}]$$

$$R_i = R_i(T, \mu, \boldsymbol{v}, \boldsymbol{q}) \quad (i = 1, 2, 3)$$

← Computable in QFT

New collision terms lead to finite fluid/cross helicities

Conclusion

- Neutrino matter = 3D topological matter
- Chiral effects can reverse the turbulent behavior from direct to inverse cascade.
- Chiral radiation transport of neutrinos should be applied in future numerical simulations.