超新星から期待される重力波: NS研究からの知見

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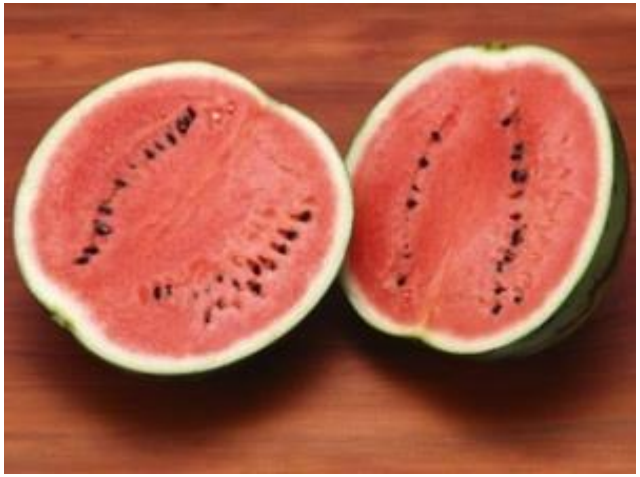
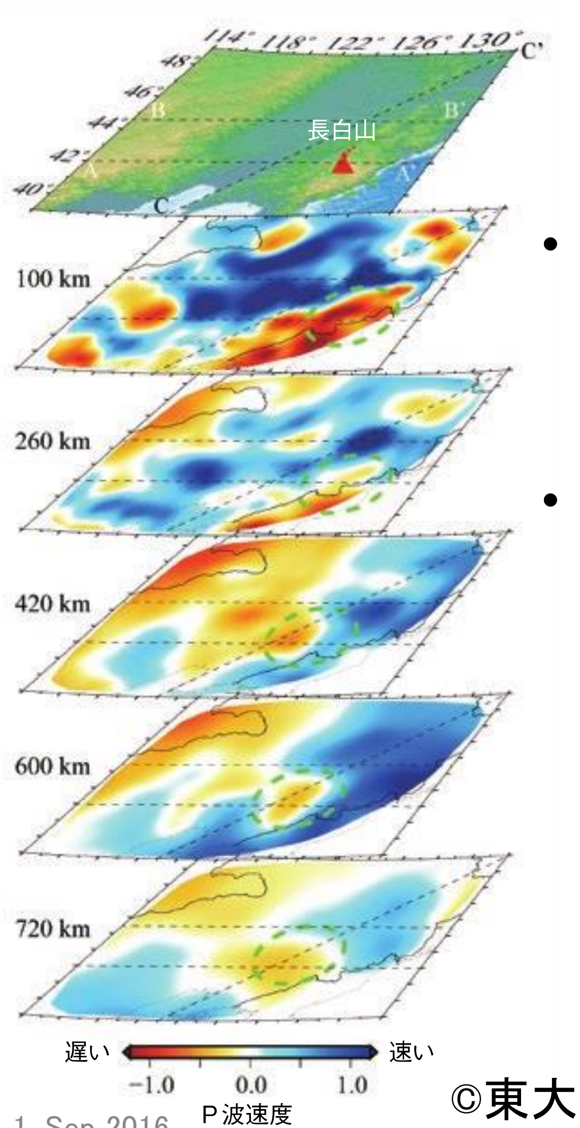
watermelon

- how to know the best time to eat a watermelon ?
  - inside can not be checked before cutting
- "empirical rule"
  - to check the best time, knock a watermelon
    - high frequency "KIN-KIN" ; too young
    - "BAN-BAN" ; best time !
    - low frequency "BON-BON" ; too old
  - need many years to get this ability



- one could see the interior with specific sound from object.
  - **asteroseismology !!**

Seismology, Helioseismology



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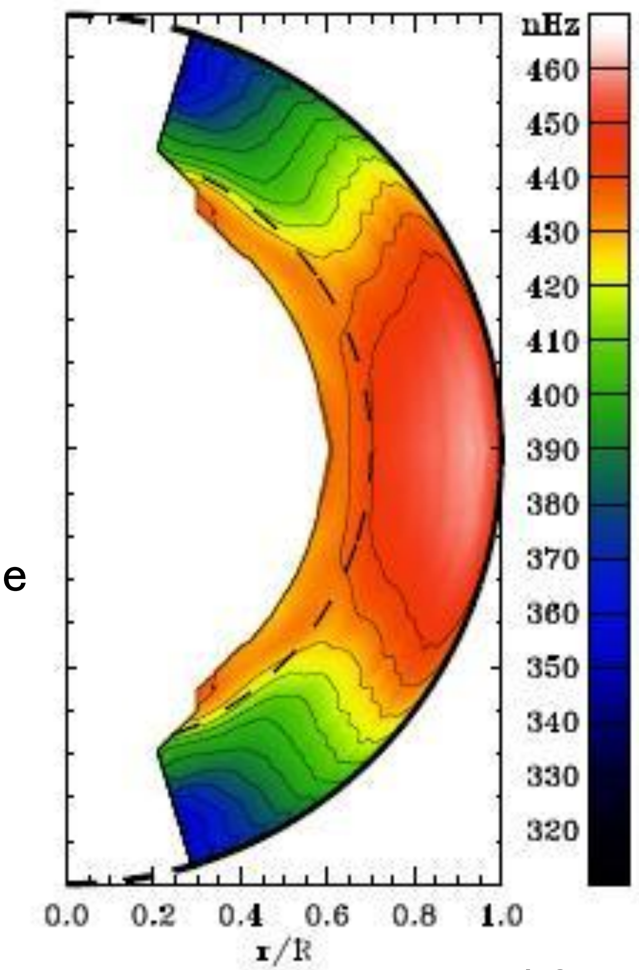
- Seismic waves tell us information inside the Earth (seismology)
- The interior of the Sun can be probed through the wave pattern on the surface (helioseismology)



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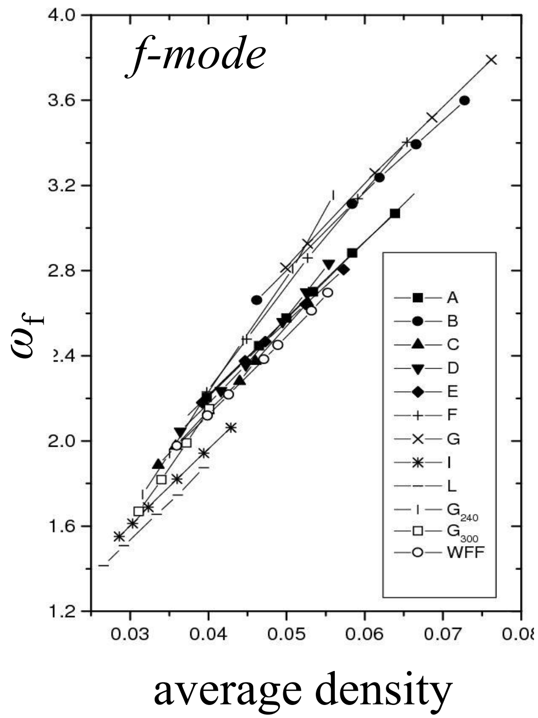
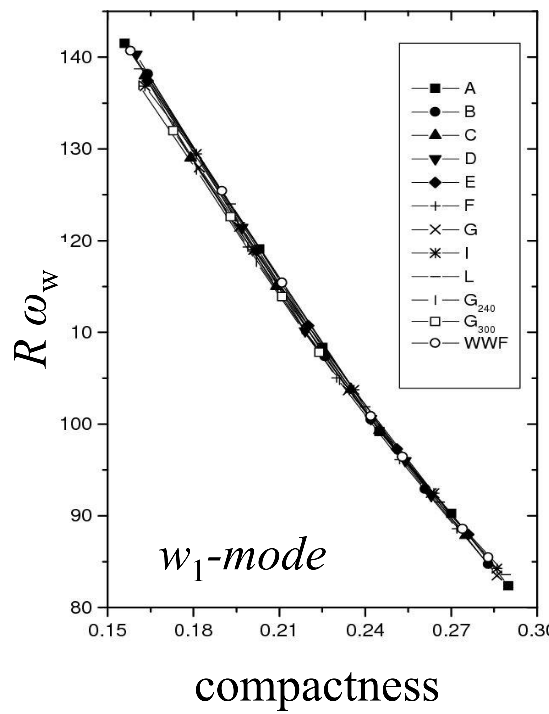
Asteroseismology in Neutron Stars

- via the observations of GW frequencies, one might be able to see the properties of NSs





Andersson & Kokkotas (1998)

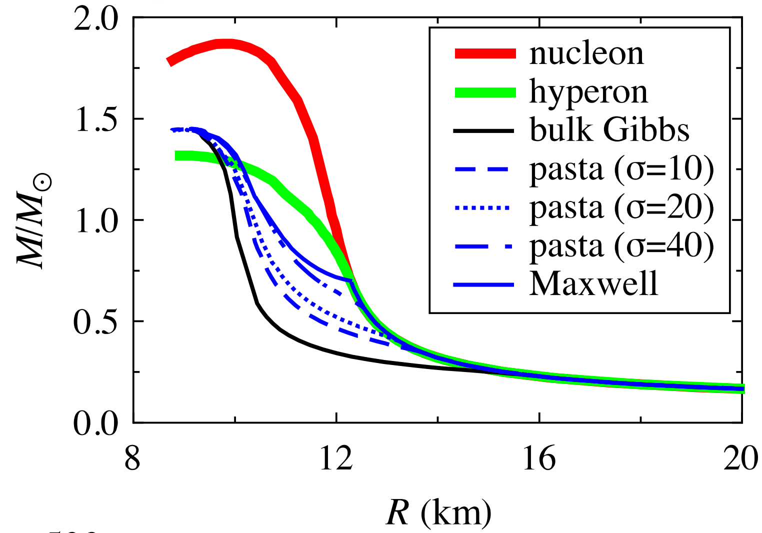
$$w_f \approx 0.78 + 1.64\left[\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10\text{km}}{R}\right)^3\right]^{1/2}$$

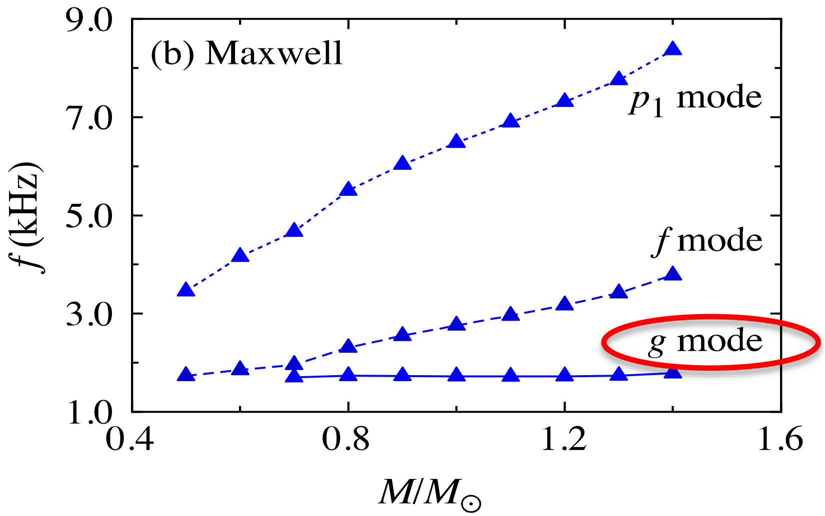
$$w_w \approx \left(\frac{10\text{km}}{R}\right)\left[20.92 - 9.14\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10\text{km}}{R}\right)\right]$$

determination of (M, R)

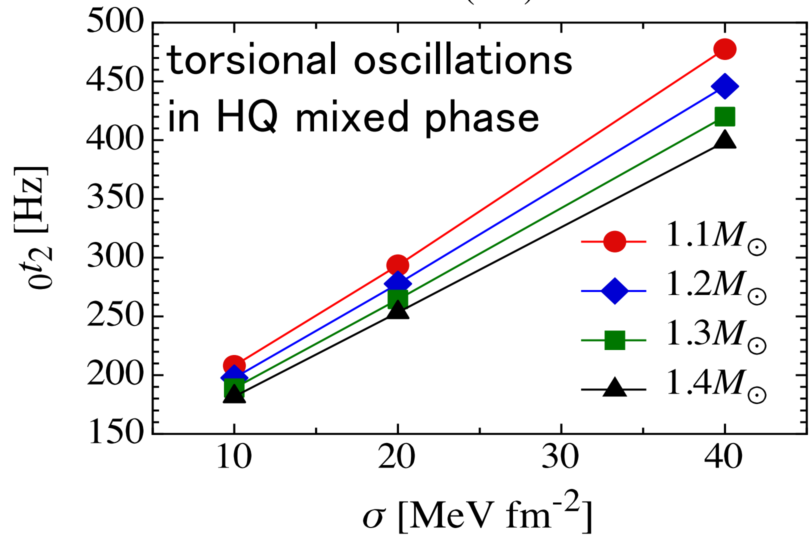
asteroseismology 2

- imprints of inside NSs
  - In deeper region of NS, quark matter could appear.
  - one might be probe the existence of density discontinuity
  - one might see the surface tension in hadron–quark mixed phase



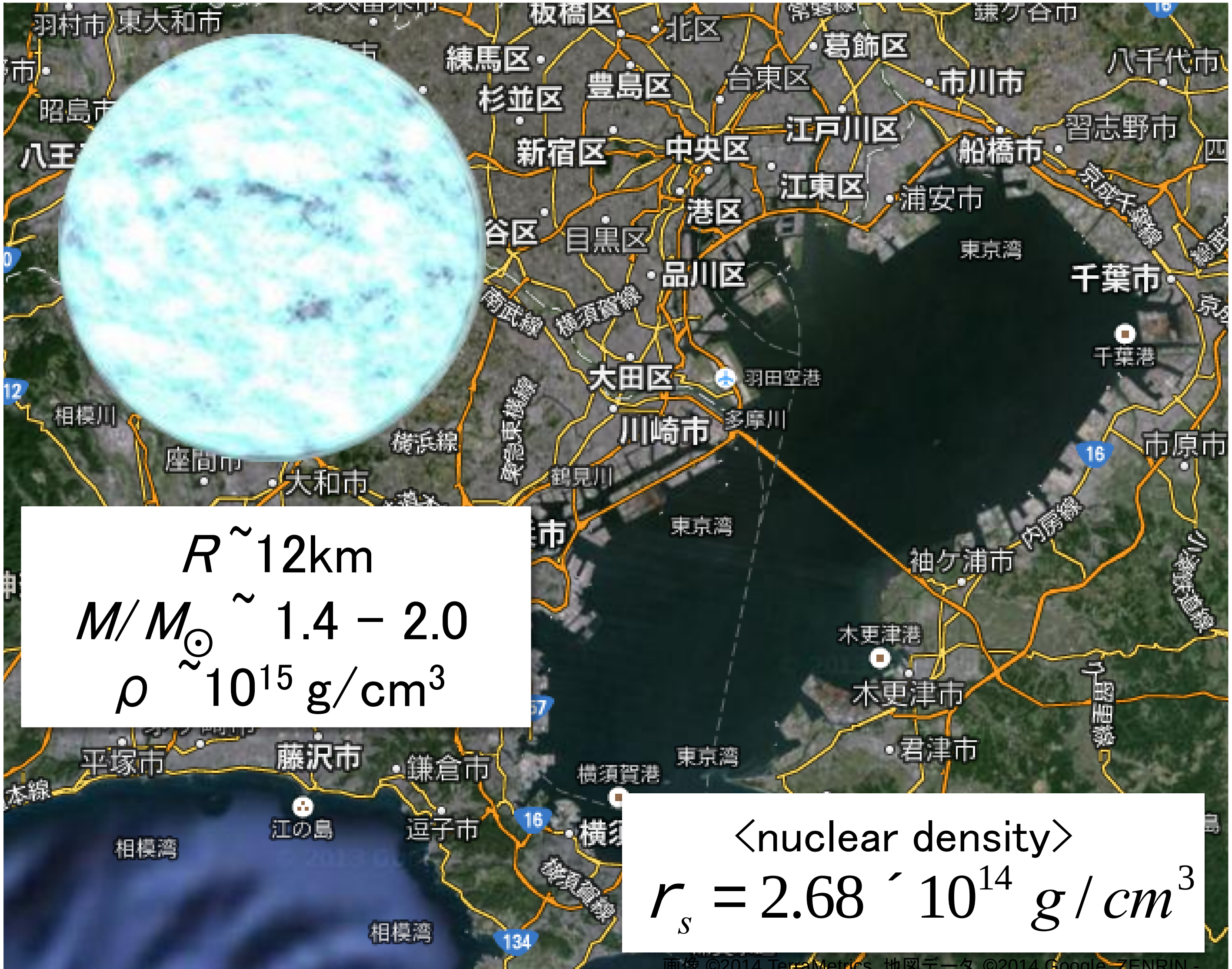


HS, Yasutake, Maruyama & Tatsumi 2011



HS, Maruyama & Tatsumi 2013

Properties of Neutron Stars



R ~12km

$M/M_{\odot}$ ~ 1.4 – 2.0

ρ ~10^{15} g/cm^3

<nuclear density>

$$\rho_s = 2.68 \times 10^{14}\ g/cm^3$$

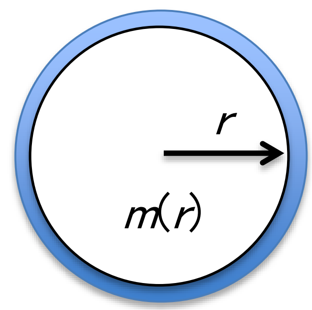
How to construct NSs

- Tolman–Oppenheimer–Volkoff (TOV) equation gives density profile of the spherically symmetric equilibrium of cold NSs.

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2} \left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 + \frac{P}{\rho c^2}\right)\left(1 - \frac{2Gm}{c^2 r}\right)^{-1}$$

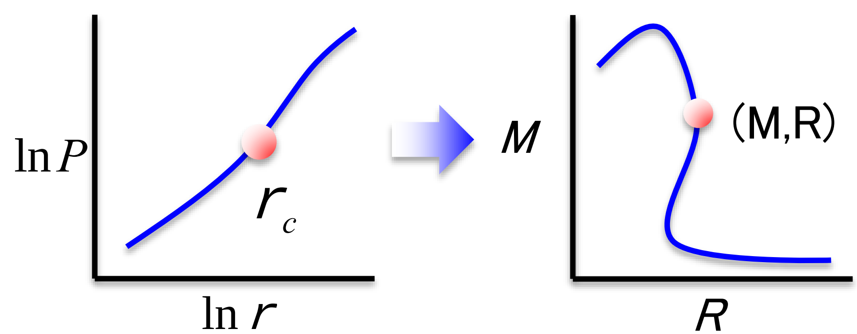
relativistic correction

$$\frac{dm}{dr} = 4\pi r^2 \rho$$



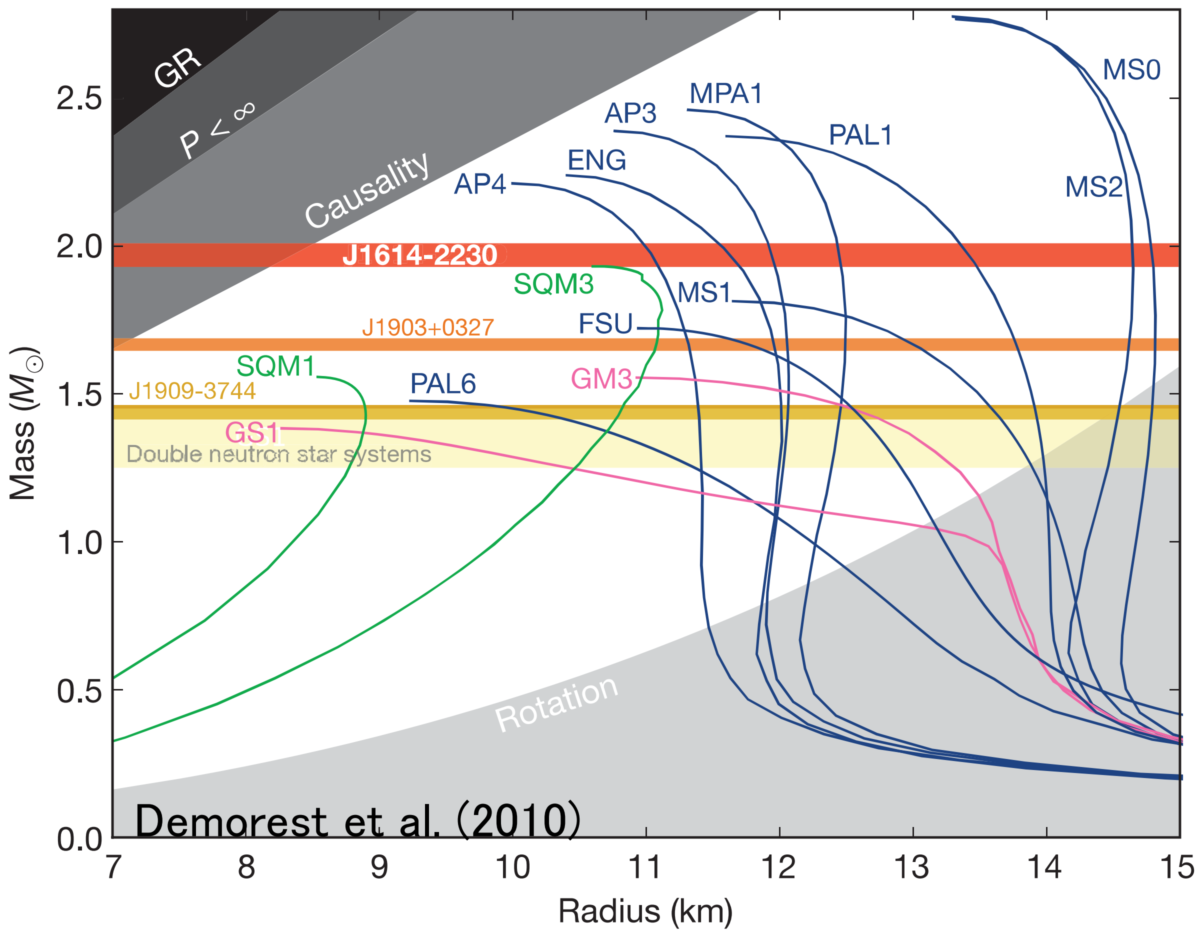
$P = P(\rho)$ equation of state (EOS)

$$R = r \quad @P = 0$$
$$M = m(R)$$



EOS is essential to construct the stellar model, but still uncertain.

too many EOSs suggested…



NS observations can make a constraint on EOS!!

nuclear saturation

- radius of an atomic nucleus with mass number A

$$R \simeq r_0 A^{1/3},\ r_0 = 1.2 \times 10^{-13} cm$$

- binding energy

$$E(A) \simeq 8A\ MeV$$

which are independent of atomic nuclei.

- density of atomic nuclei

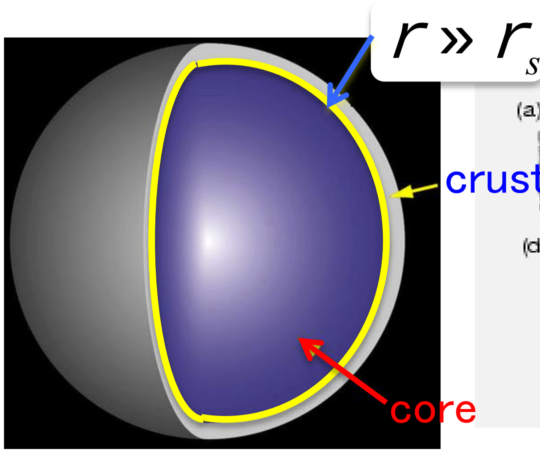
$$\rho \approx \frac{M}{R^3} = \frac{mA}{r_0^3 A} = \frac{m}{r_0^3} \cong \rho_s$$

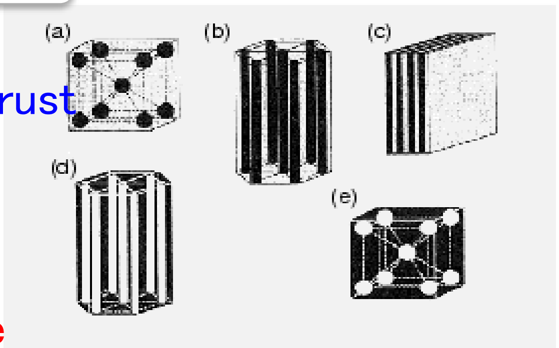
saturation density = 2.68×10^{14} g/cm^3

neutron stars

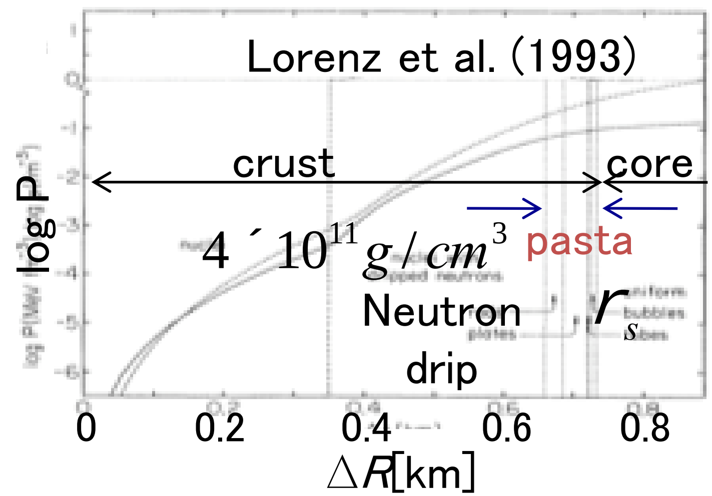
- Structure of NS
  - solid layer (crust)
  - nonuniform structure (pasta)
  - fluid core (uniform matter)
- Crust thickness ≲ 1km
- Determination of EOS for high density (core) region could be quite difficult on Earth
- Constraint on EOS via observations of neutron stars
  - stellar mass and radius
  - stellar oscillations (& emitted GWs)

    “(GW) asteroseismology”

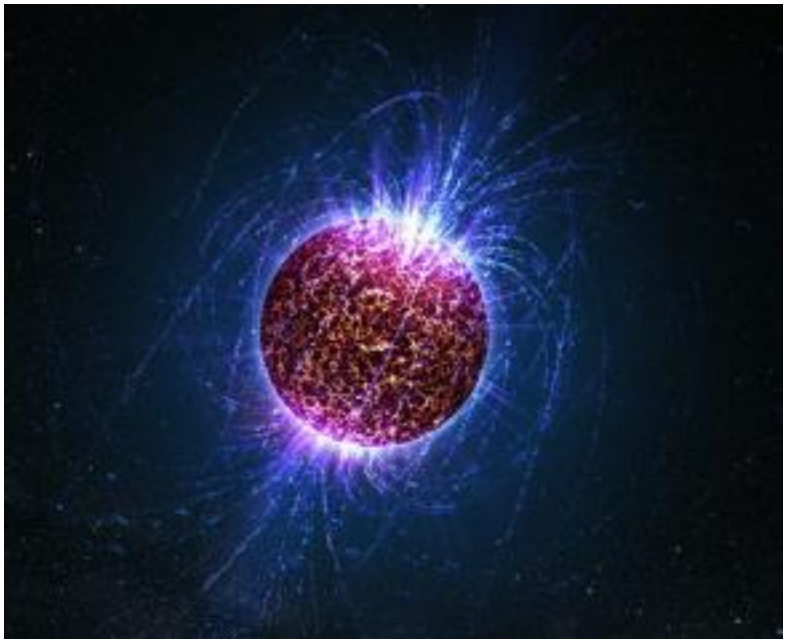




Oyamatsu (1993)



(P)NS – EOS



(1) TOV equation
(2) equation of state
– model
– nuclear interaction
– composition

- physics in NS crust
- low-mass NSs

physics in NS (core)

↓

high density region

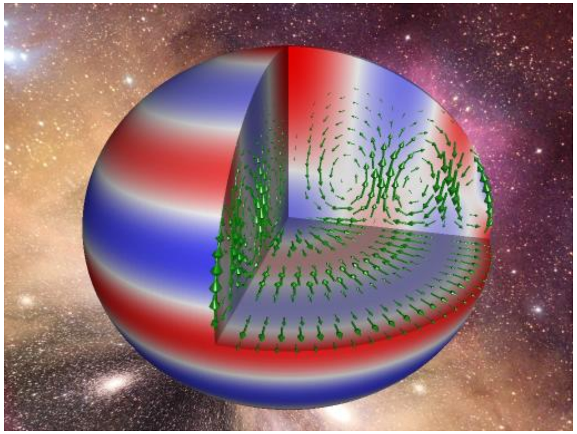
constraints from the terrestrial nuclear experiments

〜〜

properties around the saturation density

The most promising strategy for constraining the physics of neutron stars involves observing their "ringing" (oscillation modes)

- **f-mode:** scales with average density
- **p-modes:** probes the sound speed through out the star
- **g-modes:** sensitive to thermal/composition gradients
- **w-modes:** oscillations of spacetime itself.
- **s-modes:** Shear waves in the crust
- **Alfvèn modes:** due to magnetic field
- **i-modes:** inertial modes associated with rotation (r-mode)

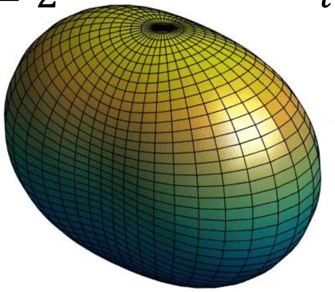


Typically **SMALL AMPLITUDE** oscillations –> weak emission of GWs

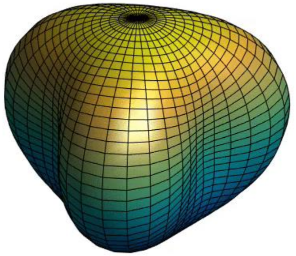
UNLESS

they become **unstable due to rotation** (r-mode & f-mode)

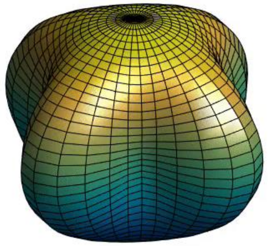
$l = 2, m = 2$



$l = 3, m = 3$



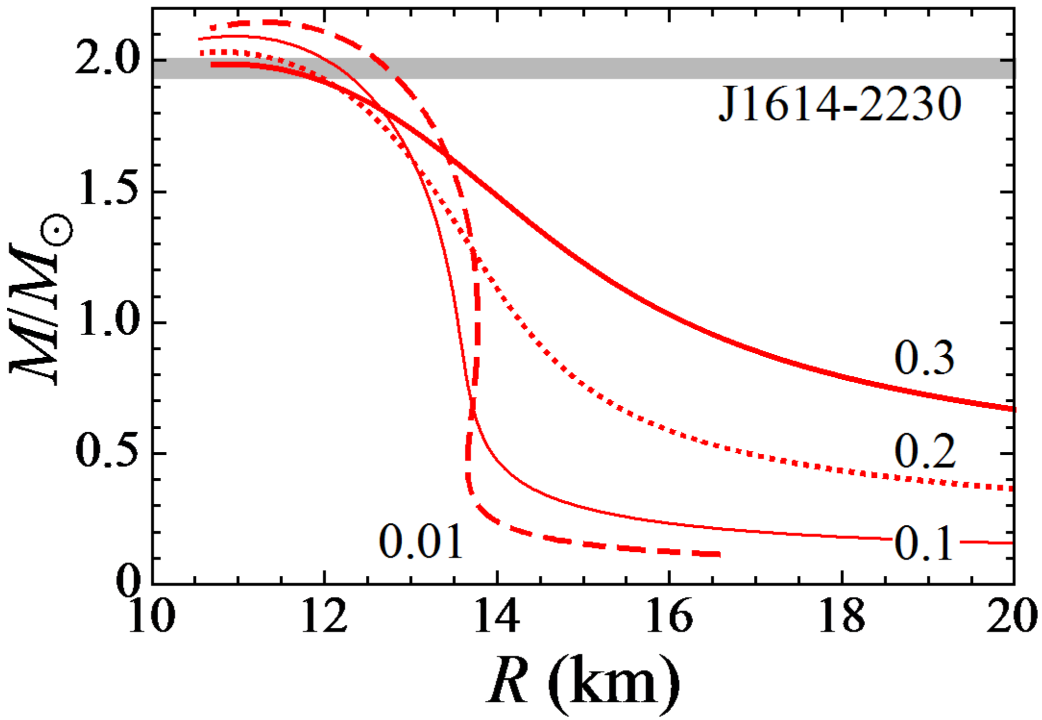
$l = 4, m = 4$



slide by Kokkotas

Protoneutron stars (PNSs)

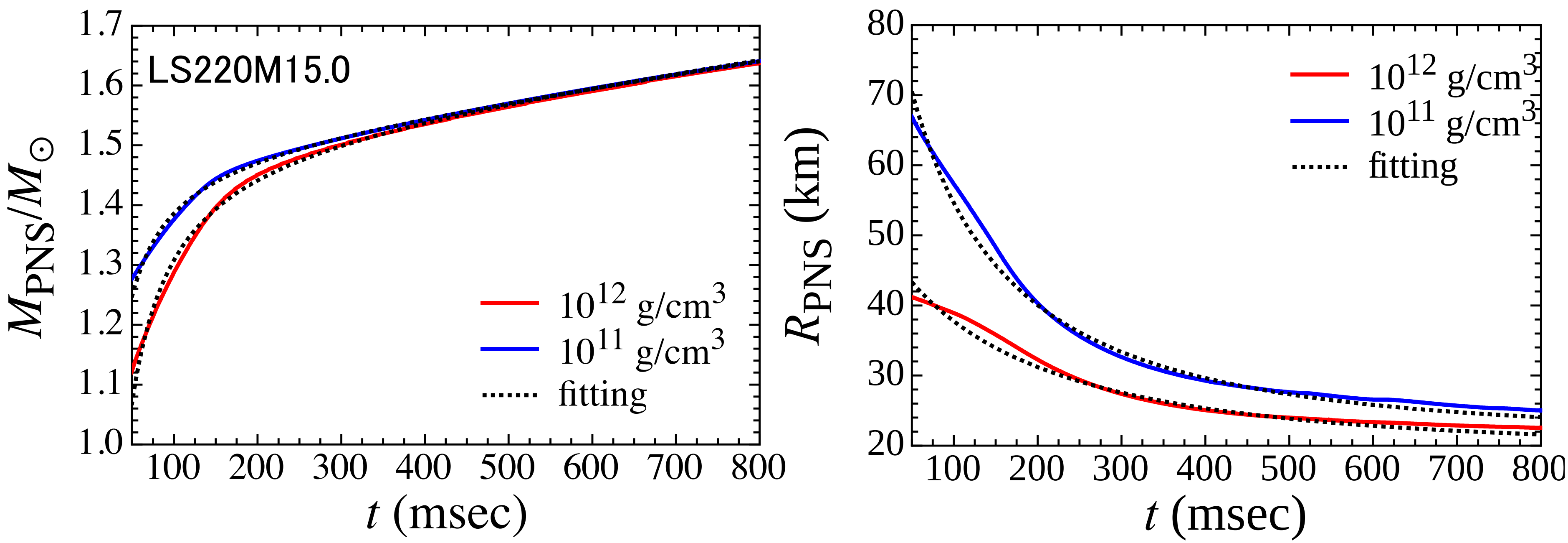
- Unlike cold neutron stars, to construct the PNS models, one has to prepare the profiles of Y_e and s.
  - for example, with LS220 and s =1.5 (k_B/baryon), but Y_e =0.01, 0.1, 0.2, and 0.3



strategy

- calculate the 1D simulation of core-collapse supernova (by Takiwaki)
  - time evolutions of radius and mass of PNS are determined
  - radius and mass of PNS are fitted by simple formula
- PNS models are constructed in such a way that the radius and mass of PNS are equivalent to the expectation from the fitting
  - with the assumption that the PNS is quasi-static at each time step
  - with the profiles of Y_e and s
- calculate the eigen-frequencies via the eigen-value problems on PNS models
  - dependence of the frequencies on the profiles of Y_e and s
  - dependence on the average density of PNS
  - dependence on the progenitor models
    - LS220 ($M_{pro}/M_\odot$ = 11.2, 15, 27, 40), Shen ($M_{pro}/M_\odot$ = 15)

evolutions of mass and radius



- fitted with

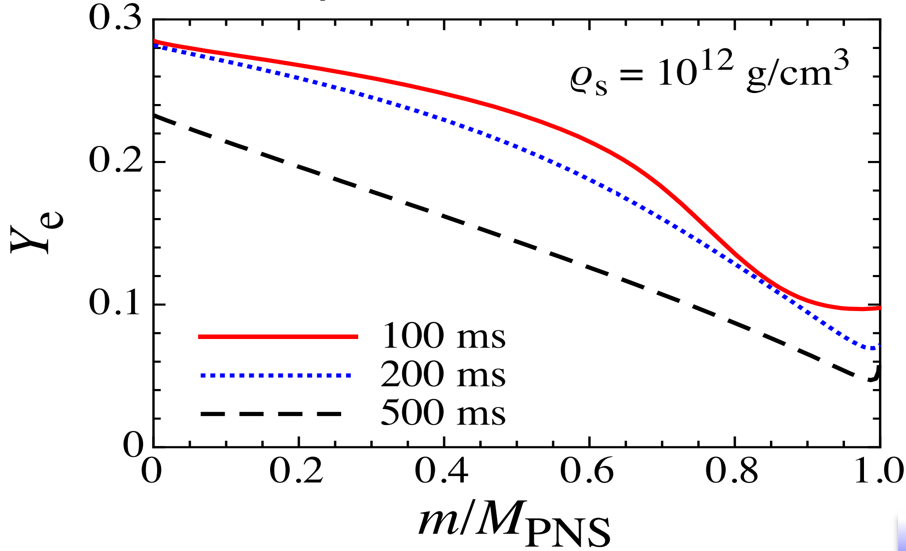
$$R_{\rm PNS}(t) = \frac{R_{\rm i}}{1 + \left[1 - \exp(-\frac{t}{\tau})\right]\left[\frac{R_{\rm i}}{R_{\rm f}} - 1\right]}$$

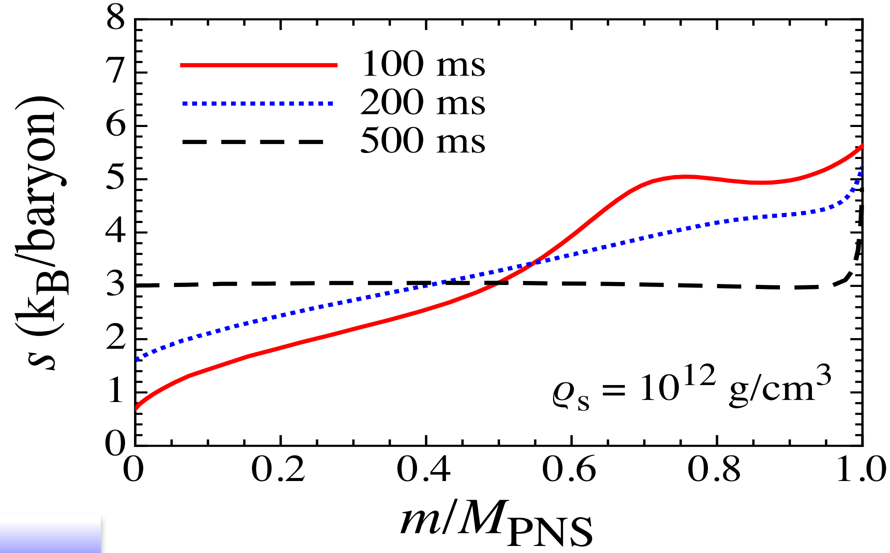
$$\frac{M_{\rm PNS}(t)}{M_\odot} = \frac{c_0}{t} + c_1 t + c_2$$

Scheck et al. (2006)

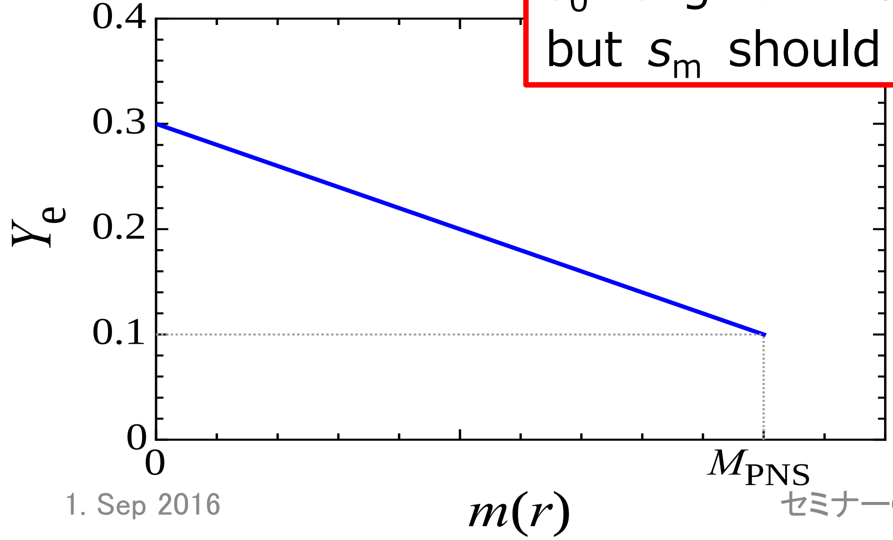
Y_e and s profiles

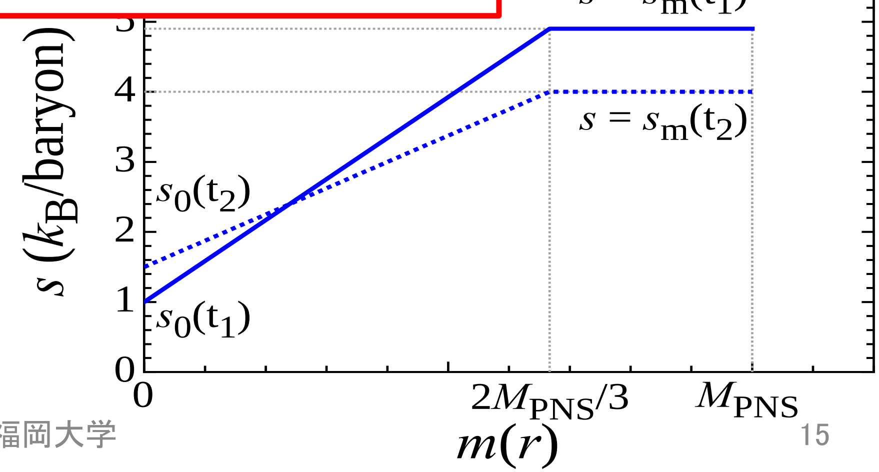
- the snap shot at t=100, 200, and 500ms after bounce





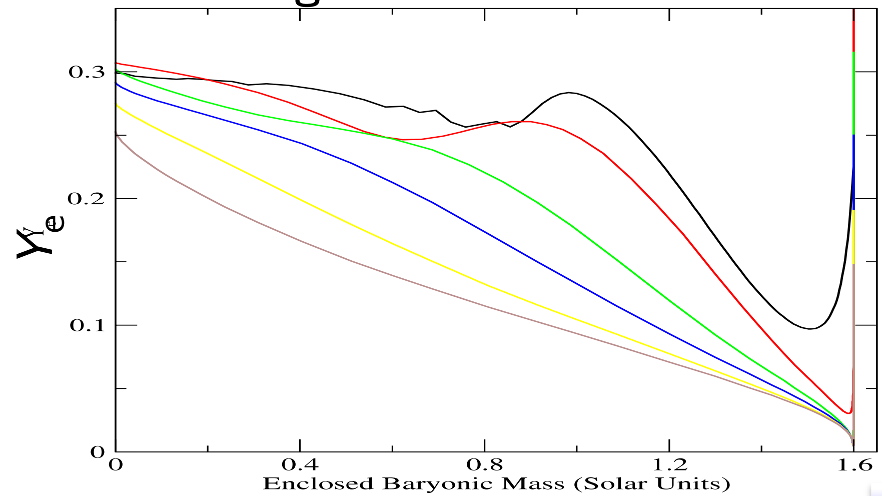
s_0 is given from numerical results,
but s_m should be determined to fit

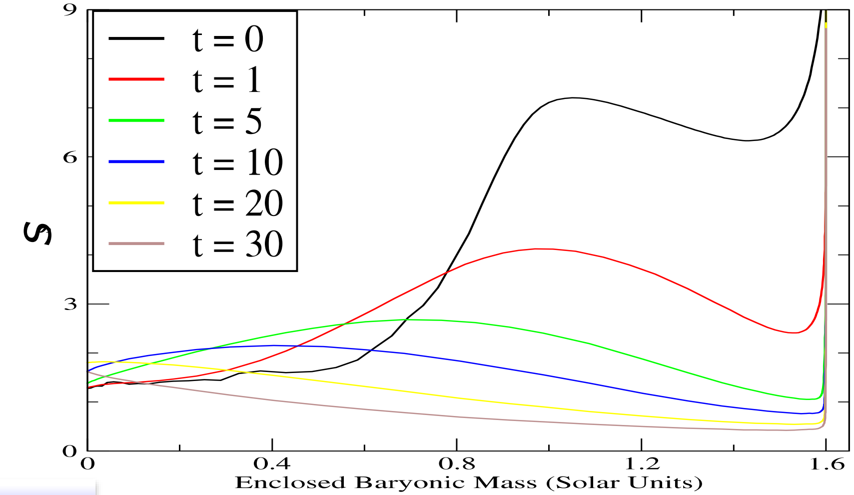


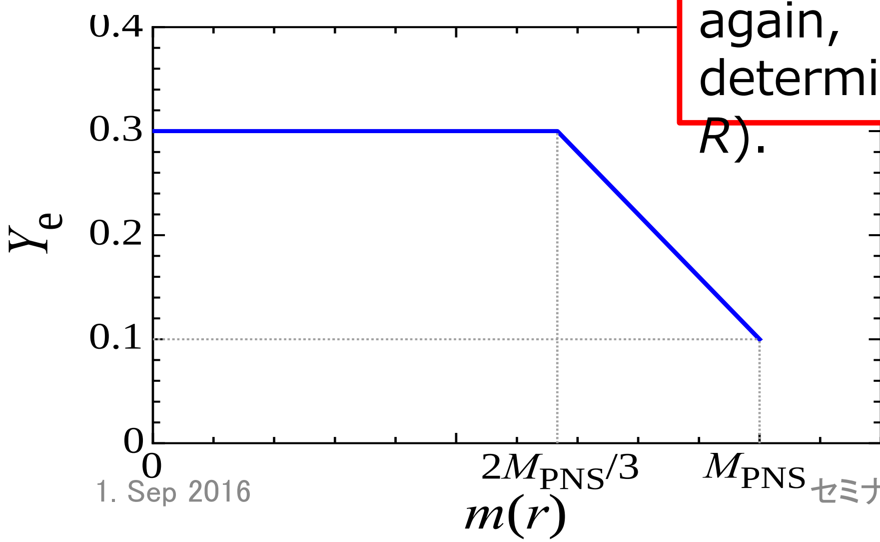


comparison with other results

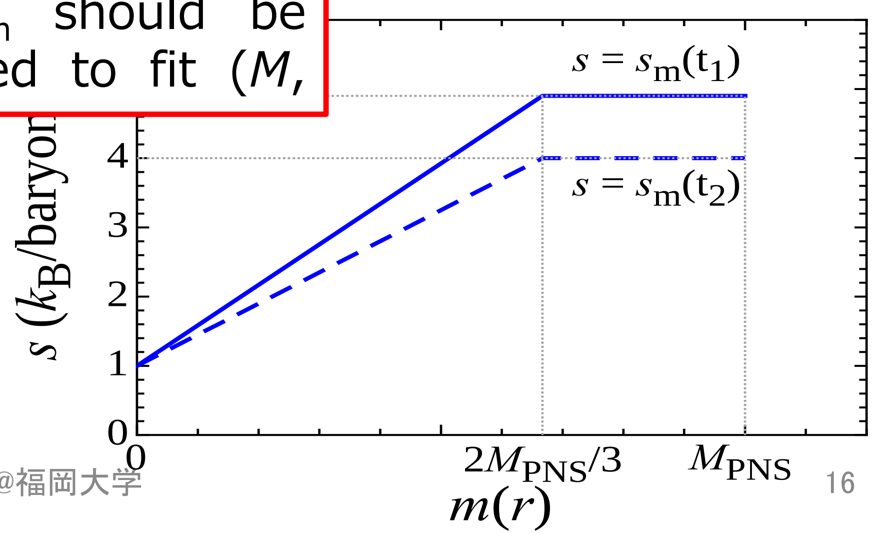
- results by Roberts (2012), where he has done the 1D simulations for long-term.





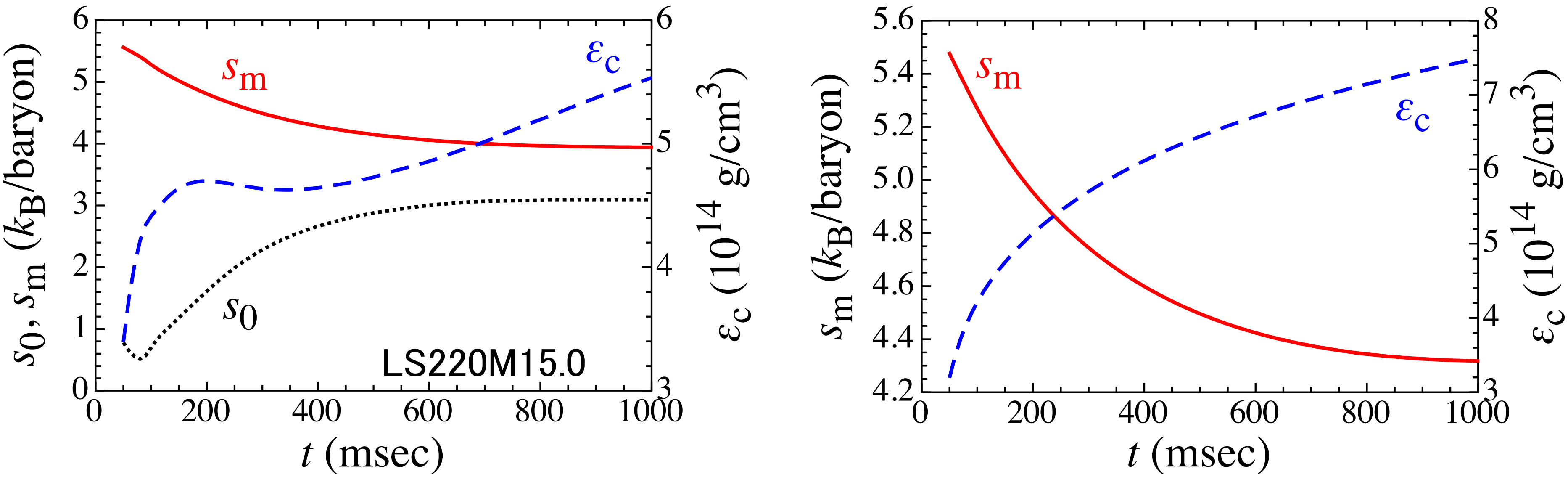


again, s_m should be determined to fit (M, R).



PNS models

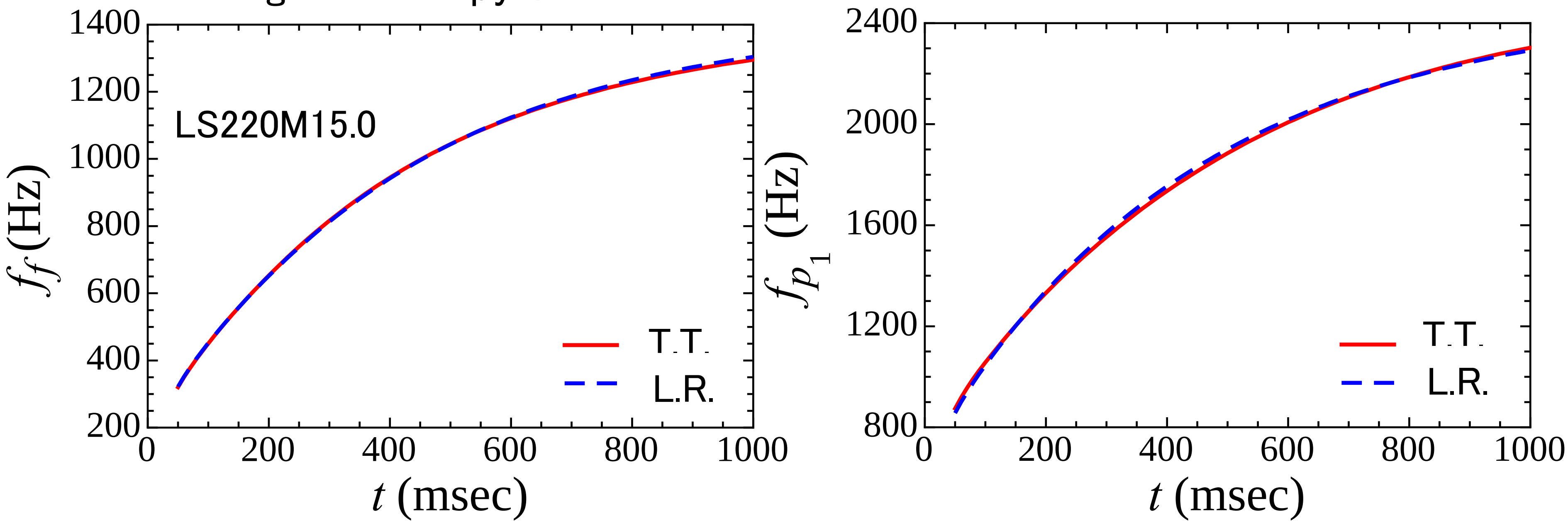
- adopting two different profiles of Y_e and s inside the PNS, we construct the PNS models.
  - unknown parameter: ε_c & s_m
  - to reproduce the PNS models with given (M, R), ε_c and s_m are fixed.



- evolutions of ε_c and s_m depend strongly on the profiles of Y_e and s.

oscillations in PNS

- with relativistic Cowling approximation
- omitting the entropy variation



- frequencies depend on mass and radius of PNS, but weakly depend on (Y_e, s) profiles.
- in the early stage, the typical frequencies of *f*-mode is ~ a few hundred hertz, which is good for gravitational wave detectors.

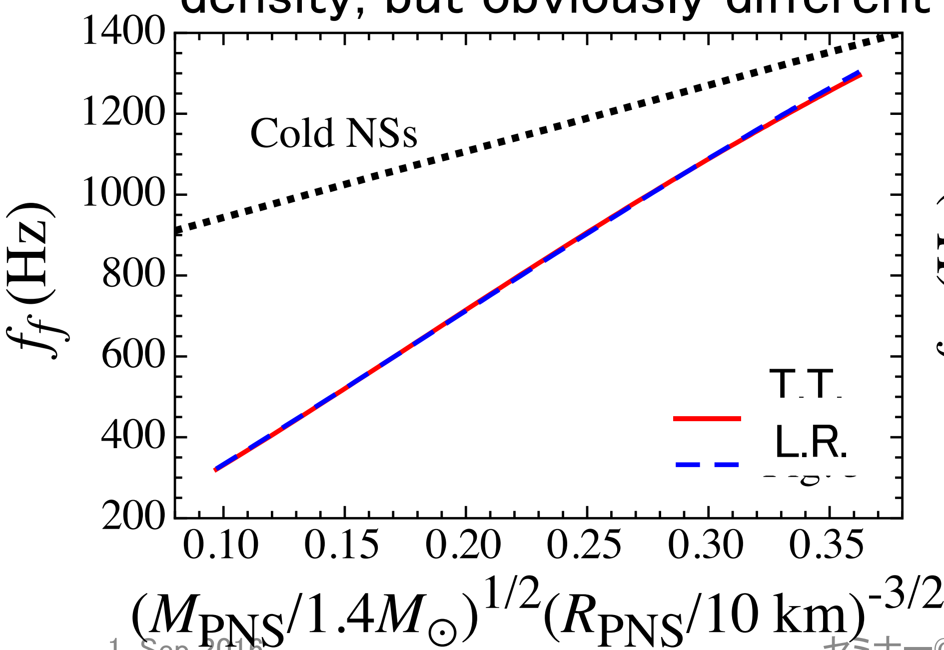
characterized by average density

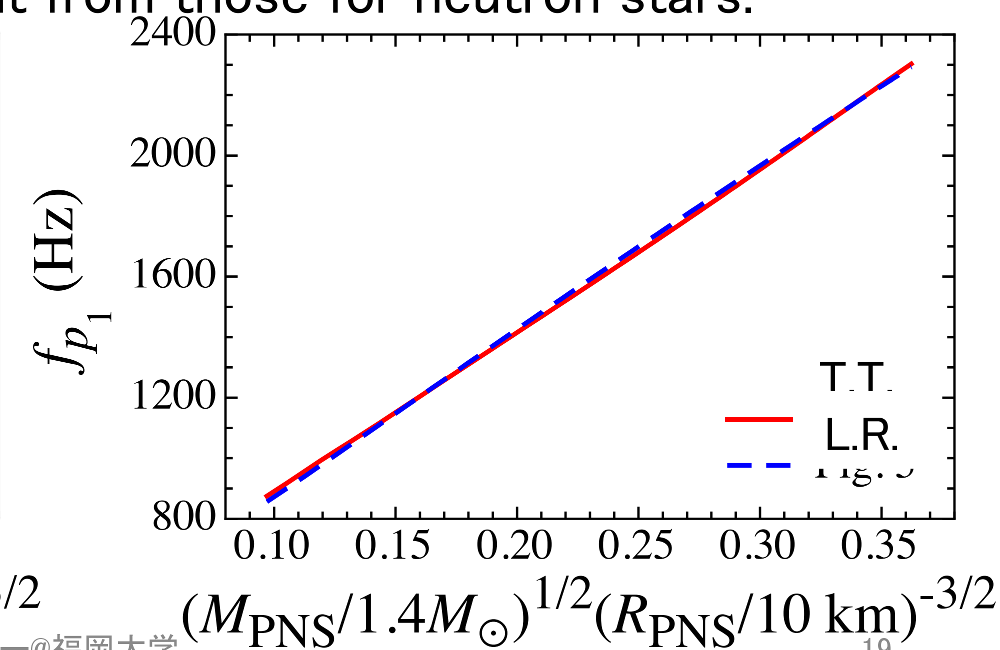
- frequencies of f-mode for cold neutron stars:

$$f_f^{(\mathrm{NS})}\ (\mathrm{kHz}) \approx 0.78 + 1.635 \left(\frac{M}{1.4M_\odot}\right)^{1/2} \left(\frac{R}{10\ \mathrm{km}}\right)^{-3/2}$$

Andersson & Kokkotas (1998)

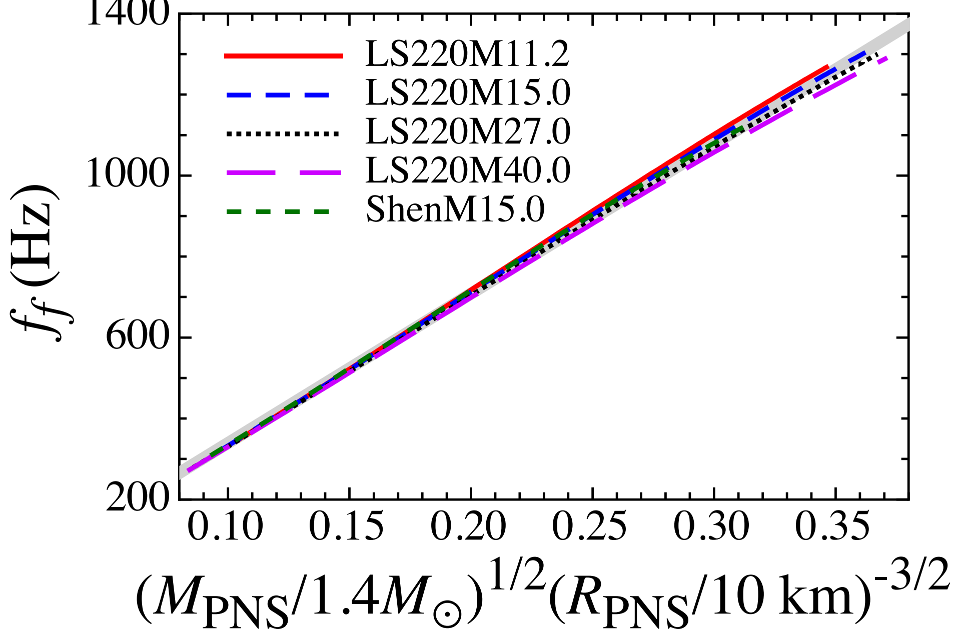
- Similarly, frequencies for PNS can be characterized by average density, but obviously different from those for neutron stars.

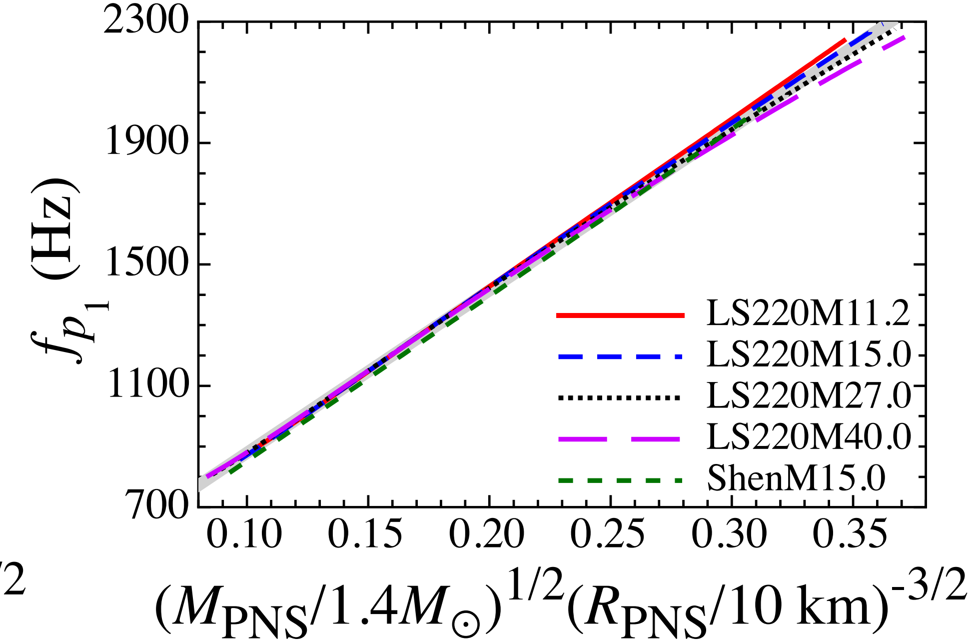




dependence on progenitor models

- results for LS220 with $M_{pro}/M_{\odot}$=11.2, 15, 27, and 40, for Shen with $M_{pro}/M_{\odot}$=15



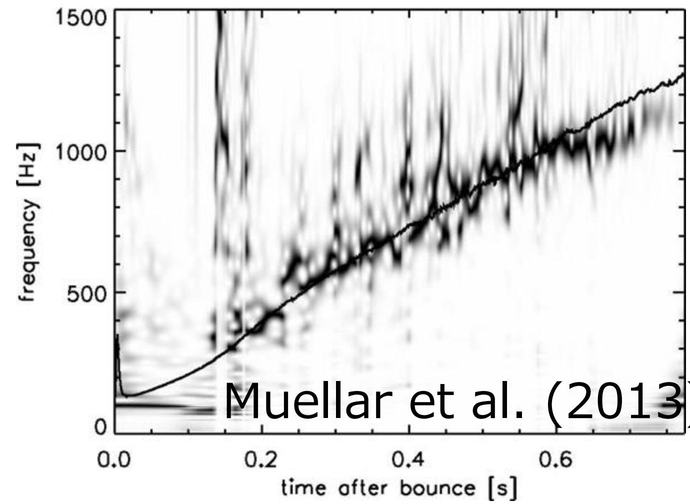


- progenitor model dependence is quite weak.

$$f_i^{(\mathrm{PNS})}(\mathrm{Hz}) \approx c_i^0 + c_i^1 \left(\frac{M_{\mathrm{PNS}}}{1.4M_{\odot}}\right)^{1/2} \left(\frac{R_{\mathrm{PNS}}}{10\text{ km}}\right)^{-3/2}$$

comparison with g-modes

- as characteristic GWs from core-collapse supernova, the excitation of g-modes around PNS has been reported (Muellar et al. (2013); Cerda-Duran et al. (2013))
  - due to the convection and the standing accretion-shock instability.

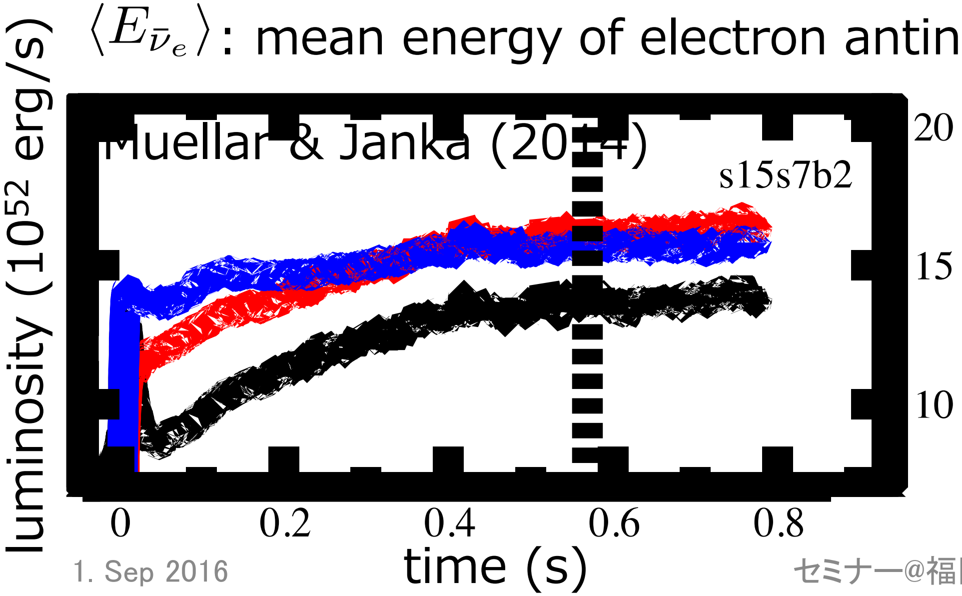


$$f_g \approx \frac{1}{2\pi}\frac{GM_{\rm PNS}}{R_{\rm PNS}^2}\left(\frac{1.1 m_n}{\langle E_{\bar{\nu}_e}\rangle}\right)^{1/2}\left(1-\frac{GM_{\rm PNS}}{c^2 R_{\rm PNS}}\right)^2$$

Muellar et al. (2013)

m_n: neutron mass

$\langle E_{\bar{\nu}_e}\rangle$: mean energy of electron antineutrinos



black: electron neutrinos

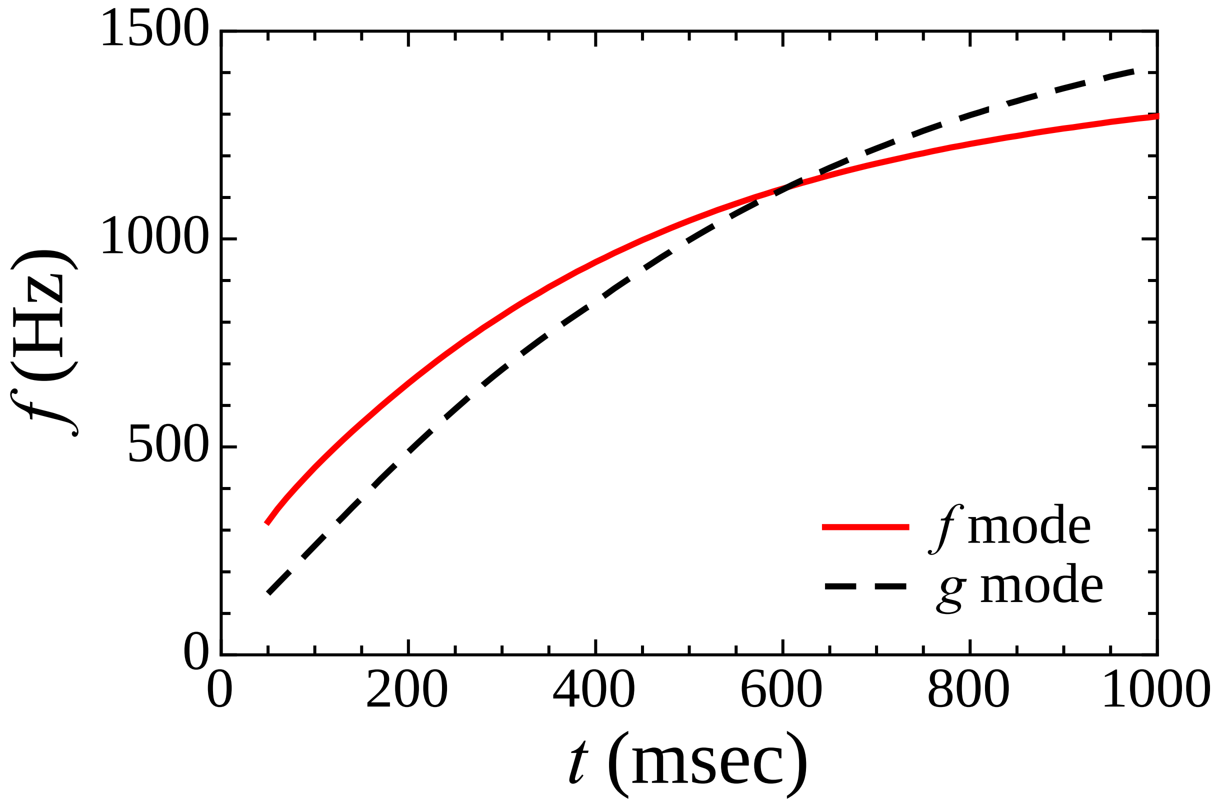
red: electron antineutrinos

blue: μ/τ neutrinos

$$\langle E_{\bar{\nu}_e}\rangle = \begin{cases} 3t/400 + 13 & (0 \le t \le 400 \text{ msec}) \\ 16 & (400 \text{ msec} \le t) \end{cases}$$

comparison with g-modes

- careful observing the gravitational wave spectra after core-collapse supernova, one might see the different sequences in spectra
  - which tells us the radius and mass of PNS



conclusion

- We examine the frequencies of gravitational waves radiating from PNS after bounce.
- The PNS models are constructed in such a way that the mass and radius obtained from 1D simulation are reconstructed.
  - two different profiles of Y_e and s are considered
- fr
  p

$$f_i^{(\mathrm{PNS})}(\mathrm{Hz}) \approx c_i^0 + c_i^1 \left(\frac{M_{\mathrm{PNS}}}{1.4M_\odot}\right)^{1/2} \left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-3/2}$$

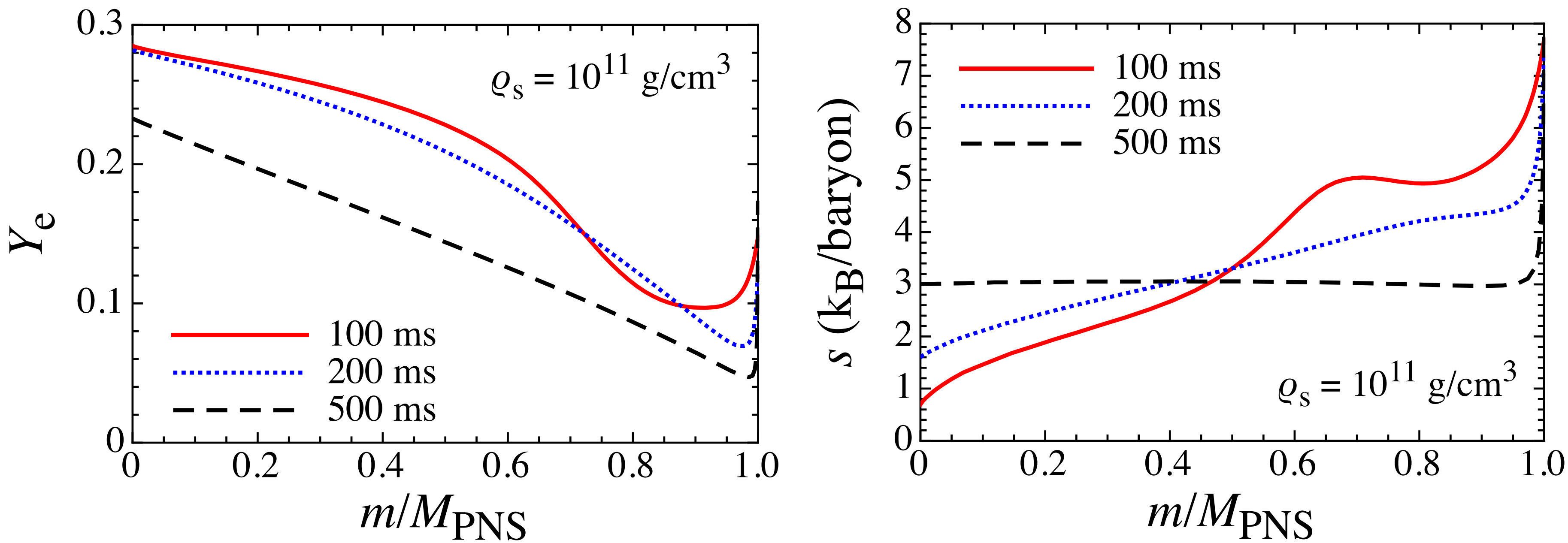
- p

$$f_g \approx \frac{1}{2\pi} \frac{GM_{\mathrm{PNS}}}{R_{\mathrm{PNS}}^2} \left(\frac{1.1 m_n}{\langle E_{\bar{\nu}_e} \rangle}\right)^{1/2} \left(1 - \frac{GM_{\mathrm{PNS}}}{c^2 R_{\mathrm{PNS}}}\right)^2$$

as a function of average density
  - different dependence for g-mode around PNS
- one might be possible to determine the mass and radius of PNS via careful observations of time evolution of gravitational wave spectra.

Y_e and s profiles

- the snap shot at t=100, 200, and 500ms after bounce



- distribution close to the surface at 10^{11} g/cm^3 abruptly increase.
- region for ~10^{11} – 10^{12} g/cm^3 could be relatively dilute